Removing gravitational lensing B-modes

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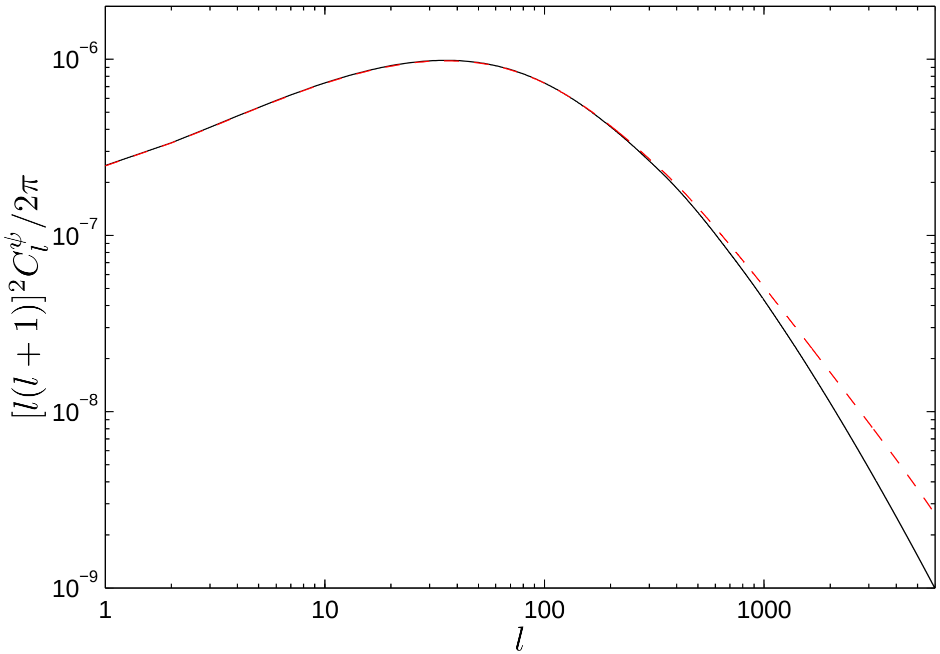
Gravitational lensing

Deflection angle

$$\boldsymbol{\alpha}(\hat{\mathbf{n}}) = \boldsymbol{\nabla}\psi(\hat{\mathbf{n}})$$

Lensing potential

$$\psi(\hat{\mathbf{n}}) \equiv -2\int_0^{r_{\rm rec}} \mathrm{d}r\, \frac{D_A(r_{\rm rec}-r)}{D_A(r_{\rm rec})D_A(r)}\Psi(r\hat{\mathbf{n}}, r)$$



Deflections of order $\sim 2'$ but coherent on degrees scales

Gravitational lensing

$$\widetilde{X}(\hat{\mathbf{n}}) = X(\hat{\mathbf{n}} + \boldsymbol{\nabla}\psi) \ , \qquad X = \{\Delta T, Q, U\}$$

Assuming $\boldsymbol{\nabla}\psi$ is small

$$\widetilde{X}(\hat{\mathbf{n}}) \approx X(\hat{\mathbf{n}}) + \nabla_i\psi(\hat{\mathbf{n}})\nabla^i X(\hat{\mathbf{n}}) + \frac{1}{2}\nabla_i\psi(\hat{\mathbf{n}})\nabla_j\psi(\hat{\mathbf{n}})\nabla^i\nabla^j X(\hat{\mathbf{n}}) + \mathcal{O}(\psi^3)$$

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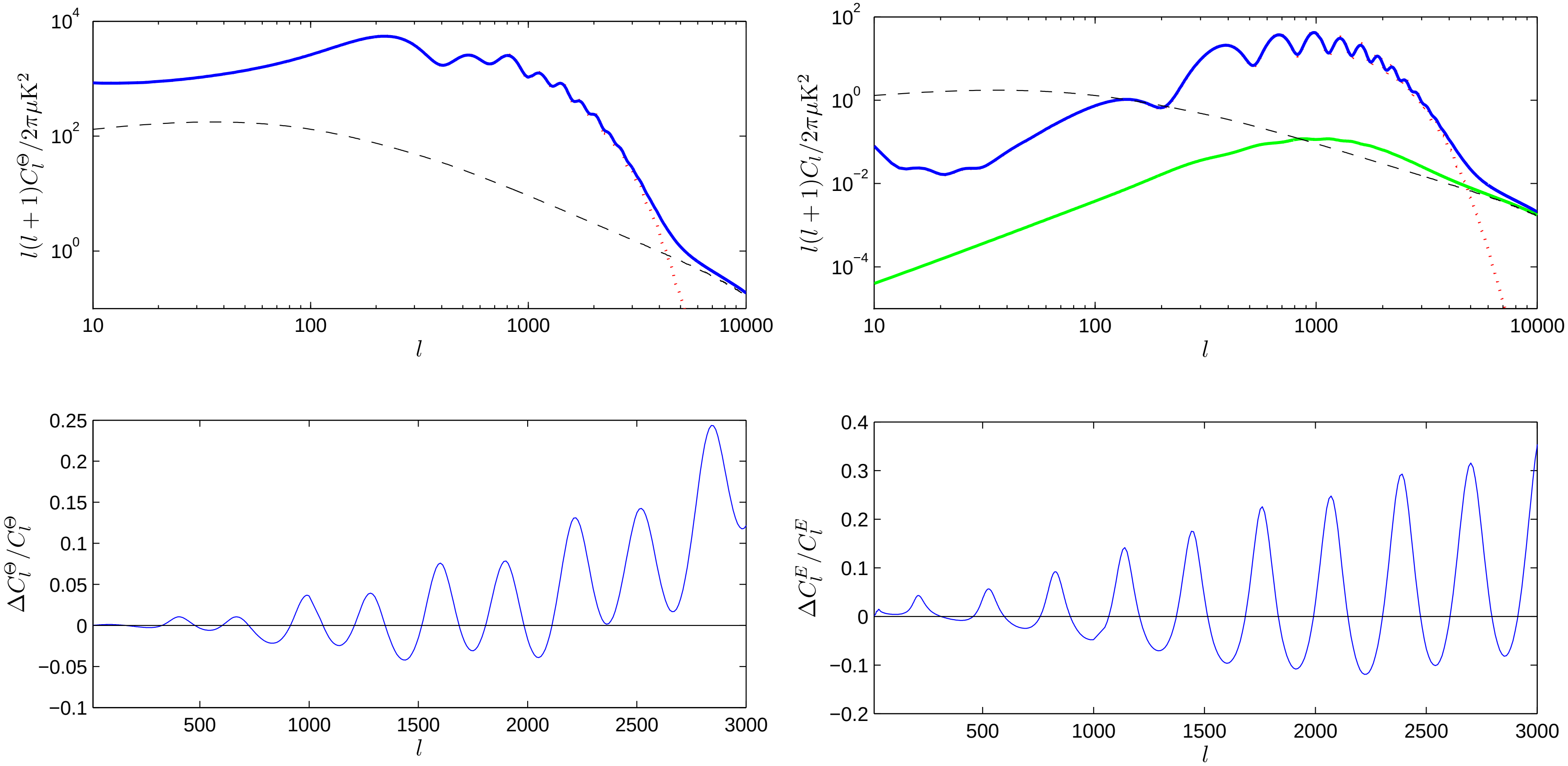
Power spectrum

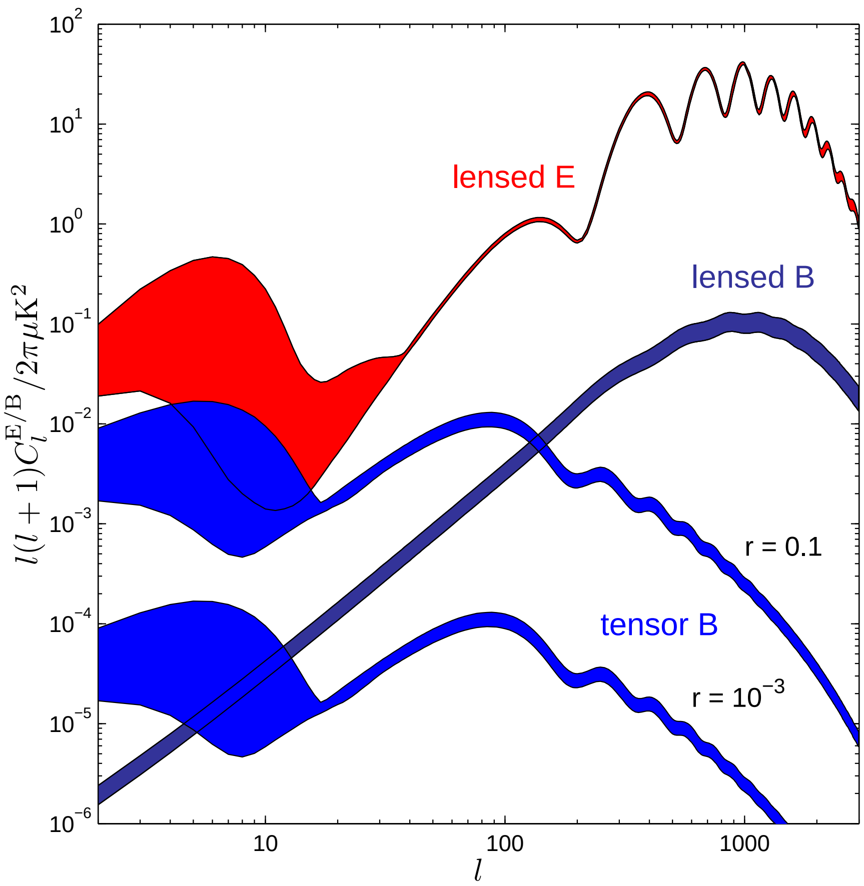
$$\begin{aligned}
\widetilde{C}_\ell^T &\approx \left(1 - \frac{\ell^2}{4\pi}\int \ell'^4 C_{\ell'}^{\psi}\frac{\mathrm{d}\ell'}{\ell'}\right) C_\ell^T + \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2}\left(\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\right)^2 C^{\psi}_{|\mathbf{l}-\mathbf{l}'|} C^T_{|\mathbf{l}'|} \\
\widetilde{C}_\ell^E &\approx \left(1 - \frac{\ell^2}{4\pi}\int \ell'^4 C_{\ell'}^{\psi}\frac{\mathrm{d}\ell'}{\ell'}\right) C_\ell^E + \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2}\left(\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\right)^2 C^{\psi}_{|\mathbf{l}-\mathbf{l}'|} C^E_{|\mathbf{l}'|} \times \\
&\qquad \cos^2 2(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}}) \\
\widetilde{C}_\ell^B &\approx \int \frac{\mathrm{d}^2\mathbf{l}'}{(2\pi)^2}\left(\mathbf{l}'\cdot(\mathbf{l}-\mathbf{l}')\right)^2 C^{\psi}_{|\mathbf{l}-\mathbf{l}'|} C^E_{|\mathbf{l}'|} \sin^2 2(\phi_{\mathbf{l}'} - \phi_{\mathbf{l}})
\end{aligned}$$

B-modes generated from lensing a purly E-mode unlensed field

Power spectrum



Power spectrum



The lowest tensor to scalar ratio that can be distinguished (without removing lensed B-modes) is $r \sim 0.2 - 1 \times 10^{-3}$

Reconstructing the lensing potential

- if we knew lensing potential ψ we could delens maps and measure tensor modes with smaller amplitude
- a given fixed lensing potential will cause break of isotropy of the CMB distribution on the sky $\Rightarrow$ use quadratic off-diagonal terms of the ψ-fixed correlation

$$\left\langle \widetilde{X}(\mathbf{l})\widetilde{Y}(\mathbf{l}')\right\rangle_{\mathrm{CMB}} = f_{XY}(\mathbf{l},\mathbf{l}')\psi(\mathbf{L}) \ , \quad \mathbf{L}=\mathbf{l}+\mathbf{l}' \ , \quad \mathbf{l}\neq\mathbf{l}'$$

to constrain the lensing potential

Quadratic estimator

$$\hat{\psi}(\mathbf{L}) = N(\mathbf{L}) \int \frac{d^2\mathbf{l}}{(2\pi)^2} \widetilde{X}(\mathbf{l})\widetilde{Y}(\mathbf{l}')g_{XY}(\mathbf{l},\mathbf{l}')$$

where normalization

$$N(\mathbf{L})^{-1} = \int \frac{d^2\mathbf{l}}{(2\pi)^2} f_{XY}(\mathbf{l},\mathbf{l}')g_{XY}(\mathbf{l},\mathbf{l}')$$

is chosen such that

$$\left\langle \hat{\psi}(\mathbf{L}) \right\rangle_{\mathrm{CMB}} = \psi(\mathbf{L})$$

To optimize estimator the weights $g_{XY}(\mathbf{l},\mathbf{l}')$ are chosen by minimizing the variance $\left\langle \hat{\psi}^*(\mathbf{L})\hat{\psi}(\mathbf{L}) \right\rangle$ subject to the normalization constraint

$$g_{XY}(\mathbf{l},\mathbf{l}') = \frac{\widetilde{C}_\ell^{YY}\widetilde{C}_{\ell'}^{XX} f_{XY}(\mathbf{l},\mathbf{l}') - \widetilde{C}_\ell^{XY}\widetilde{C}_{\ell'}^{XY} f_{XY}(\mathbf{l}',\mathbf{l})}{\widetilde{C}_\ell^{XX}\widetilde{C}_{\ell'}^{YY}\widetilde{C}_\ell^{YY}\widetilde{C}_{\ell'}^{XX} - (\widetilde{C}_\ell^{XY}\widetilde{C}_{\ell'}^{XY})^2}$$

Quadratic estimator

- for XY = TT

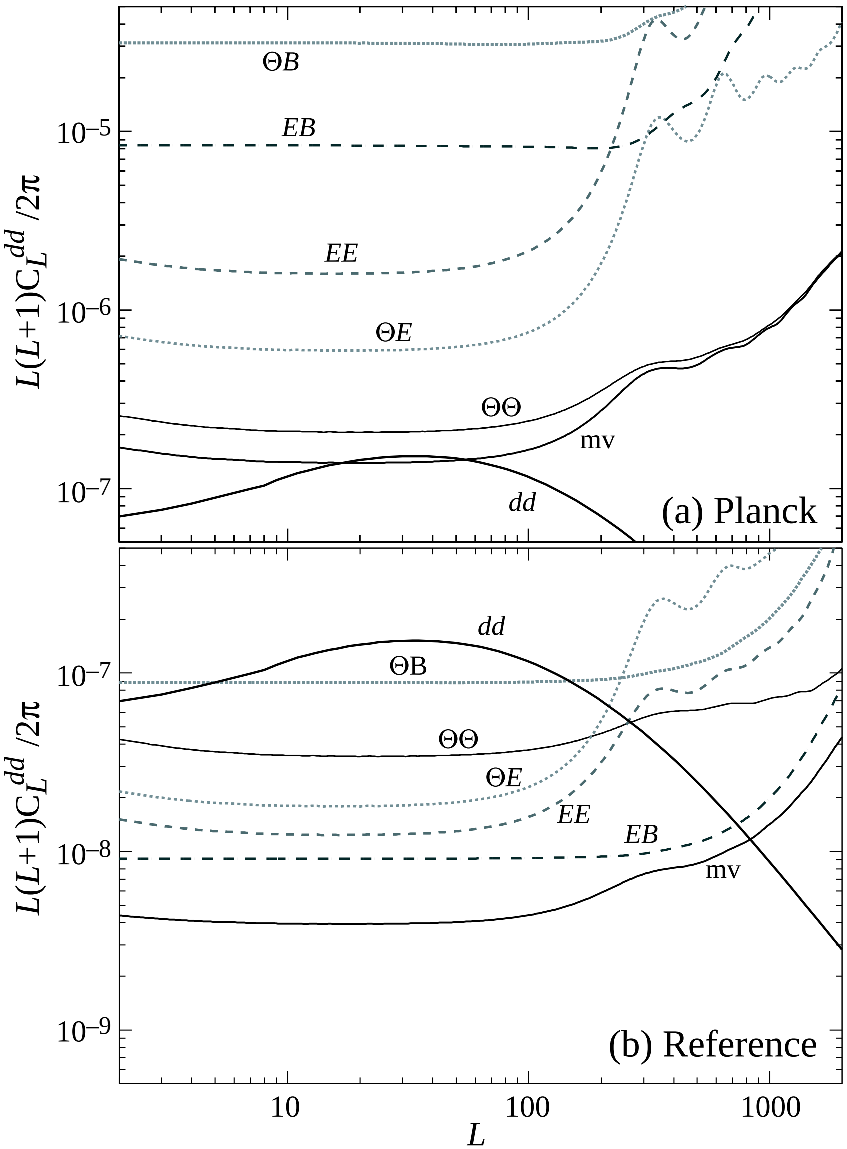
$$\hat{\psi}_{TT}(\mathbf{L}) = N_{TT}(\mathbf{L}) \int \frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2} \frac{\mathbf{L}\cdot\mathbf{l}\, C^T_{|\mathbf{l}|}}{\widetilde{C}^T_\ell\ \widetilde{C}^T_{|\mathbf{L}-\mathbf{l}|}} \Delta\widetilde{T}(\mathbf{l})\Delta\widetilde{T}(\mathbf{L}-\mathbf{l})$$

- for XY = EB

$$\hat{\psi}_{EB}(\mathbf{L}) = N_{EB}(\mathbf{L}) \int \frac{\mathrm{d}^2\mathbf{l}}{(2\pi)^2} \frac{\mathbf{L}\cdot\mathbf{l}\, C^E_{|\mathbf{l}|} + (\mathbf{L}-\mathbf{l})\cdot\mathbf{L}\, C^B_{|\mathbf{L}-\mathbf{l}|}}{\widetilde{C}^E_{|\mathbf{l}|}\ \widetilde{C}^B_{|\mathbf{L}-\mathbf{l}|}} \sin 2(\phi_{\mathbf{l}} - \phi_{\mathbf{l}'}) \times \widetilde{E}(\mathbf{l})\widetilde{B}(\mathbf{L}-\mathbf{l})$$

Quadratic estimator

- estimator $\hat{\psi}_{TT}$ better for noisy experiments (like PLANCK)
- estimator $\hat{\psi}_{EB}$ better if noise is sufficiently low
- estimator $\hat{\psi}_{EB}$ allows reconstruction of the lensing potential out to smaller scales



Quadratic estimator

Advantages:

- reduces gravitational lensing noise by up to a factor of 7
- fast method of estimation

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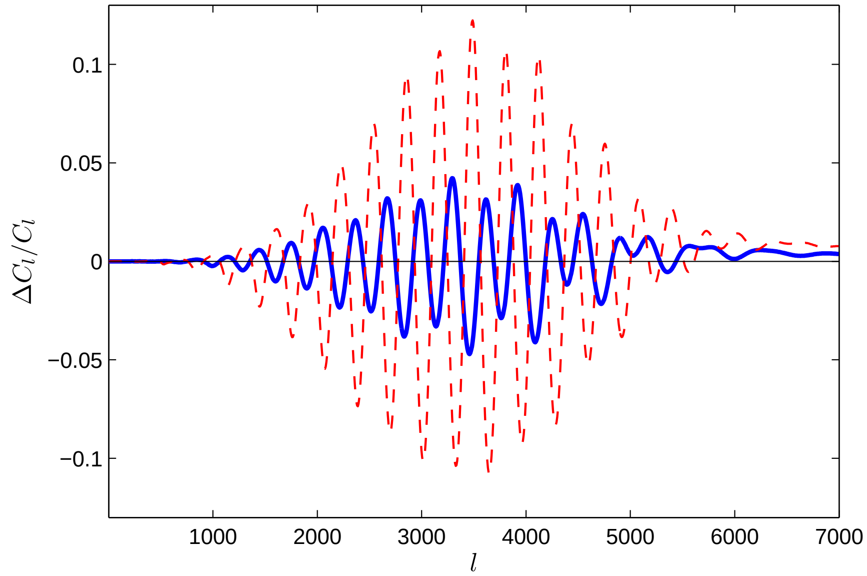

Maximum likelihood estimator

- log likelihood function

$$\mathcal{L}[\psi] \equiv -2\log P(\psi|\mathbf{X}^{\rm tot},\mathbf{S},\mathbf{N}) = \mathbf{X}^{\rm tot\,\dagger}\mathbf{C}^{-1}\mathbf{X}^{\rm tot} + \log\det\mathbf{C}$$

  where $\mathbf{C} \equiv \left\langle \mathbf{X}^{\rm tot}\mathbf{X}^{\rm tot\,\dagger}\right\rangle_{\mathbf{X},\mathbf{n}} = \mathbf{\Lambda}(\psi)\mathbf{S}\mathbf{\Lambda}(\psi)^{\dagger} + \mathbf{N}$, $\mathbf{X}^{\rm tot} = \mathbf{\Lambda}(\psi)\mathbf{X} + \mathbf{n}$
  To solve

$$\frac{\delta\mathcal{L}}{\delta\psi}[\hat{\psi}] = 0$$

  one uses iterative method

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$$\mathcal{P}[\psi] \quad\equiv\quad \mathcal{L}[\psi] - 2\log P(\psi|\mathbf{C}_{\psi}) = \mathcal{L}[\psi] + \psi^{\dagger}\mathbf{C}_{\psi}^{-1}\psi + \log\det\mathbf{C}_{\psi}$$

  Solution

$$\hat{\psi} = -\mathbf{C}_{\psi}\frac{\delta\mathcal{L}}{\delta\psi}[\hat{\psi}]$$

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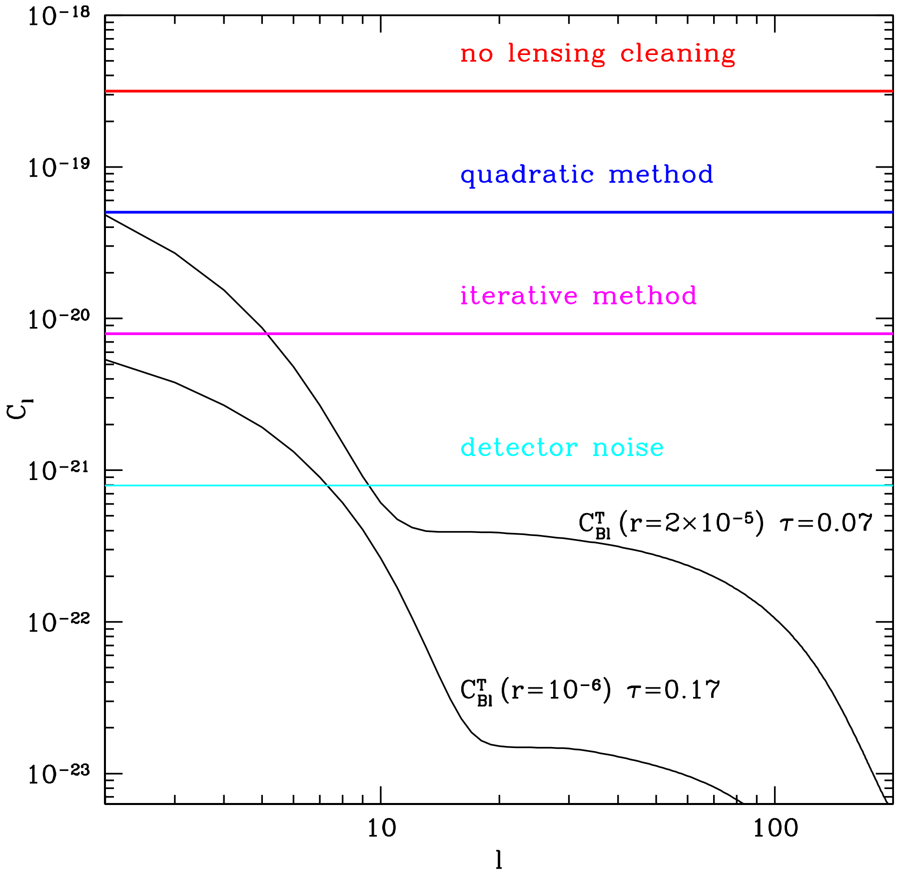
$$\hat{\psi} = -\mathbf{C}_\psi\frac{\delta\mathcal{L}}{\delta\psi}[\hat{\psi}]$$

- if we approximate

$$\frac{\delta\mathcal{L}}{\delta\psi} \approx \frac{\delta\mathcal{L}}{\delta\psi}_{|\psi=0} + \frac{\delta^2\mathcal{L}}{\delta\psi\delta\psi}\,\psi$$

  to linear order in ψ one gets quadratic estimator

Maximum likelihood estimator



Maximum likelihood estimator

Advantages:

- reduces gravitational lensing noise by up to a factor of 40
- reconstruction limited by cosmic variance of unlensed fields

Maximum likelihood estimator

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- reduces gravitational lensing noise by up to a factor of 40
- reconstruction limited by cosmic variance of unlensed fields

Disadvantages:

- computationally more demanding than quadratic estimator