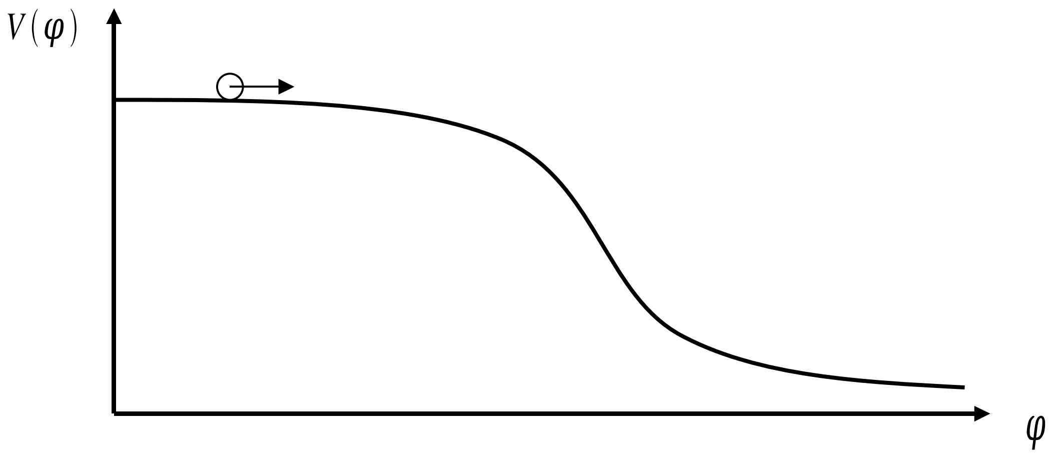
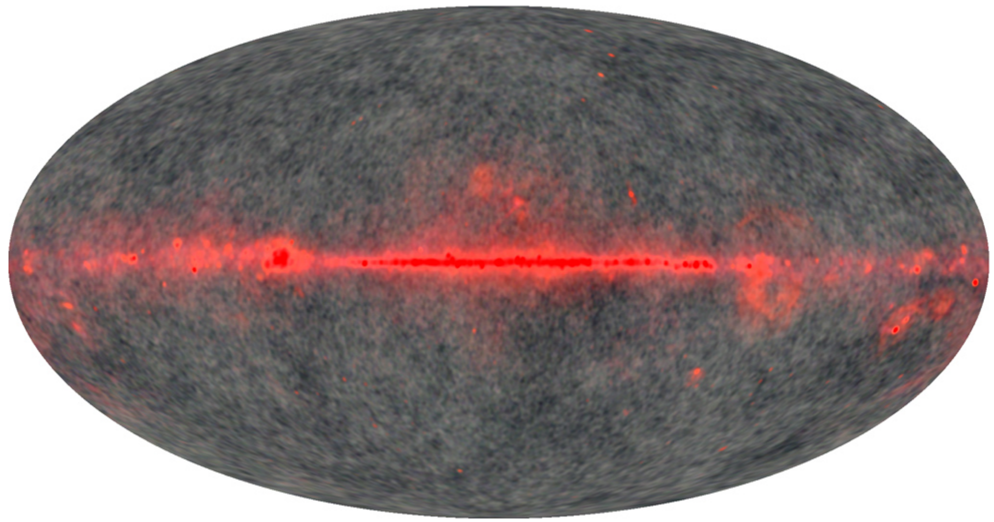
B-modes and inflation





WMAP science team

Samuel Leach (SISSA)

Tuesday 3 July 2007, APC Paris

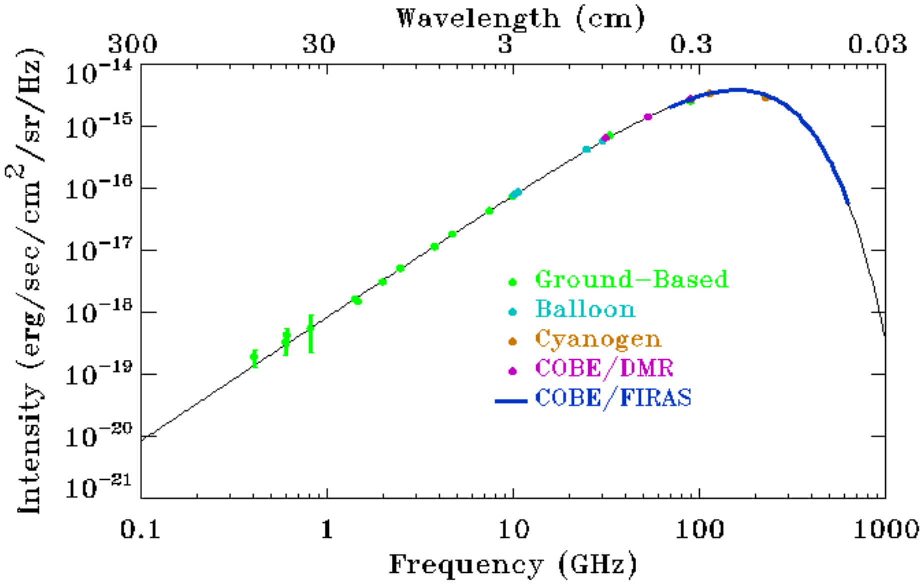
Outline

- Basics physics of inflation.

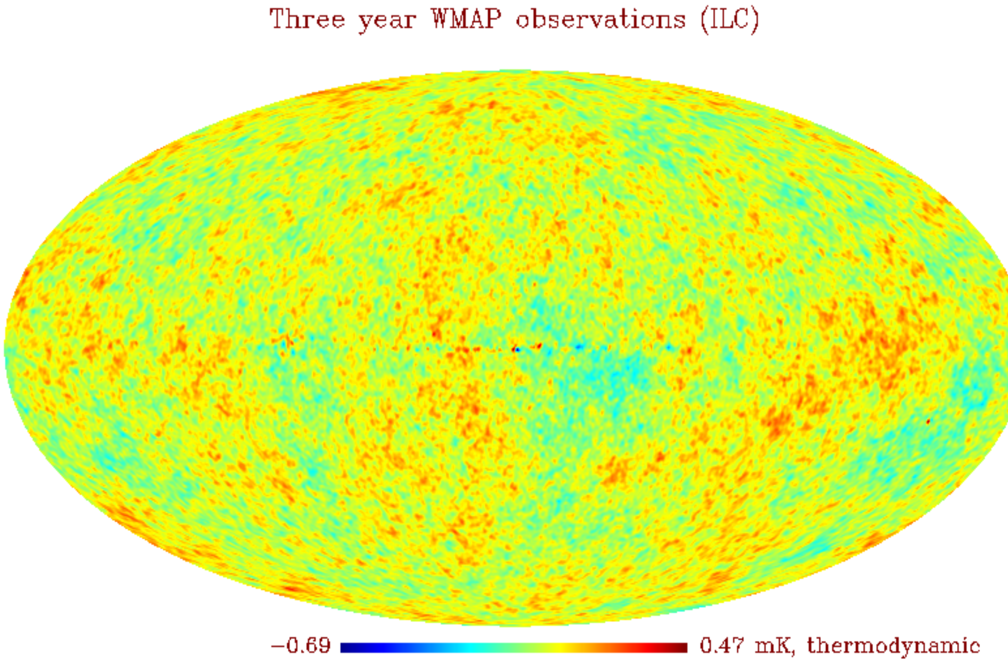
- Current observational picture.

- Data analysis strategies for constraining inflation.

Motivating inflation – CMB observations



Courtesy ARCADE



CMB spectrum:

Probe of:

-Rescattering.

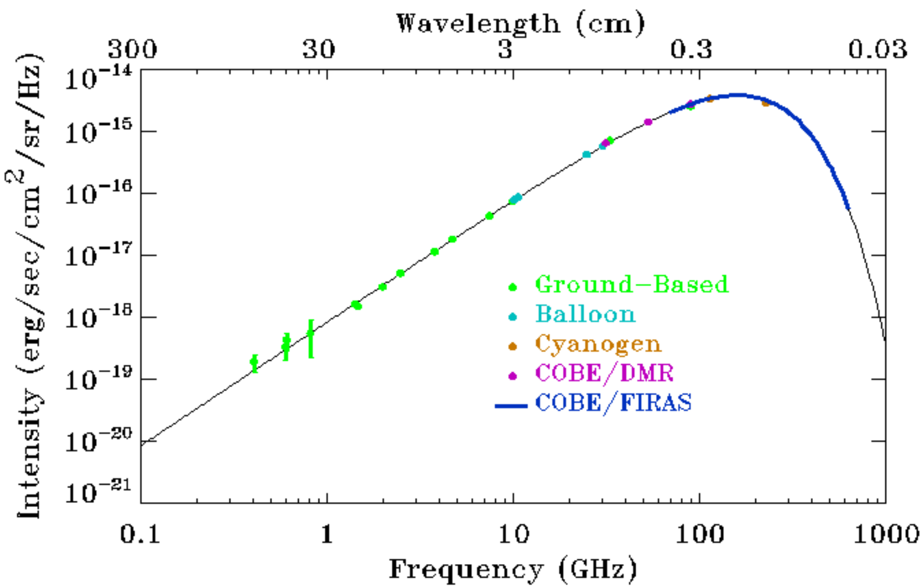
-Relic decays.

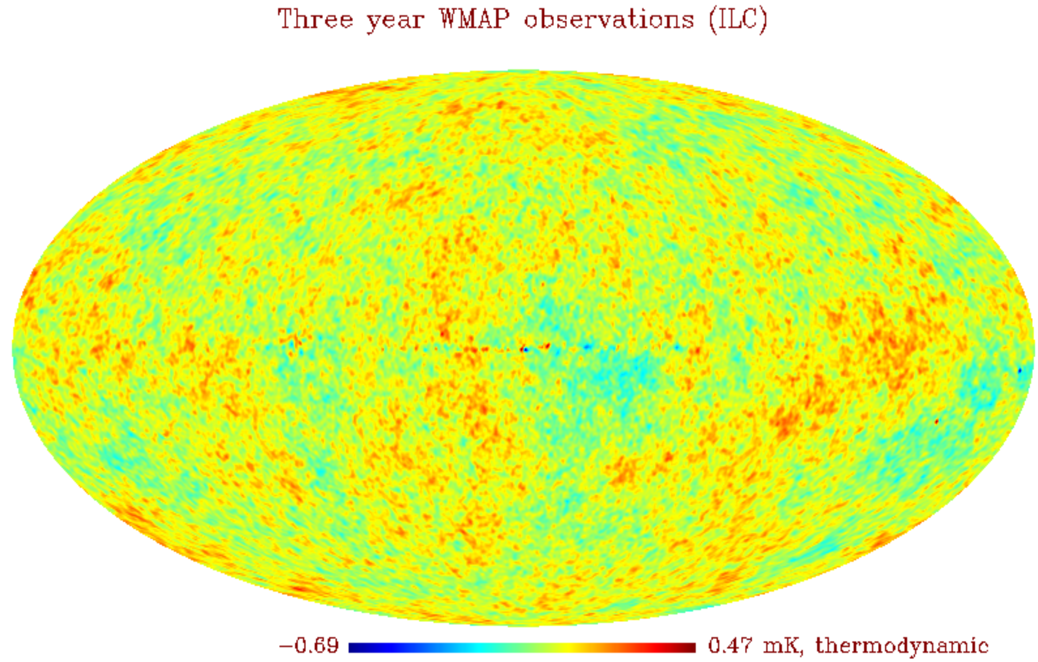
CMB anisotropies

Probe of:

- Global cosmological parameters.
- Initial seed fluctuations.
- Reionization history.

Motivating inflation – CMB observations





Courtesy ARCADE

The problem: Initial conditions:

- Separate regions of the universe share the same black body spectrum and fluctuations, even though they are apparently outside one another's sound horizon in the standard hot big bang.

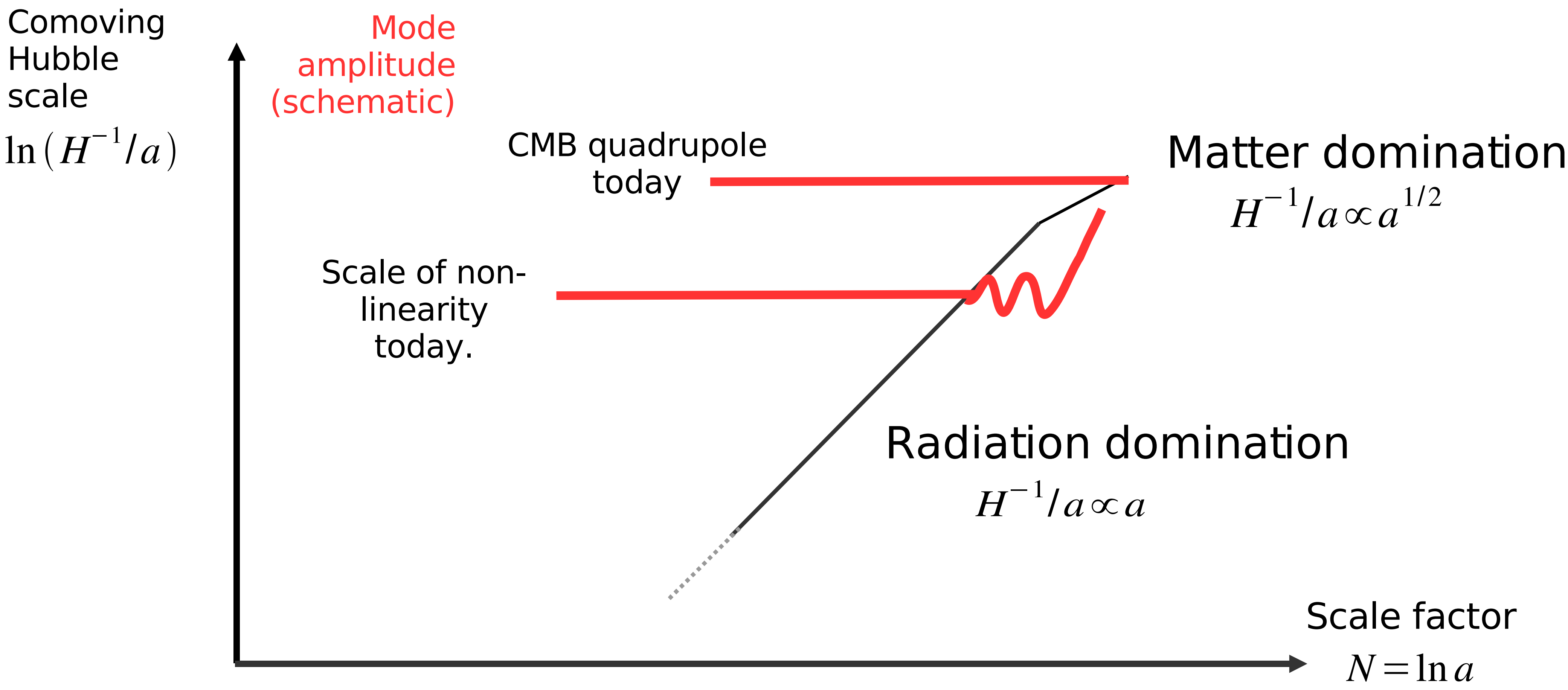
Inescapable conclusion:

- New physics beyond the Standard Big Bang required to explain this.

Perturbation problem

- Modes with a given comoving wavelength remain frozen while they are "outside the horizon". Smaller scales enter the horizon first and immediately begin to oscillate and collapse. Largest scales remain frozen.
- The Sachs-Wolfe effect and acoustic peaks are striking manifestations of this physics.

Problem: How to excite these modes when the horizon is so much smaller than the comoving wavelength?



Physics of inflation

- Based on the perturbation (and the horizon and flatness) problem, we will speculate that H^{-1}/a decreased in the early universe.

- Then explore the consequences and characteristics of an epoch where H^{-1}/a is decreasing.

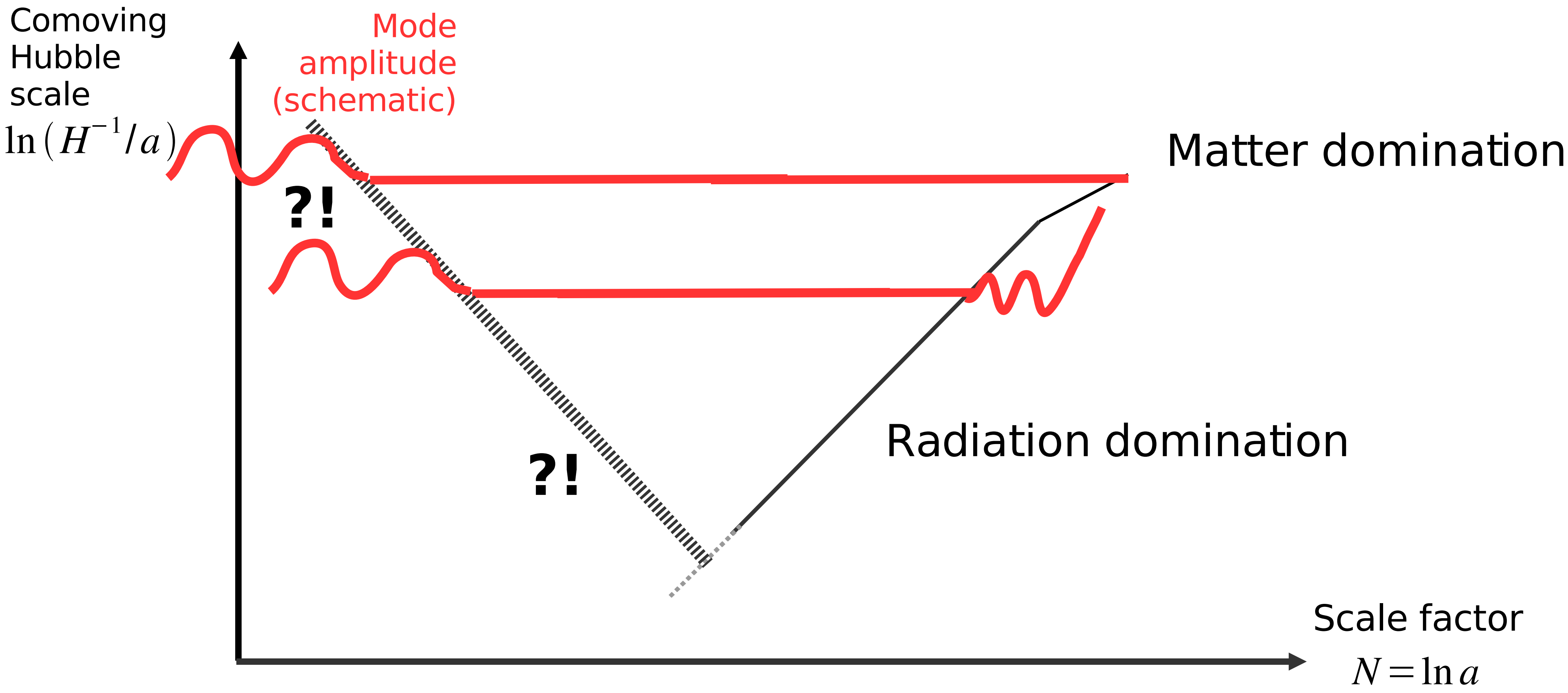
- Implement this within the big-bang model using a scalar field.

- Tensor perturbations from inflation.

Key speculation:

What if H^{-1}/a decreases in the early universe?

$$\frac{d}{dt}\left(H^{-1}/a\right)<0 \Leftrightarrow \ddot{a}>0$$ The idea of accelerated expansion follows directly.



Scalar field implementation of inflation

- Up until now, no mention of the matter responsible for inflation.
- Typically consider "rolling" scalar field, to avoid problems associated with scalar field trapped in a false vacuum:

$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$ Energy density.

$P = \frac{1}{2}\dot{\phi}^2 - V(\phi)$ Pressure.

Acceleration equation: $\ddot{a} = \frac{-4\pi G}{3} a(\rho + P)$

=> Require **negative pressure** for accelerated expansion.

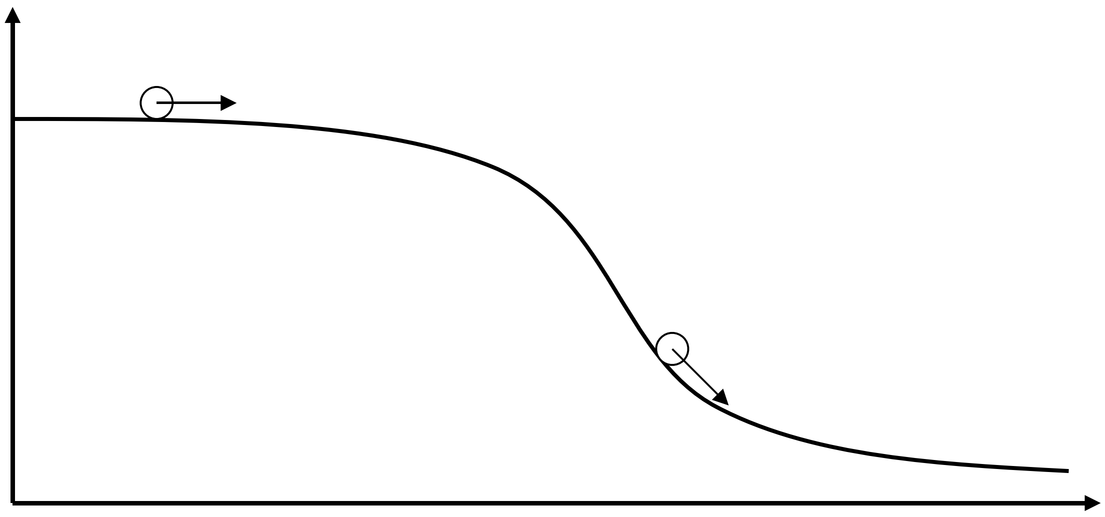
Desired accelerated expansion given by the potential energy dominated regime:

$$\dot{\phi}^2 < V(\phi) \Rightarrow \ddot{a} > 0$$

$$\dot{\phi}^2 \ll V(\phi) \Rightarrow a \propto e^{Ht}$$

Exponential expansion

Equations of motion - background:



Background equations:

$$H^2 = \frac{8\pi}{3 m_{Pl}^2}\left(\frac{\dot{\phi}}{2} + V(\phi)\right)$$

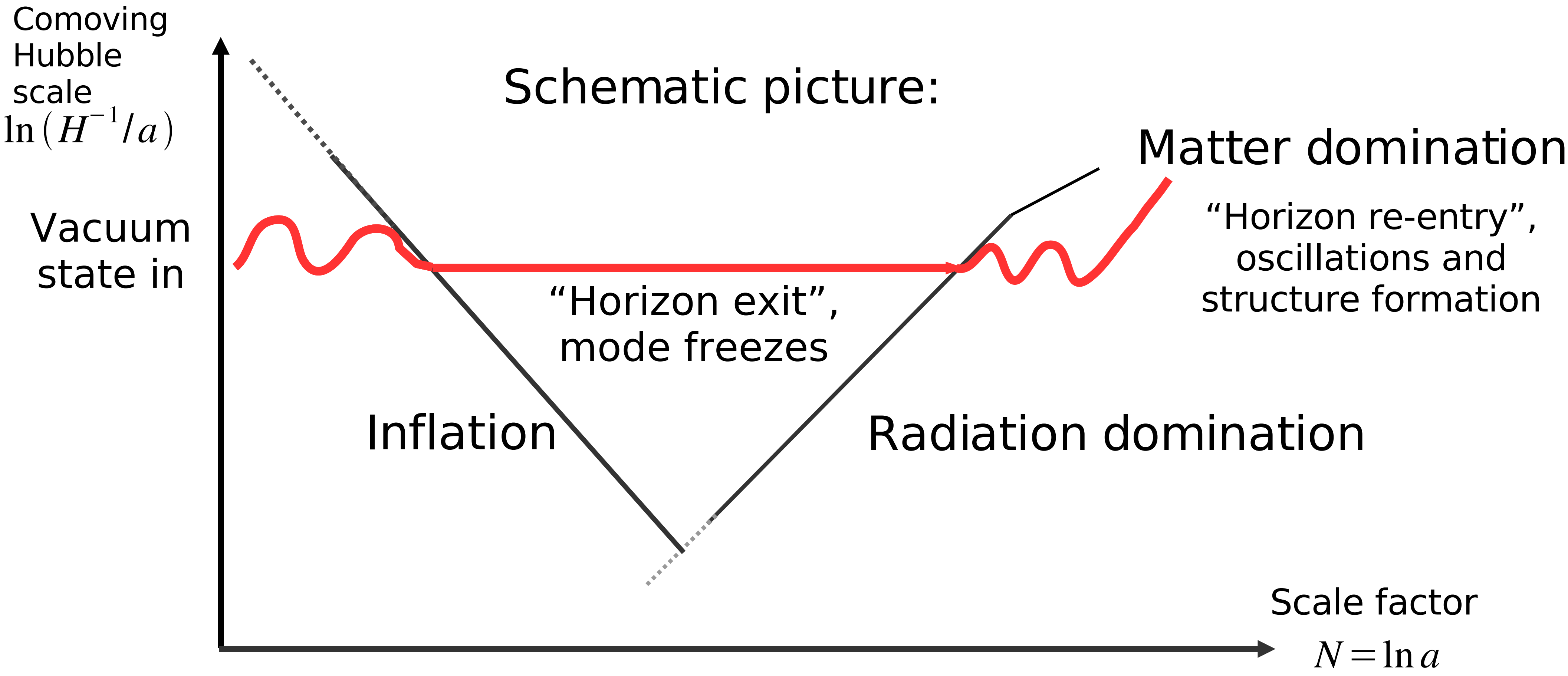
Friedmann equation: potential energy domination, H=constant.

$$\ddot{\phi} + 3\mathrm{H}\,\dot{\phi} + \frac{dV}{d\phi} = 0$$

Scalar field evolution: analogy with rolling on a potential, with friction from $3\mathrm{H}\,\dot{\phi}$ term.

Equations of motion - perturbations:

- Here we will cover the tensor perturbations.
- Analogous equations for the scalar sector, but requires more involved cosmological perturbation theory.



Equations of motion - perturbations

- Skip the derivation and quote the equation of motion for tensor perturbations:

$$\ddot{h} + 2\frac{\dot{a}}{a}\dot{h} + k^2 h = 0$$

Encodes the fact that gravitational waves decay once they enter the horizon, à la *CAMB*.

Sketch of the power spectrum calculation:

Usually rewritten and solved using a conformal time formalism:

$$v \equiv ah$$

$$v'' + \left(k^2 - a''/a\right)v = 0$$

During inflation:

$$v'' + \left(k^2 - (aH)^2\right)v = 0$$

“Horizon crossing” defined by the epoch $k = aH$

Equations of motion - perturbations

$$v''+\left(k^2-(aH)^2\right)v=0 \qquad v''+\left(k^2-2/\eta^2\right)v=0$$

Equation of motion has a similar form to the quantum harmonic oscillator. As such, v is now considered as a complex amplitude and is normalised via QFT:

$$v(k,\eta)=\frac{e^{-ik\eta}}{\sqrt{2\mathrm{k}}}\left[1-\frac{i}{k\eta}\right]$$

Oscillatory solution at early times, growing solution after horizon crossing.

Finally we want to calculate the variance in h, the original perturbation, at late times:

Result:

$$P_h(k)=\frac{2\mathrm{k}^3}{\pi^2}\left|\frac{v}{a}\right|^2=2\left(\frac{H_{inf}}{2\pi}\right)^2$$

Scale invariant tensor power spectrum
Amplitude is proportional to the energy scale of inflation.

For exact spectra, we can perform mode-by-mode integration.

Power spectrum from inflation – slow-roll approximation

Reminder: “slow-roll inflation” means $H, \dot{\phi}$ vary slowly

Scalars:

Power spectrum $$P(k) = \left(\frac{H}{2\pi}\right)^2 \left(\frac{H}{\dot{\phi}}\right)^2_{k=aH}$$

Spectral index $$n_S - 1 \equiv \frac{d\ln P(k)}{d\ln k} = -2\epsilon - \frac{d\ln\epsilon}{d\ln a}$$ ← “slow-roll parameters”

Tensors:

Power spectrum $$P_T(k) = 2\left(\frac{H}{2\pi}\right)^2_{k=aH}$$

Spectral index $$n_T \equiv \frac{d\ln P_T(k)}{d\ln k} = -2\frac{d\ln H}{d\ln a}$$

Key result:

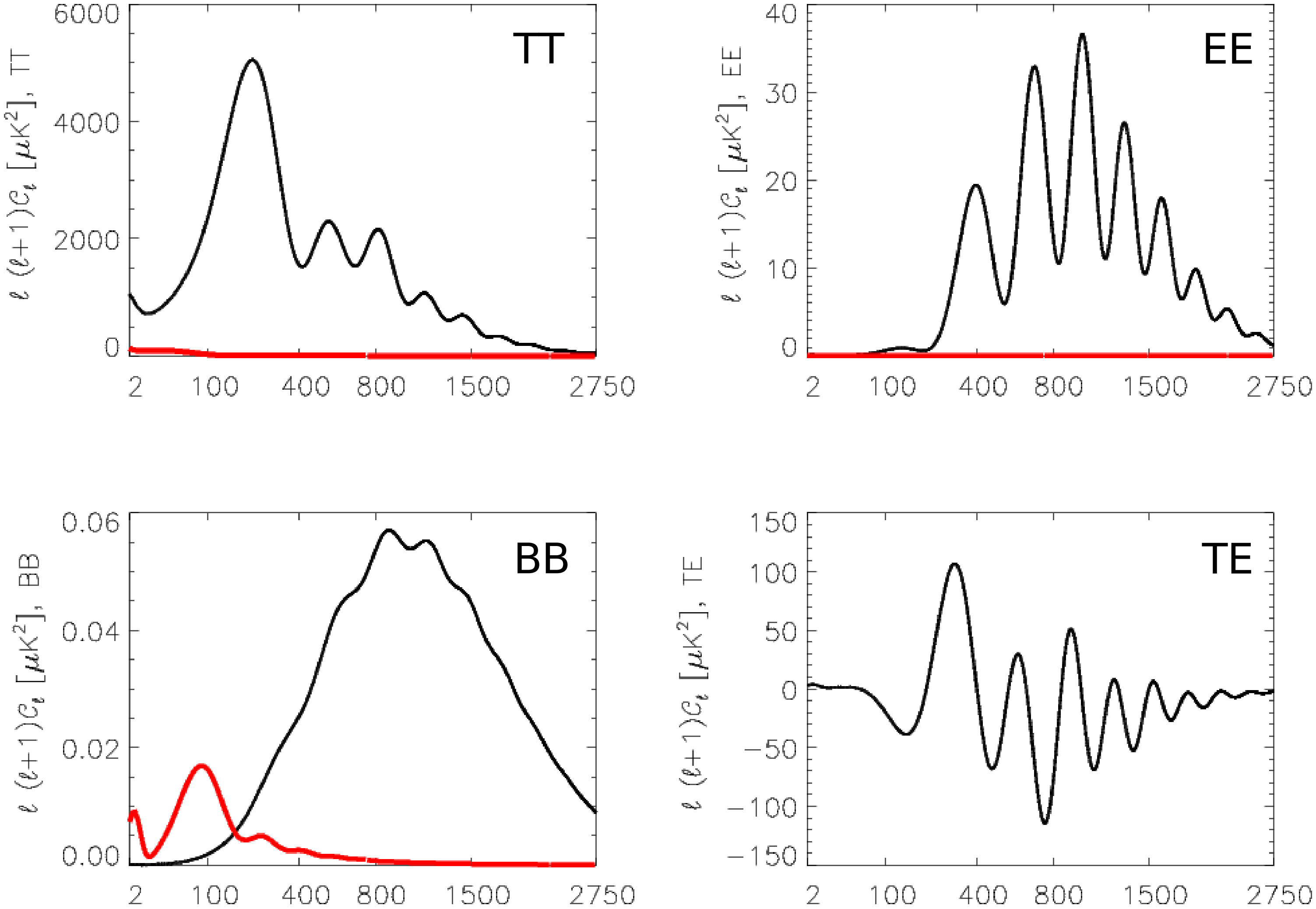
Shape of primordial power spectrum probes the dynamics of the early universe.

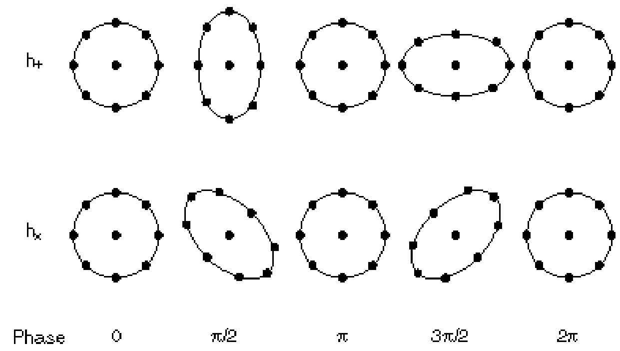
Inflation and CMB polarization

Inflation at high energies produces a tensor spectrum, leading to a B-mode peak on large scales:

Standard model prediction

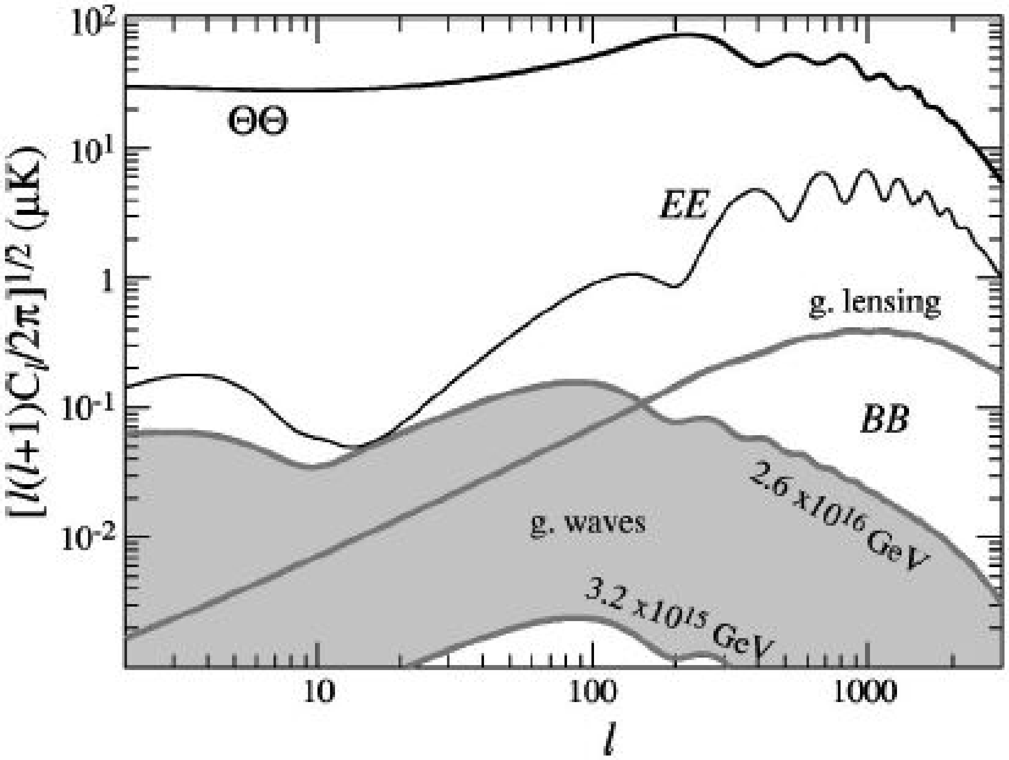
Contribution from inflation T/S = 0.3





- Gravitational waves have two polarization states. This extra freedom allows to distort the CMB to produce a B mode pattern, unlike scalars perturbations.

B-modes and inflation



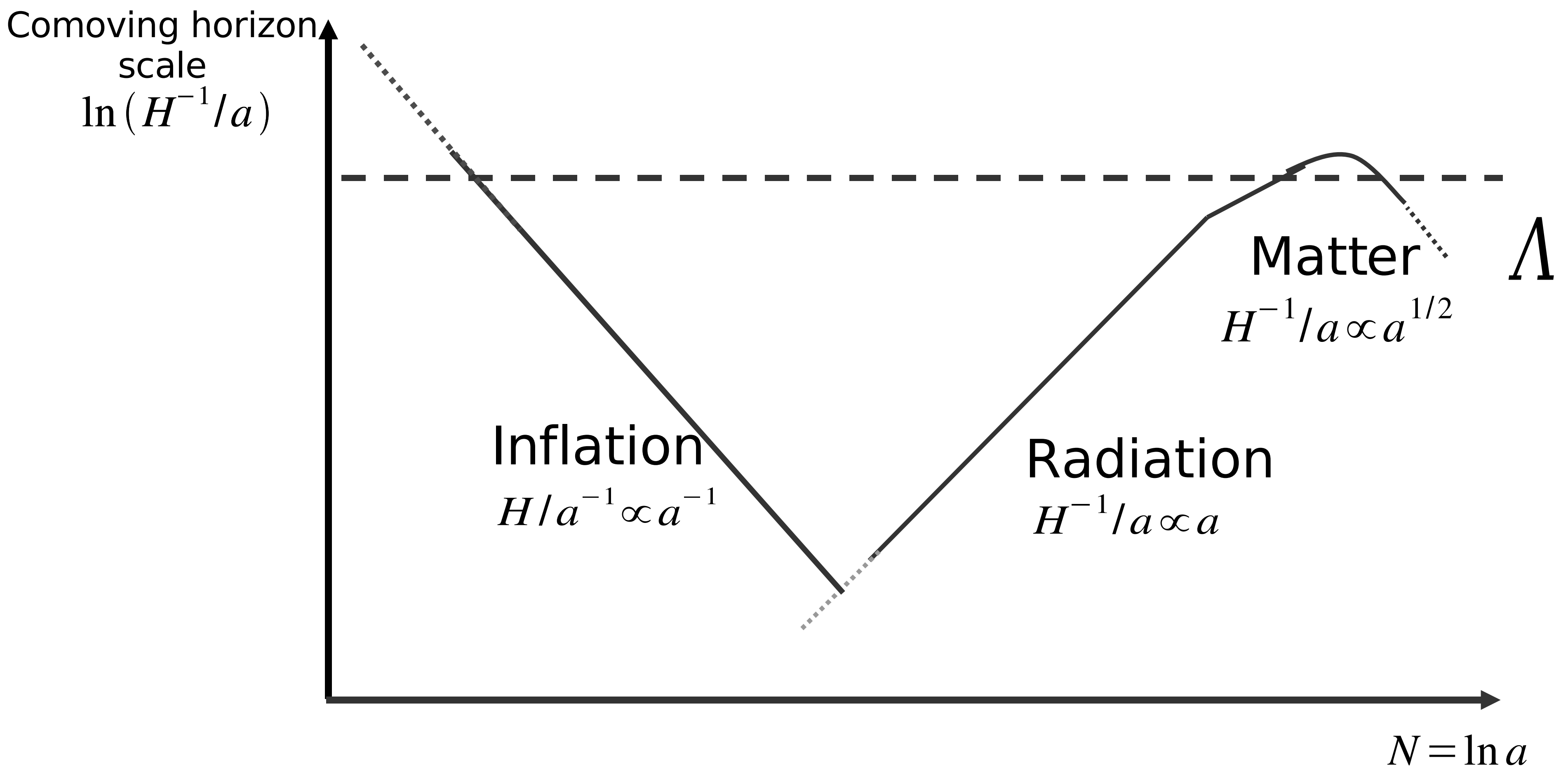
BICEP collaboration

- A small window exists for observing B modes from inflation.

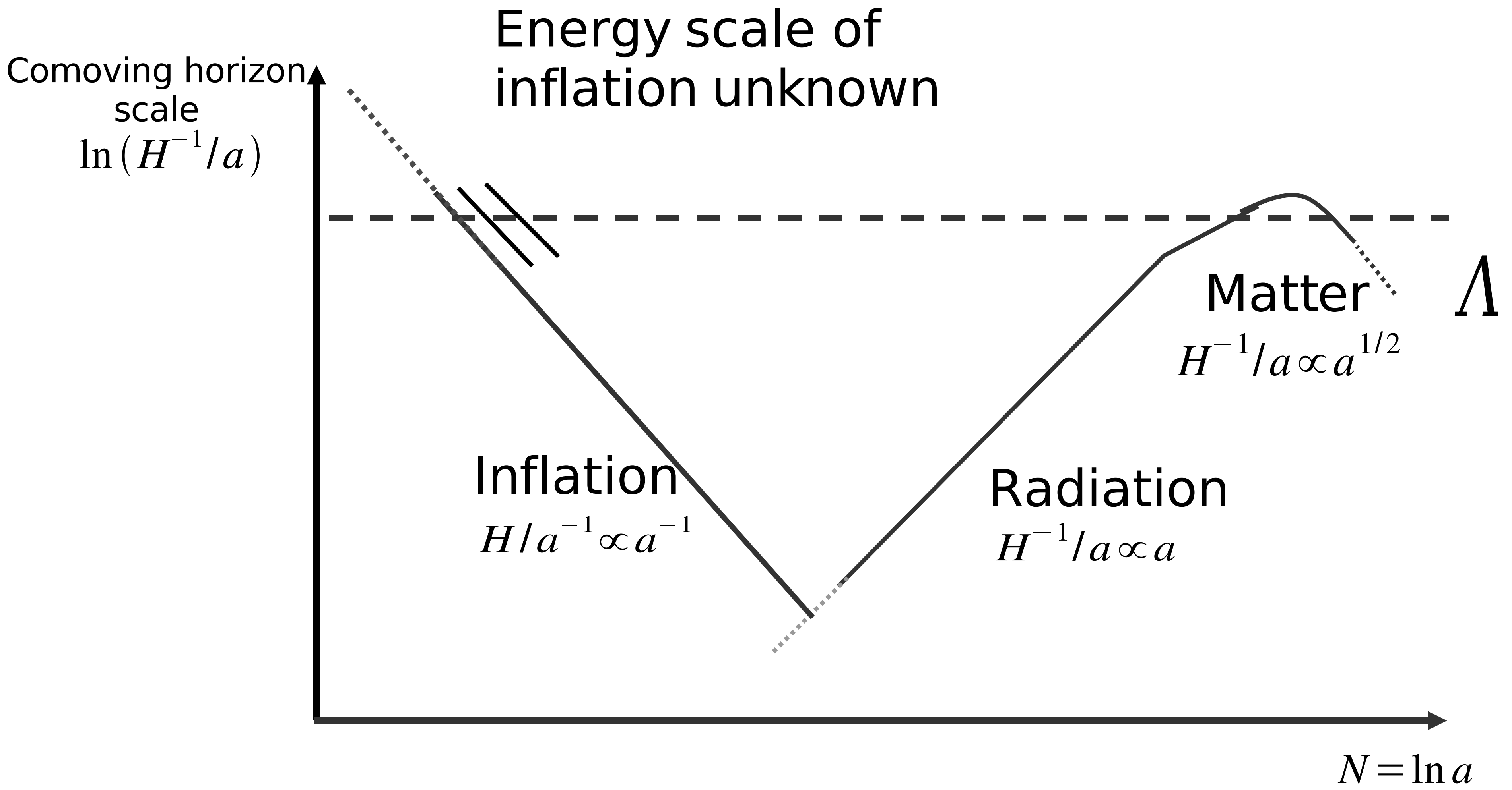
- Reality check : Energy scale of inflation is completely unknown. Assuming energy scale of inflation may be a low as the electroweak scale, then r may be 54 orders of magnitude below the current bound r<0.3

Nevertheless: B-modes provide a direct constraint on the inflationary hot-big bang model, and physics at very high energies.

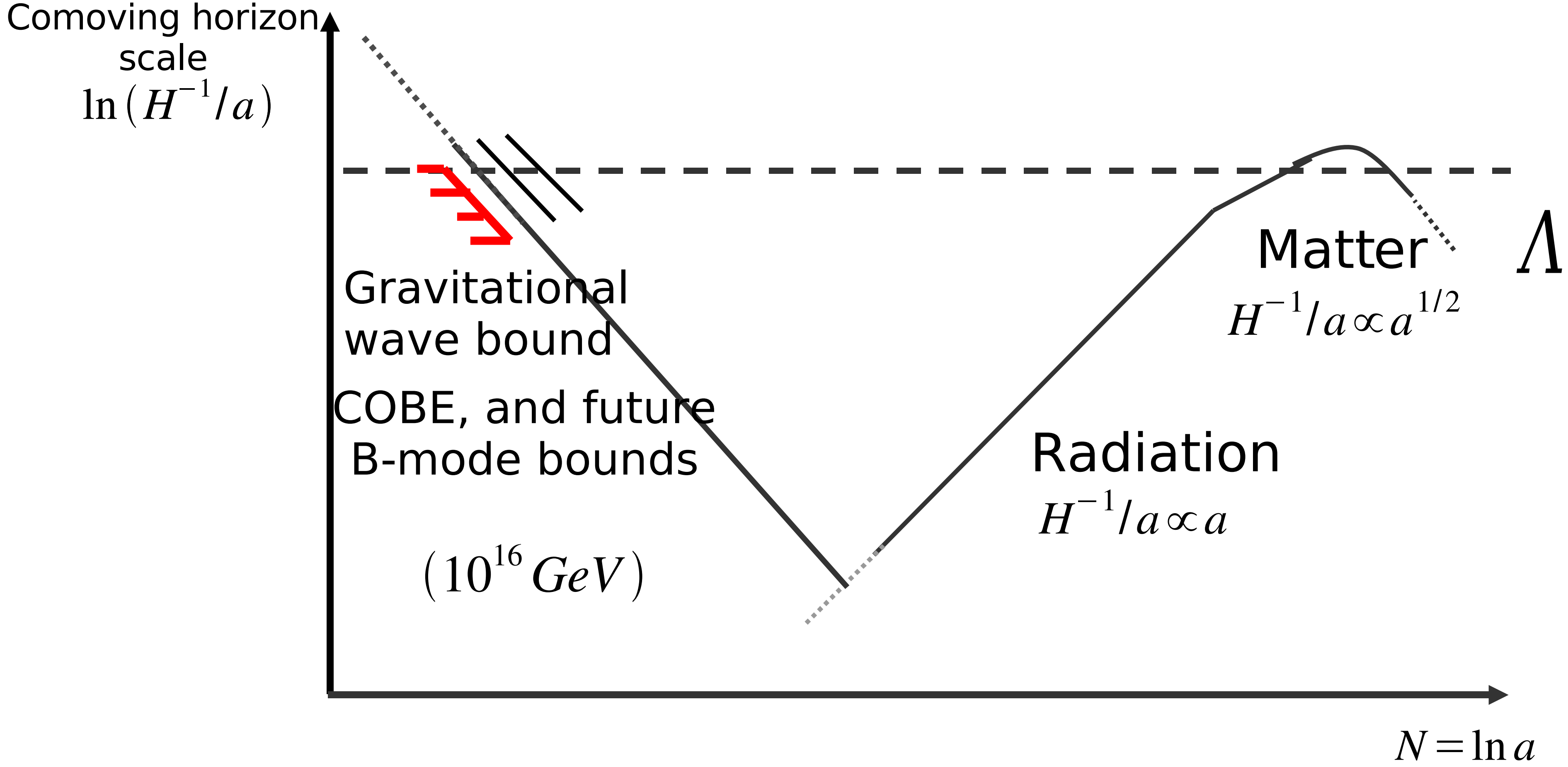
Constraints on the expansion history



Constraints on the expansion history



Constraints on the expansion history



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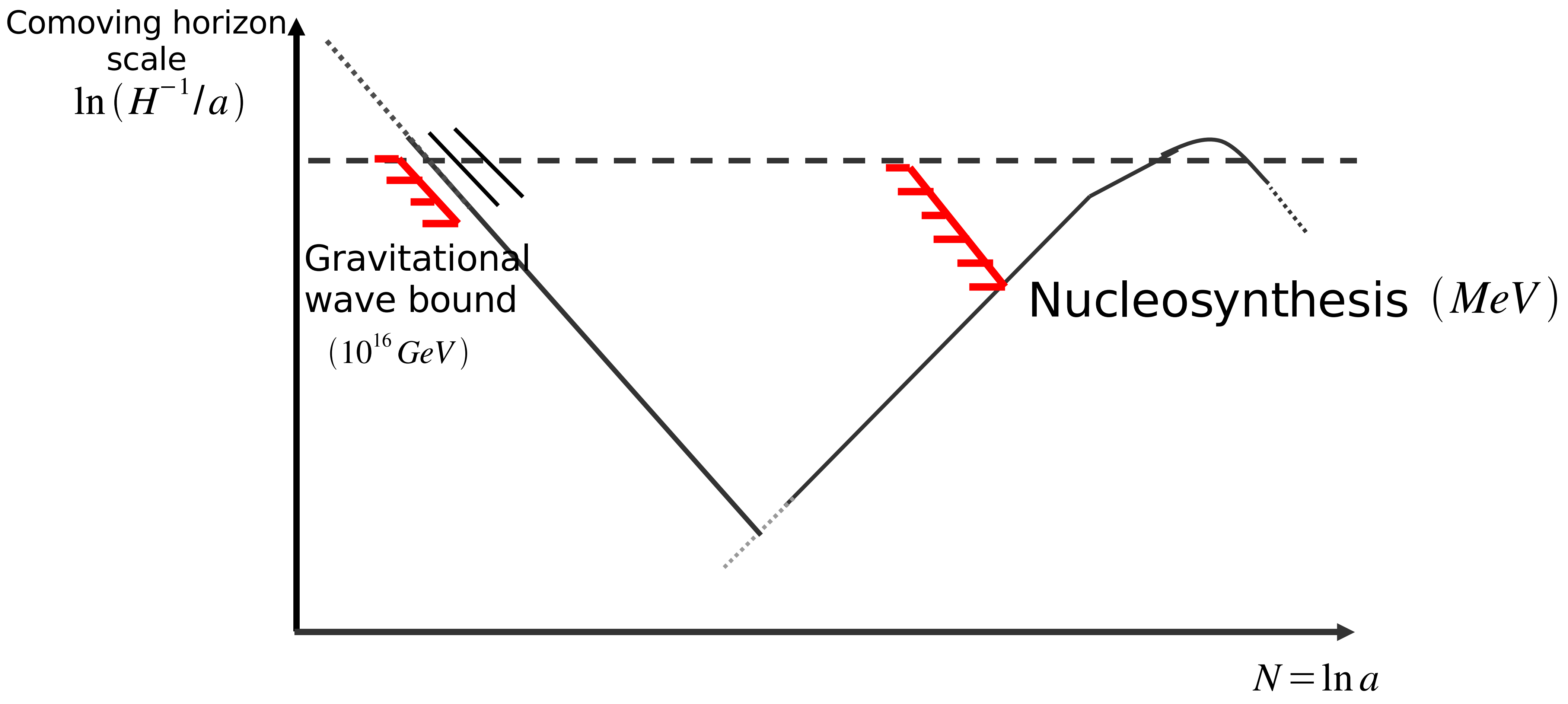
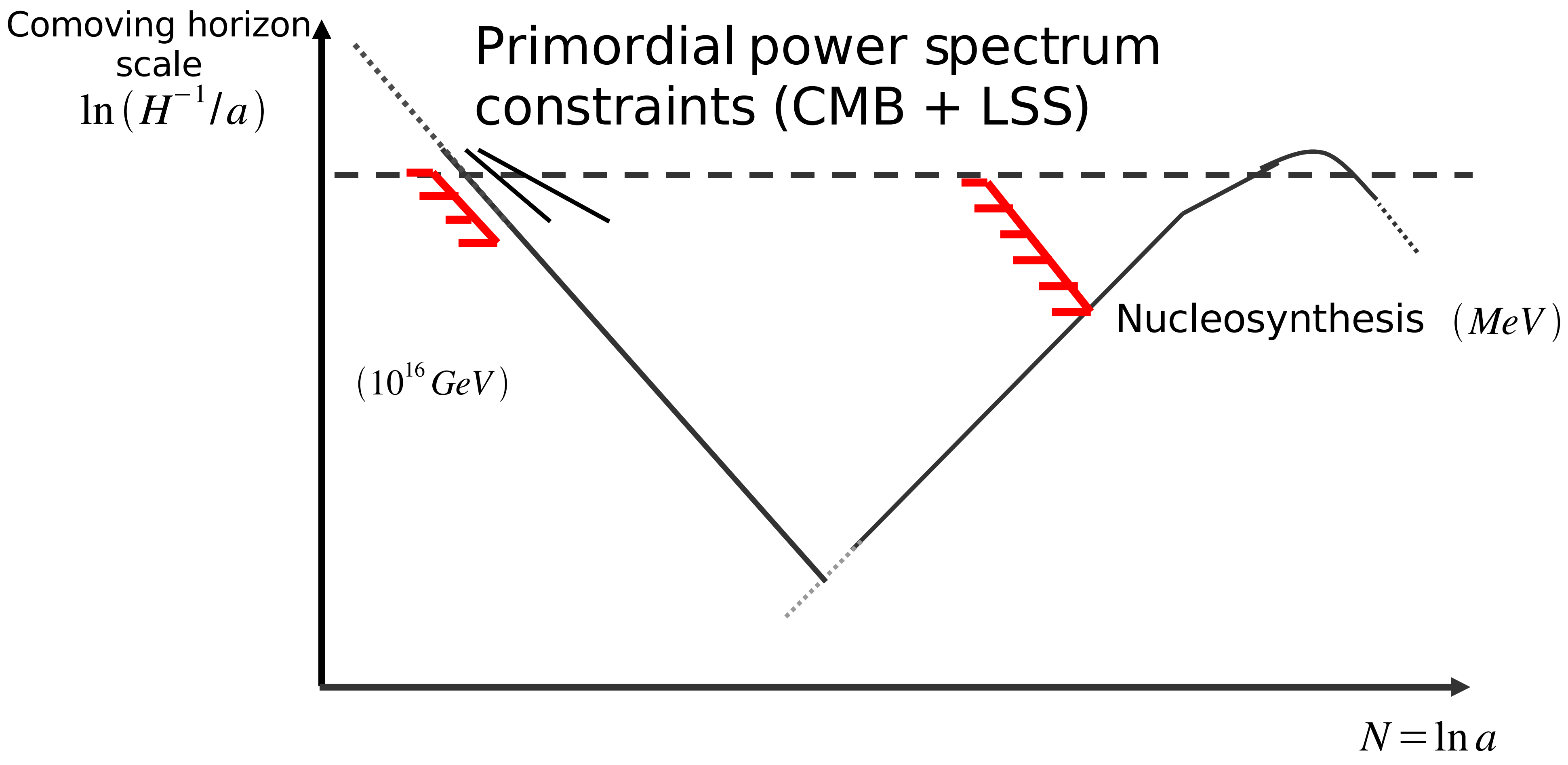
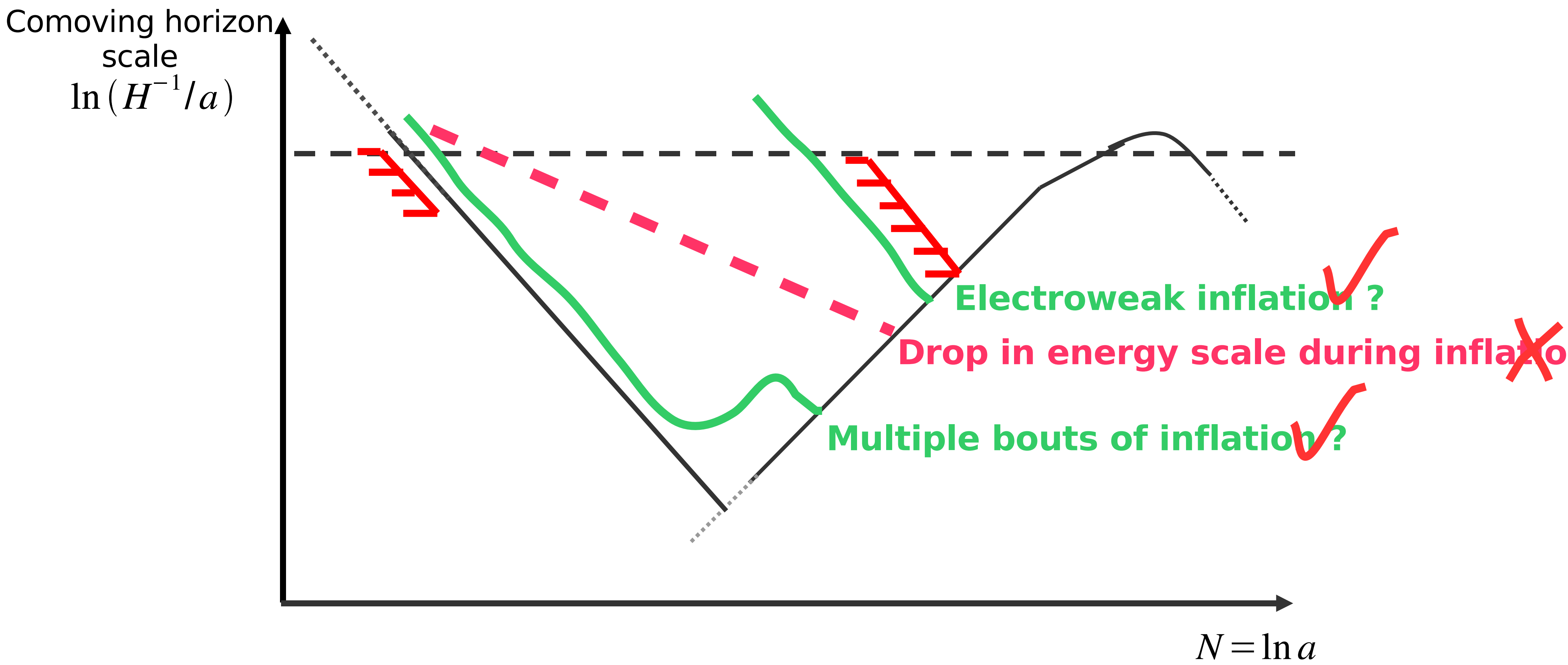
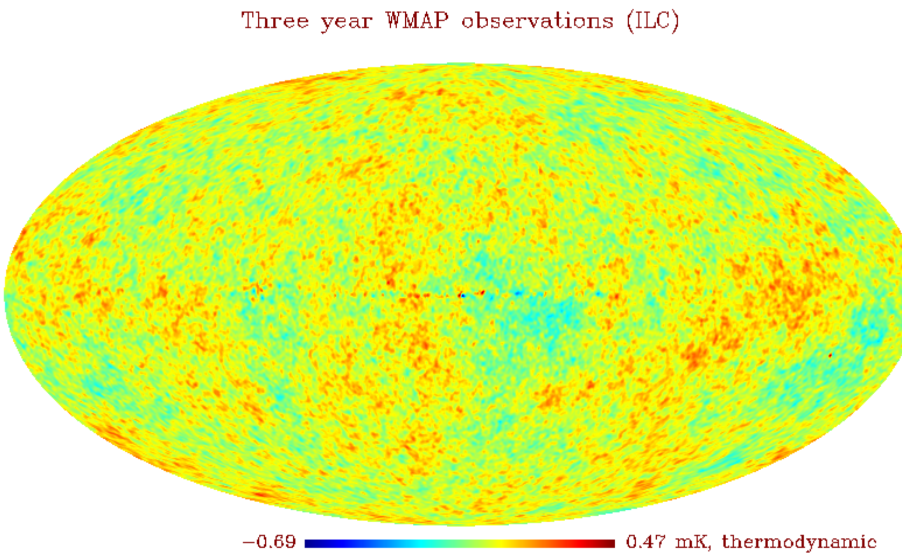


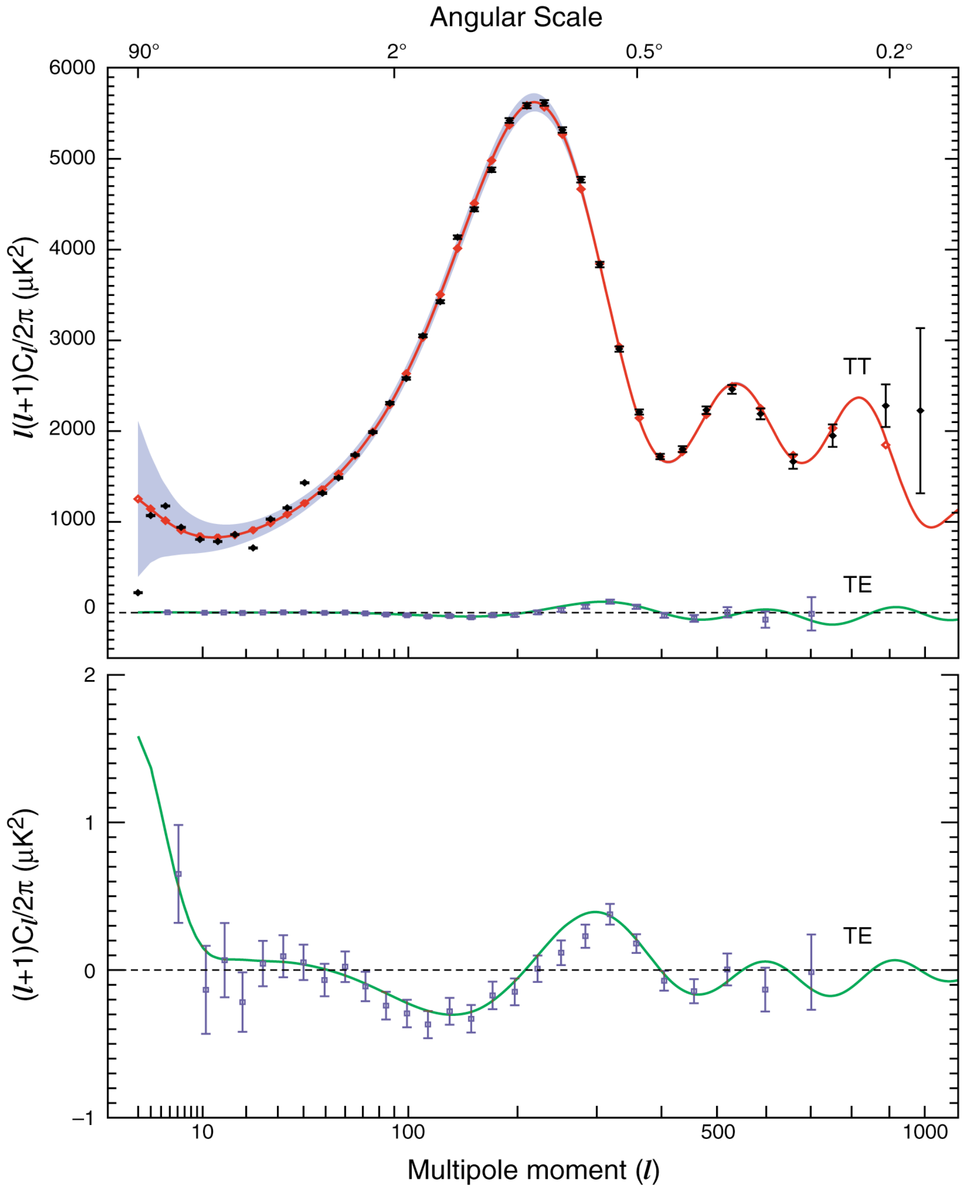
Constraints on the expansion history



Constraints on the expansion history







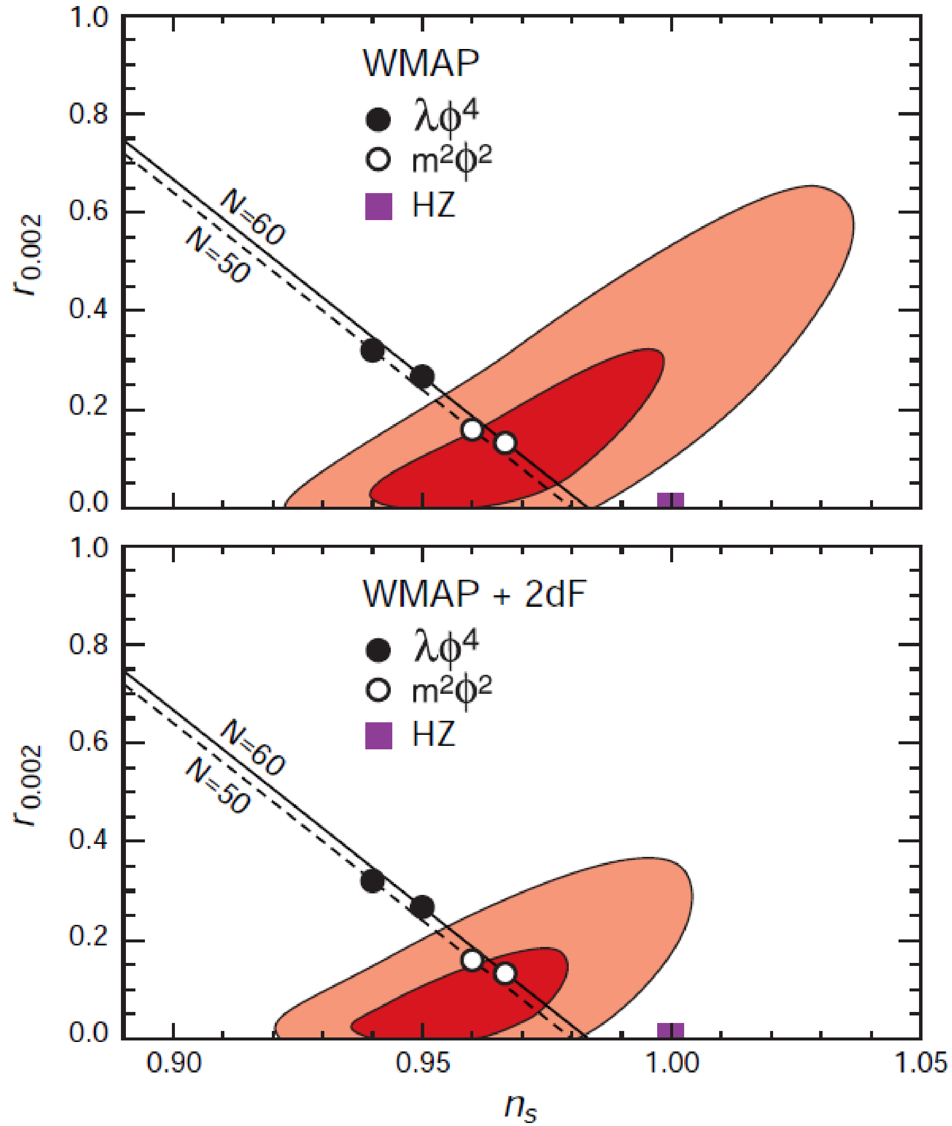
- WMAP vindicates the picture of photon-baryon oscillations, gravitationally coupled to dark matter potentials.

- Initial conditions are roughly scale invariant adiabatic Gaussian perturbations.

- WMAP tentatively indicates that we may need to go beyond the long-standing Harrison-Zeldovich scale-invariant spectrum.

- They found that the spectral index seems to be a little less that 1. This sits well with the idea of a dynamical origin to the perturbations.

- CMB constraints beginning to make an impact on toy models of inflation like $V = \lambda \phi^4$. These models basically over predict the tensor component.



Spergel et al 2006

Primordial power spectrum

The basic picture: primordial fluctuations act as initial conditions for CMB anisotropies and later seed structure formation via gravitational instability.

The basic model: primordial fluctuations are gaussian adiabatic density perturbations described by a power spectrum P(k) taken to be nearly scale-invariant:

$$C_l = 4\pi \int d\ln k\, P(k)\, \Delta_l^2(k, \{\theta_i\}) + N_l$$

Measured anisotropy spectrum

Power-law ? $P = P_0 (k/k_0)^{n_s - 1}$

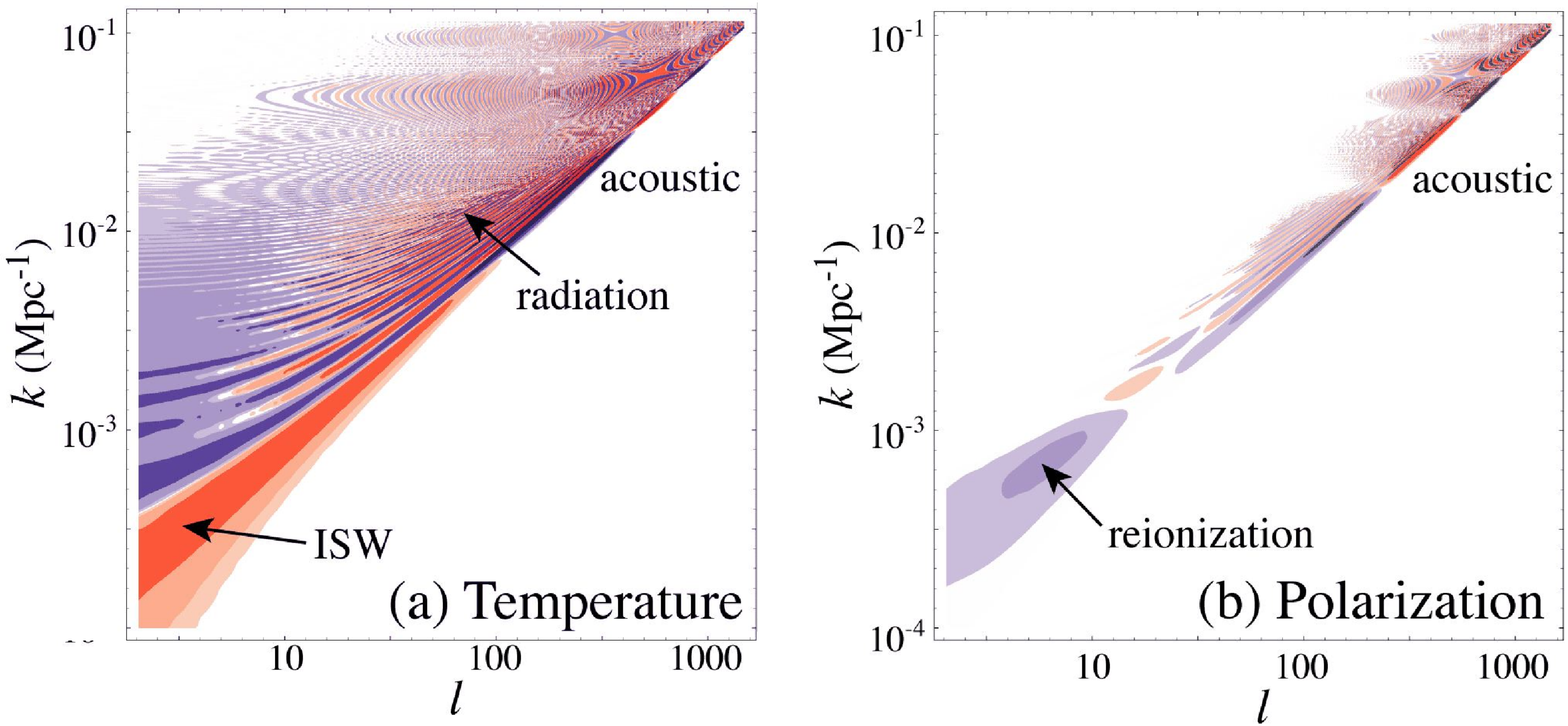
CMB transfer functions. Sensitive to a handful of cosmological parameters $\{\theta_i\}$

Currently: Normalization $P_0 \simeq 23 \pm 5 \times 10^{-10}$

Spectral index $n_s \simeq 0.97 \pm 0.03$

The basic problem: exploit C_l and our understanding of the CMB transfer functions to determine the details of P(k).

- Information from polarization is important in this context. Polarization is a cleaner probe of the CMB transfer functions.

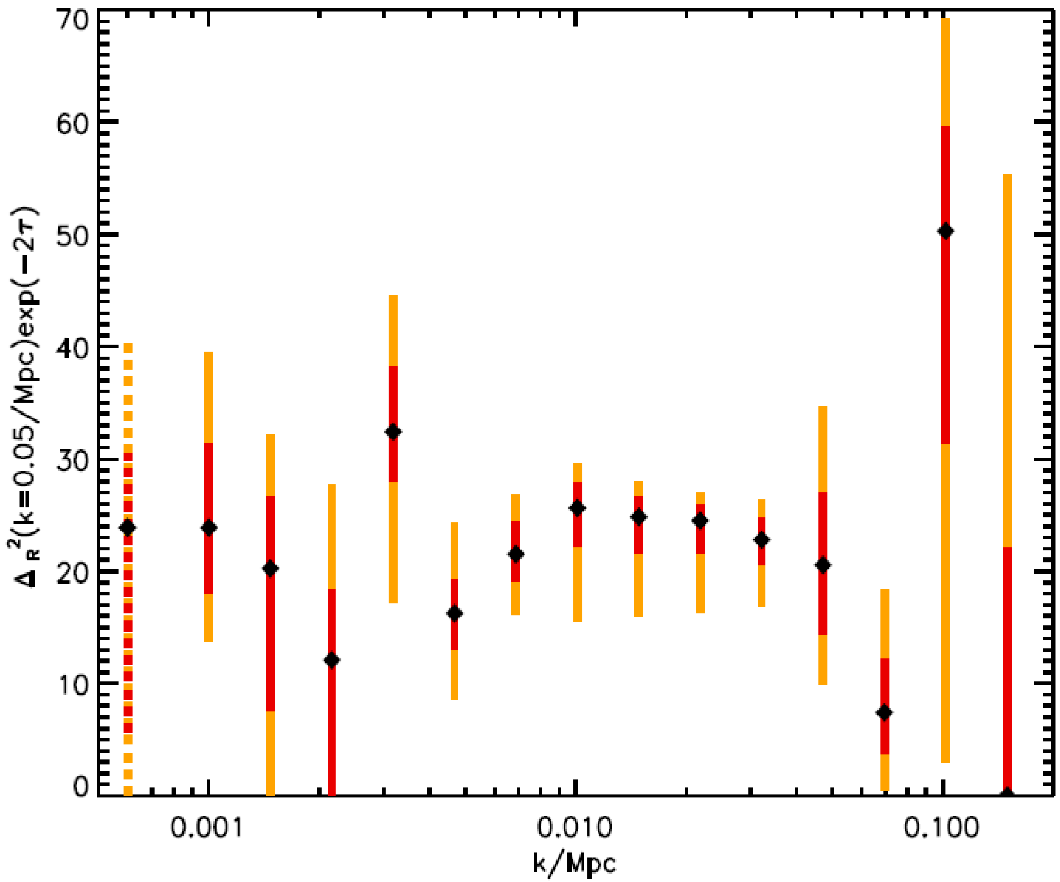


Hu and Okamoto 2006

- Typically assume a piecewise constant primordial power spectrum.

- Integrate out the usual cosmological parameters along with the power spectrum amplitudes.

- We can see that the basic scale-invariant model fits quite well.



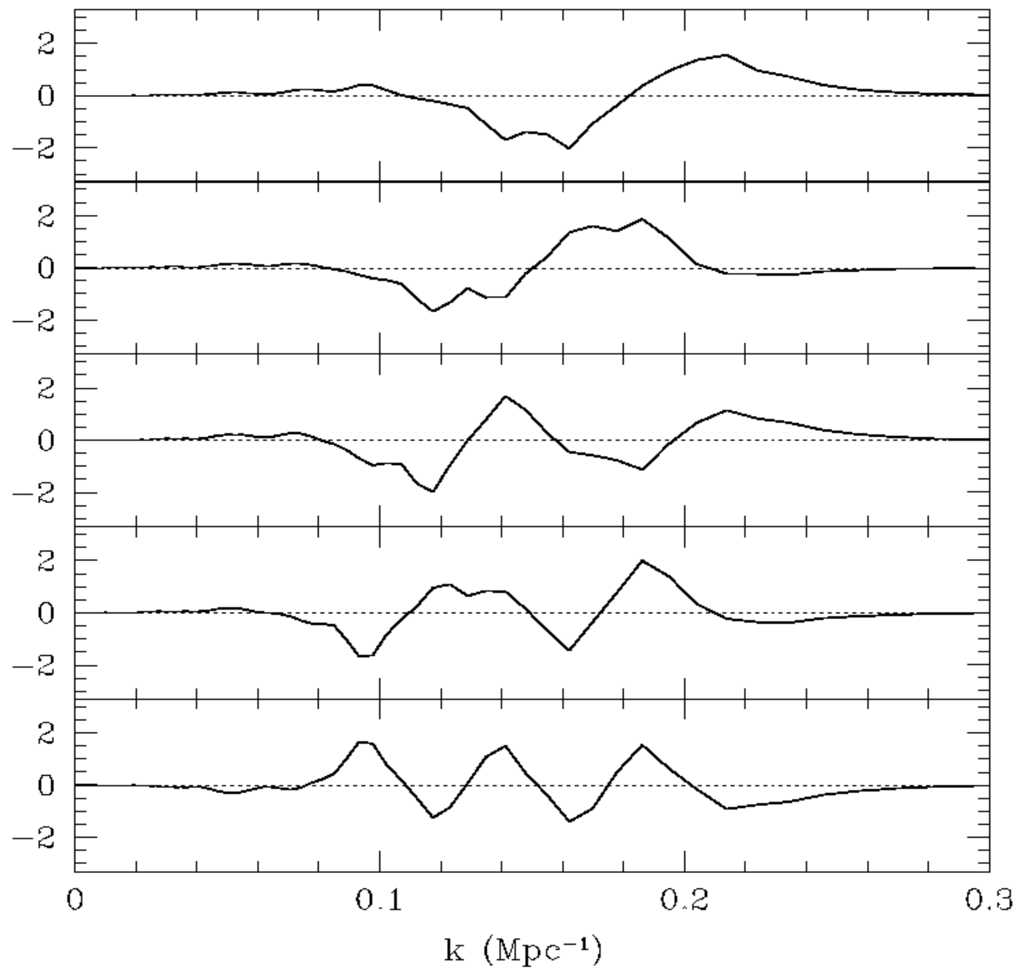
Spergel et al 2006

- Alternatively, can construct an orthonormal power spectrum model.

$$\frac{P(k)}{P_0} = m_0 + \sum_{a=1}^{a_{max}} m_a S_a(k)$$

$$m_a = \int d\ln k \frac{P(k)}{P_0} S_a(k)$$

- Append the mode amplitudes $\{m_a\}$ to the usual list of cosmological parameters and integrate out the parameter space using MCMC.



Hu and Okamoto 2004

- Advantages: automation; optimization of the model to a given noise level; PCA modes can also be constructed to be orthogonal to the effect of cosmological parameters on the CMB spectrum => PCA constraints give a primordial power spectrum likelihood function.

Details of the construction

- First construct the Fisher information matrix for the instrument, using a set of power spectrum test spikes as parameters p_μ

$$F_{\mu\nu} = \sum_{l=2}^{l_{max}} \sum_{X,Y} \frac{\partial C_l^X}{\partial p_\mu} cov^{-1}(C_l^X, C_l^Y) \frac{\partial C_l^Y}{\partial p_\nu}$$

X, Y = {TT, TE, EE, BB}

$$= \sum_{l=2}^{l_{max}} \frac{2l+1}{2} Tr\left[D_{l\mu} C_l^{-1} D_{l\nu} C_l^{-1} \right]$$

$$C_l^{-1} = (S_l + N_l)^{-1}$$

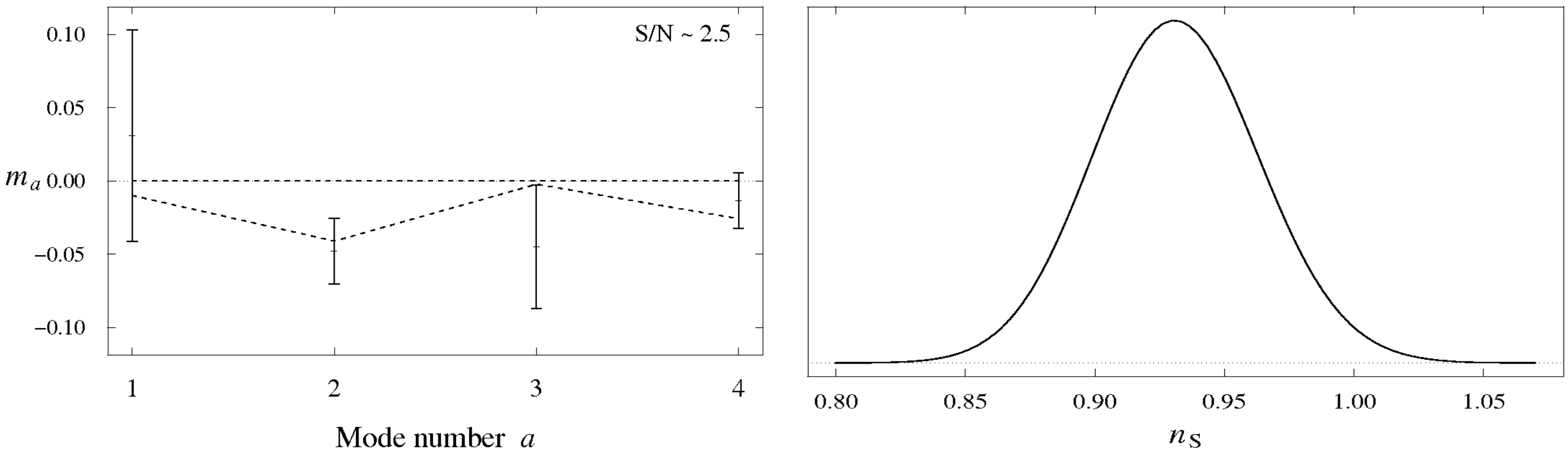
Encodes signal plus noise model

Encodes transfer of power from k space to l space

- Then invert F to obtain a covariance matrix and diagonalise to obtain the orthonormal eigenvectors S_{ia} which are the PCA power spectrum modes.

- Kadota, Dodelson, Hu and Stewart 2006 go one step further to propose what can be described as “principal component analysis of the inflationary potential”.

Current power spectrum PCA constraints are fairly weak:



Leach 2006

- Large data sets, and unknowns on the theory side drives this type of detailed empirical study.

Summary

- CMB polarization will provide important constraints on the inflationary hot big bang model.

- E-mode spectrum constrains the dynamics of inflation via the primordial scalar power spectrum.

- B-mode spectrum constrains the energy scale of inflation via the tensor spectrum.