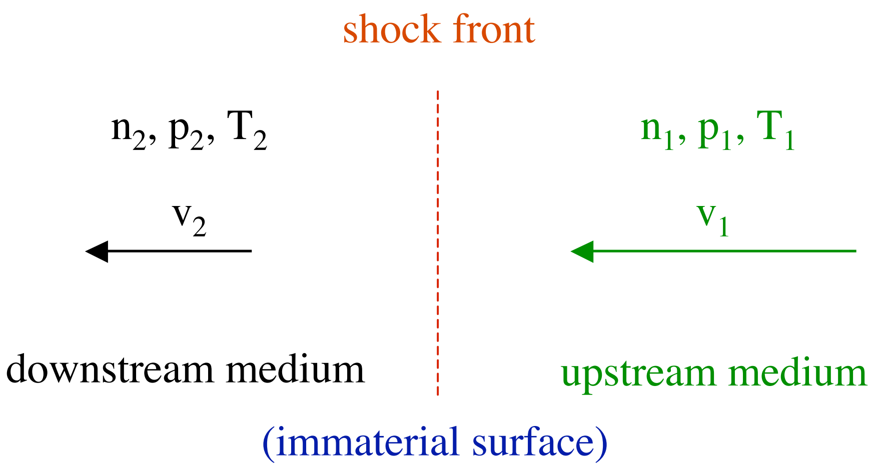
Diffusive shock acceleration:
a first order Fermi process

Shock waves

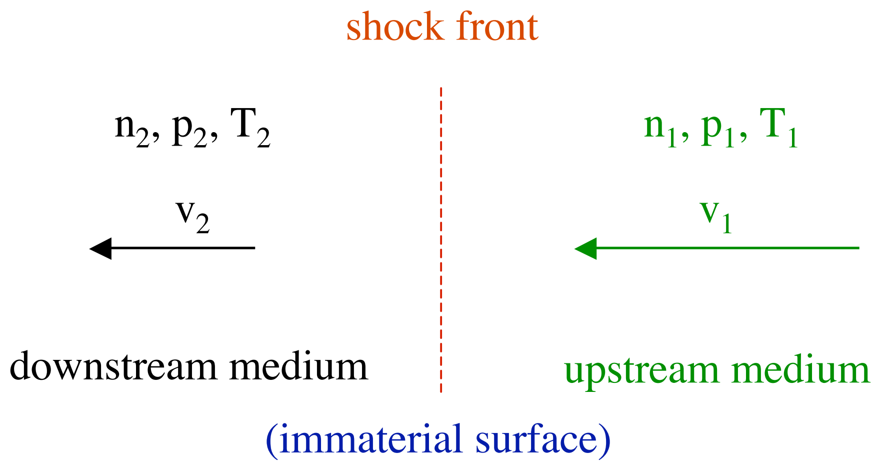
- Discontinuity in physical parameters



- What can we tell from the purely macroscopic point of view?

Shock waves "jump conditions"

- Conservation equations



mass

$$\rho_2 v_2 = \rho_1 v_1$$

momentum

$$p_2 + \rho_2 v_2^2 = p_1 + \rho_1 v_1^2$$

energy

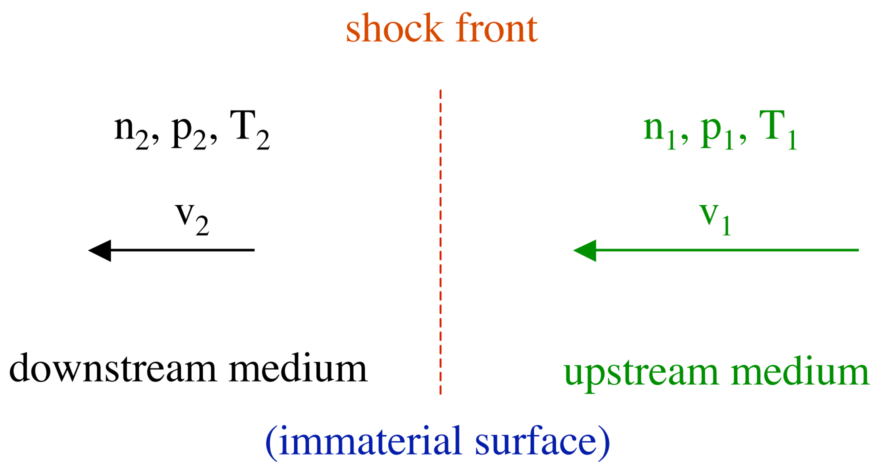
$$\rho_2 v_2 (v_2^2/2 + p_2/\rho_2 + e_2) = \rho_1 v_1 (v_1^2/2 + p_1/\rho_1 + e_1)$$

$$e_i = \frac{1}{\gamma - 1}(p_i/\rho_i)$$

γ → adiabatic index

Shock waves "jump conditions"

- Solve macroscopic conservation equations



$$\frac{v_2}{v_1} = \frac{\gamma + M_1^{-2} \pm (1 - M_1^{-2})}{\gamma + 1} \qquad \frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$

$$\frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1 + 2M_1^{-2}} \qquad \frac{T_2}{T_1} = \frac{[2\gamma M_1^2 - (\gamma - 1)][\gamma - 1 + 2M_1^{-2}]}{(\gamma + 1)^2}$$

$M_1 = v_1/c_1$

- Solution:

$$\frac{v_2}{v_1} = \frac{\gamma + 1/M_1^2 \pm (1 - 1/M_1^2)}{\gamma + 1}$$

- Trivial solution: $v_2 = v_1$!

- Shock wave solution:

$$\frac{v_2}{v_1} = \frac{n_1}{n_2} = \frac{\gamma - 1 + 2/M_1^2}{\gamma + 1} \qquad \frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma - 1)}{\gamma + 1}$$

- NB: M_1 can be either > 1 or < 1, but

$$[\, M_1^2 - (\gamma - 1)/2\gamma \,] \times [\, M_2^2 - (\gamma - 1)/2\gamma \,] = ((\gamma + 1)/2\gamma)^2$$

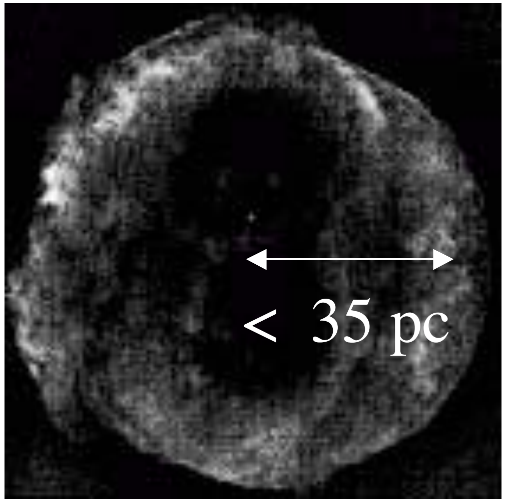
$M_1 > 1 \Leftrightarrow M_2 < 1$

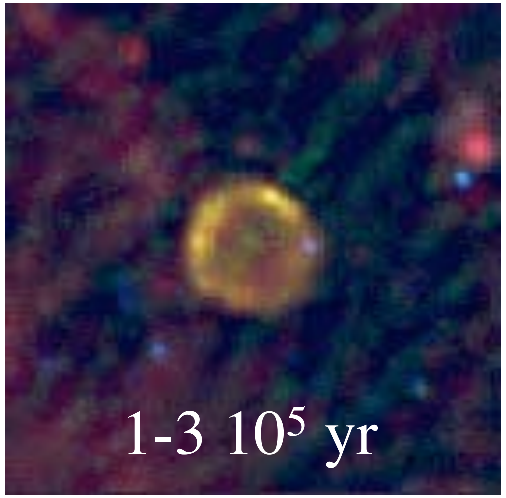
But entropy must increase!

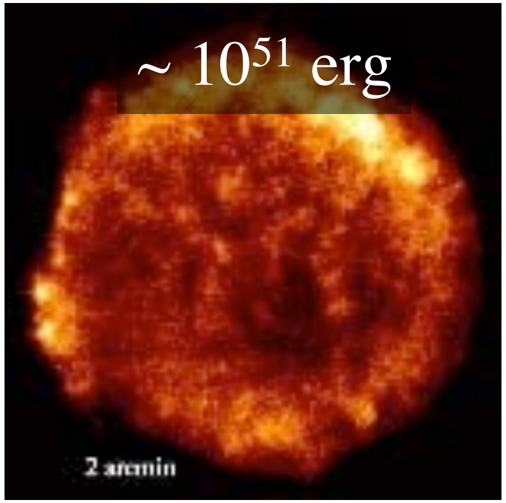
Astrophysical shocks

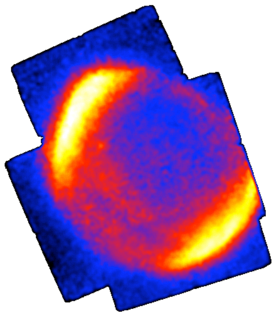
- Supernovae eject supersonic material

S
N
R

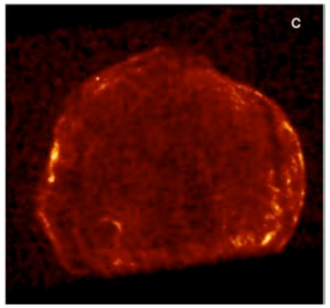








SN 1006

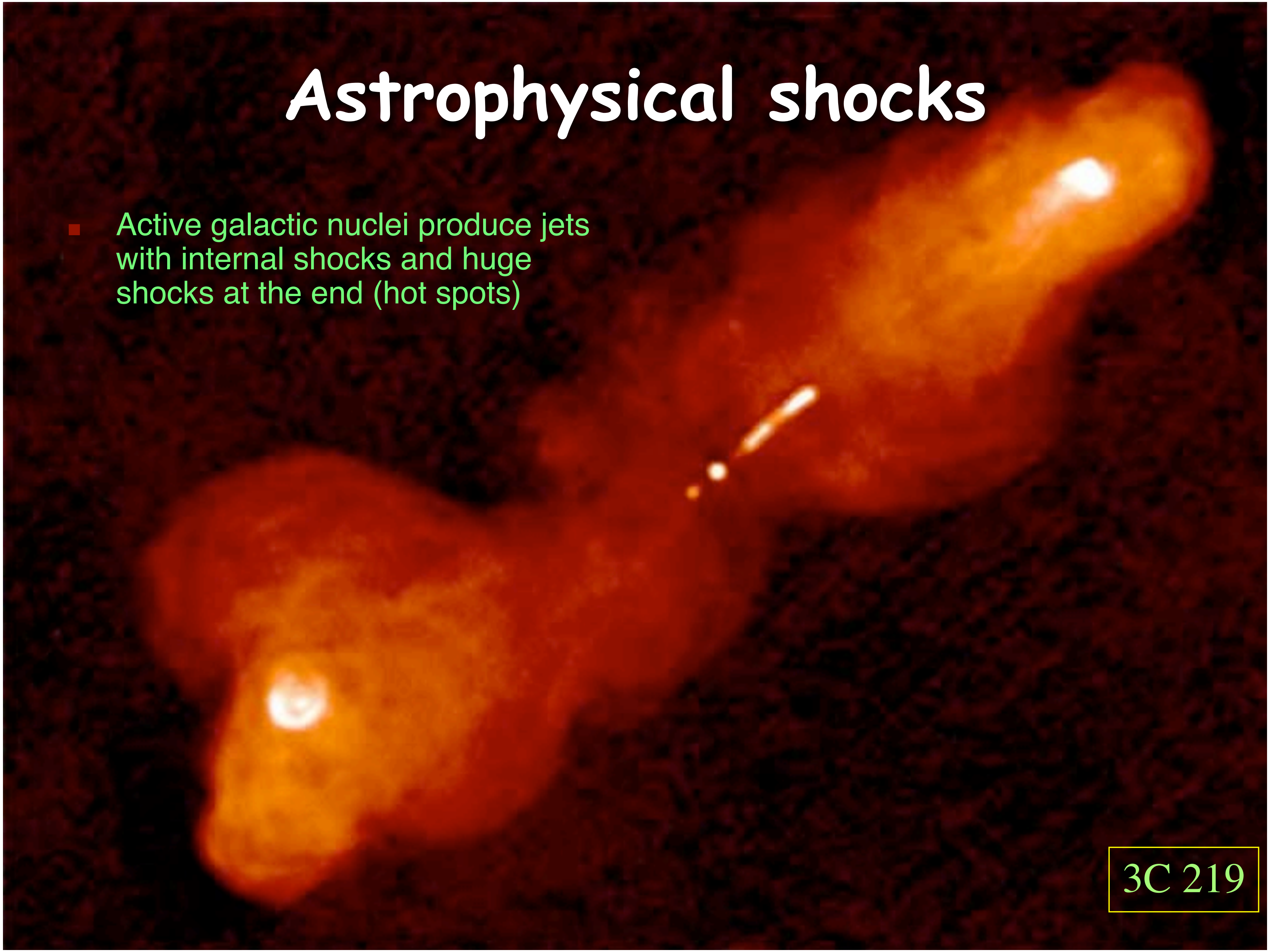


Tycho

- + gamma-ray bursts (relativistic fireballs)
- Stellar mass black holes emit plasma blobs

Astrophysical shocks

- Active galactic nuclei produce jets with internal shocks and huge shocks at the end (hot spots)

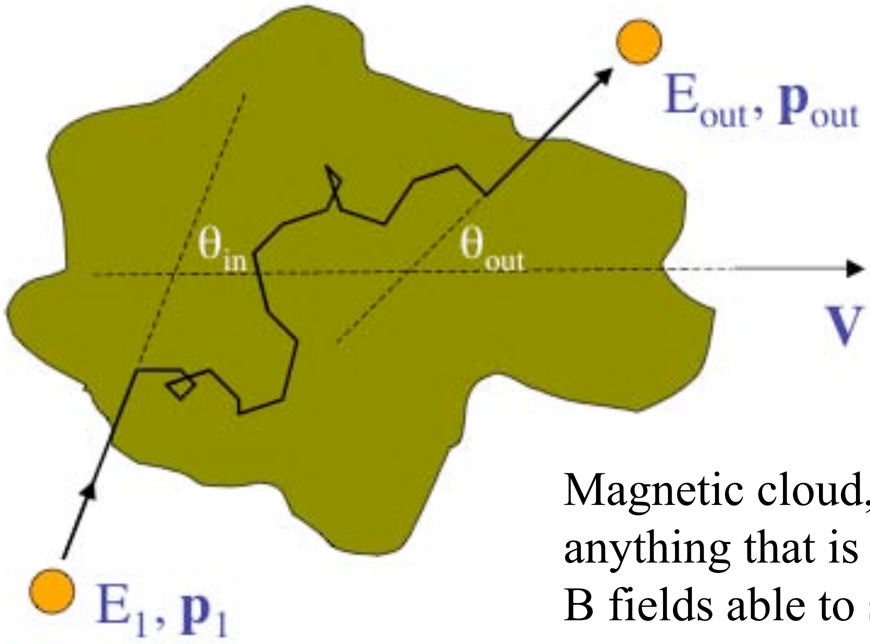


- In the interstellar or intergalactic medium, all the shocks are “collisionless”!

What is a collisionless shock?!

- SN shocks: V ~ 10 000 km/s ; E ~ 2 MeV/proton
- Stopping length ~ 1 kpc !
- But: interaction with B and E fields
  - $R_L = p/qB \sim 10^{-8}$ pc
  - Streaming with $v > c_A$ “impossible”.

"Acceleration by change-of-frame"

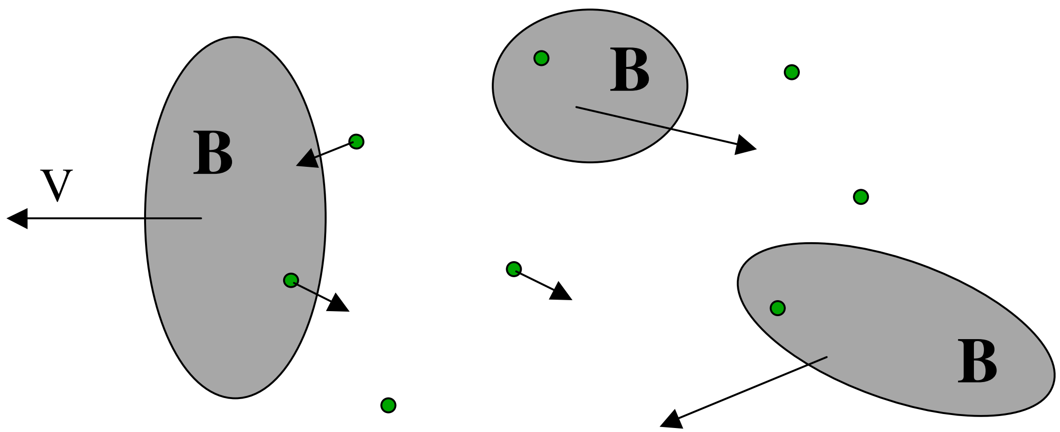


Magnetic cloud, MHD wave, etc.: anything that is moving are carrying B fields able to scatter particles…

$$\frac{\Delta E}{E} = \frac{\beta(\cos\theta'_{out} - \cos\theta_{in}) + \beta^2(1 - \cos\theta_{in}\cos\theta'_{out})}{1-\beta^2}$$

Second order, stochastic Fermi accéleration

- More head-on collisions than overtaking collisions



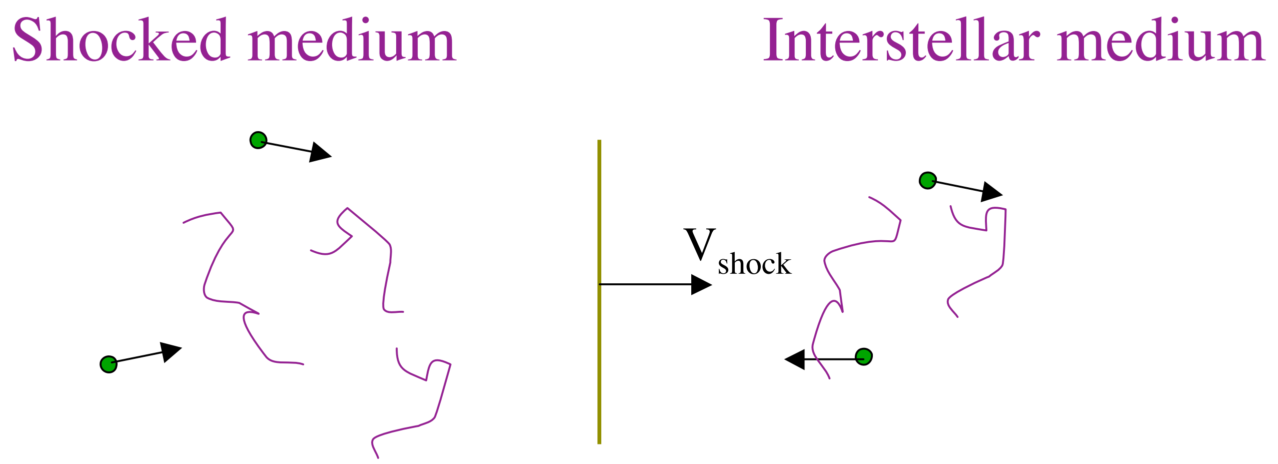
- The energy gain is only due to the difference in both collision rates, which is $\propto V/c$

- Energy change at each "collision": $\propto V/c$

- Resulting average energy gain: $\propto (V/c)^2$

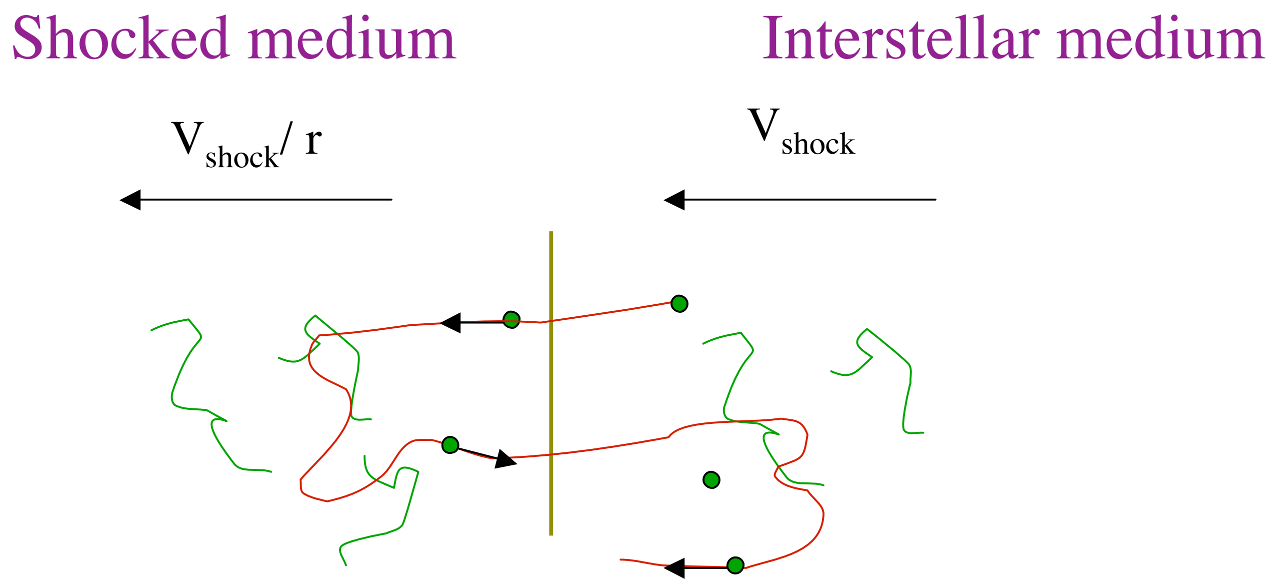
Diffusive shock acceleration

- Shock wave (e.g. supernova explosion)



- Magnetic wave production
  - **Downstream** : by the shock (compression, turbulence, hydro and MHD-instabilities, shear flows, etc.)
  - **Upstream** : by the cosmic rays themselves !
- → ‘isotropisation’ of the distribution (in local rest frame)

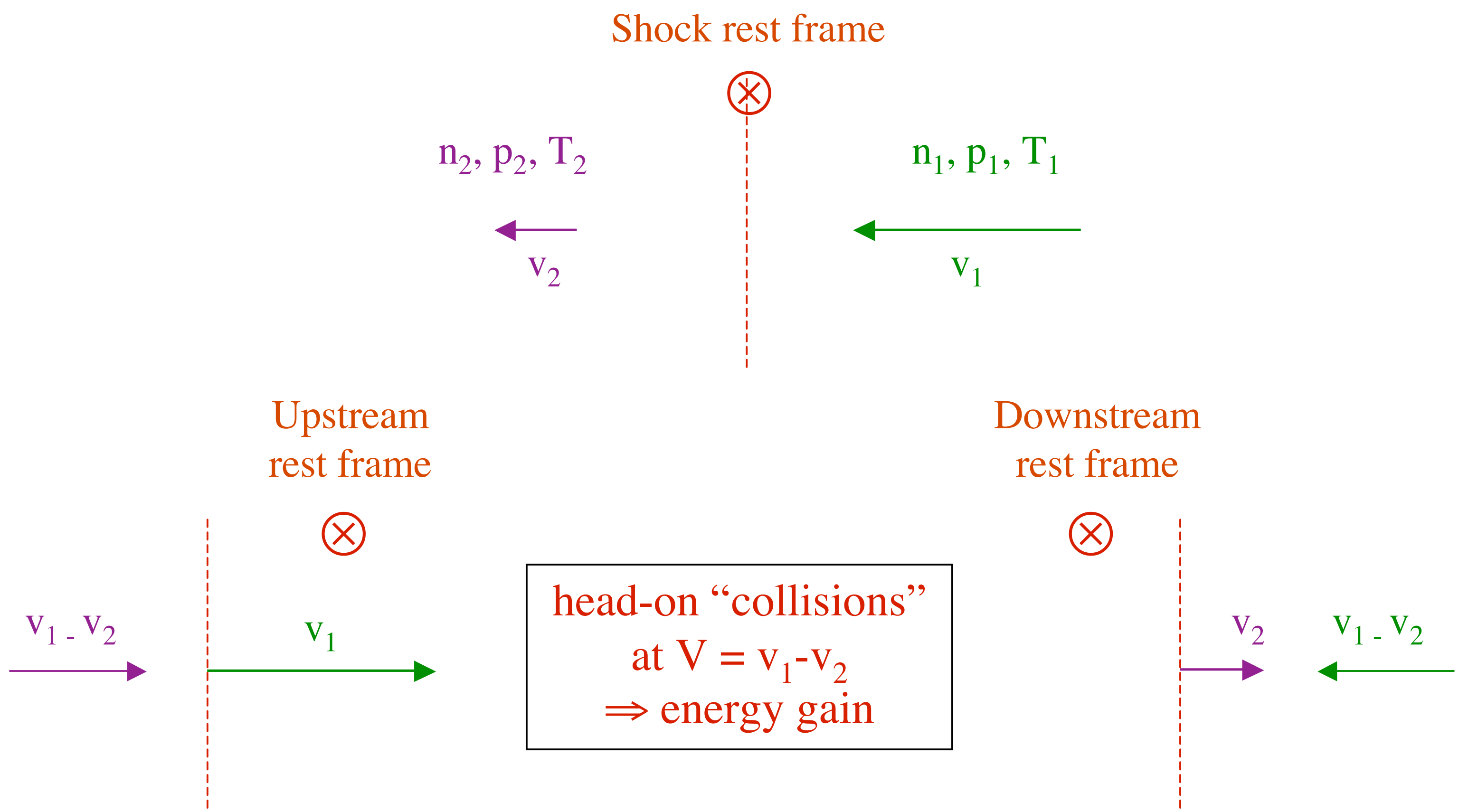
You're always lucky!



- At each crossing, the particle sees a 'magnetic wall' at V = (1-1/r)
- → only head-on collisions…

Shock acceleration

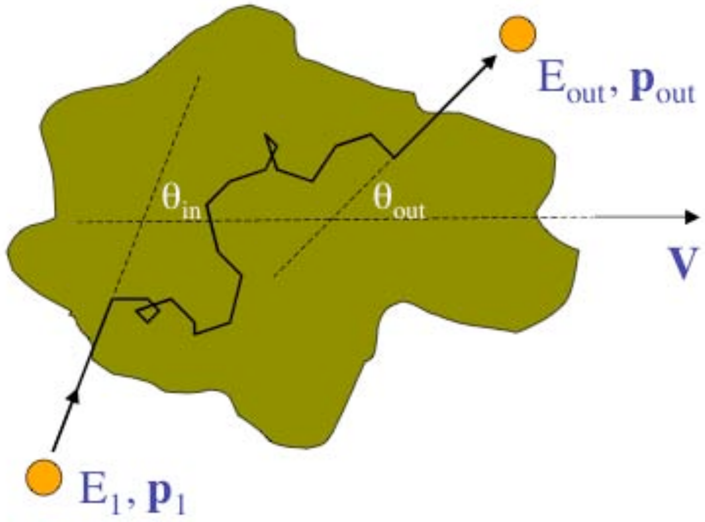
- 3 points of view...



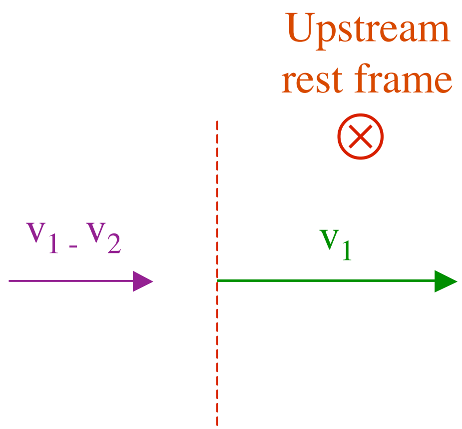
As seen from upstream, the downstream medium is coming closer

As seen from downstream, the upstream medium is coming closer

New version of the same calculation



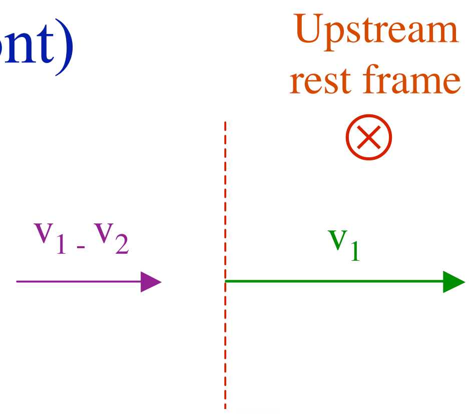
- Magnetic cloud replaced by the plasma on the other side of the shock front



$$\frac{\Delta E}{E} = \frac{\beta(\cos\theta'_{out} - \cos\theta_{in}) + \beta^2(1 - \cos\theta_{in}\cos\theta'_{out})}{1-\beta^2}$$

- Angular distribution at the crossing of the shock

(isotropy on either side of the shock front)



$$\mathrm{d}n(\theta) = \frac{n_0}{4\pi}\mathrm{d}\Omega(\theta) = \frac{1}{2}n_0 \sin\theta \mathrm{d}\theta$$

⬇

$$\mathrm{d}^4N = \underbrace{v\cos\theta}_{\text{relative velocity}}\mathrm{d}n(\theta)\mathrm{d}t\mathrm{d}^2S = \frac{1}{2}n_0 v\cos\theta\sin\theta\mathrm{d}\theta\mathrm{d}t\mathrm{d}^2S$$

- Average crossing angles

$$<\cos\theta_{\mathrm{in}}>= \frac{\int_{\pi}^{\pi/2}\cos\theta_{\mathrm{in}}^2\sin\theta_{\mathrm{in}}\mathrm{d}\theta_{\mathrm{in}}}{\int_{\pi}^{\pi/2}\cos\theta_{\mathrm{in}}\sin\theta_{\mathrm{in}}\mathrm{d}\theta_{\mathrm{in}}} = -\frac{2}{3}$$

and $<\cos\theta'_{\mathrm{out}}>= 2/3$

Average energy gain

$$\frac{\Delta E}{E} = \frac{\beta(\cos\theta'_{\text{out}} - \cos\theta_{\text{in}}) + \beta^2(1 - \cos\theta_{\text{in}}\cos\theta'_{\text{out}})}{1-\beta^2}$$

$$\Rightarrow \quad <\Delta E> = \frac{4}{3}\beta E = \frac{4}{3}\frac{\Delta v}{c}E$$

$$\Delta v = v_1 - v_2 = \frac{r-1}{r}V_{\text{choc}} \quad \Rightarrow \quad \frac{<\Delta E>}{E} = \frac{4}{3}\frac{r-1}{r}\frac{V_{\text{choc}}}{c}$$

Shock acceleration:

$$\frac{<\Delta E>}{E} = \frac{4}{3}\frac{r-1}{r}\frac{V_{\rm choc}}{c}$$

$\Delta E > 0$ — Acceleration process

$\Delta E \propto E$ $(\Delta E / E = \mathrm{cst})$ — Fermi process

$\Delta E / E \propto \Delta v / c$ — First order process

- Acceleration cycles

$$E_n = (1+k)^n E_0 \quad \text{where} \quad k = \frac{4}{3}\frac{r-1}{r}\beta_{\rm choc}.$$

Shock acceleration cycles

$$\frac{<\Delta E>}{E} = \frac{4}{3}\frac{r-1}{r}\frac{V_{\text{choc}}}{c}$$

- After *n* cycles

$$E_n = (1+k)^n E_0 \quad \text{where} \quad k = \frac{4}{3}\frac{r-1}{r}\beta_{\text{choc}}.$$

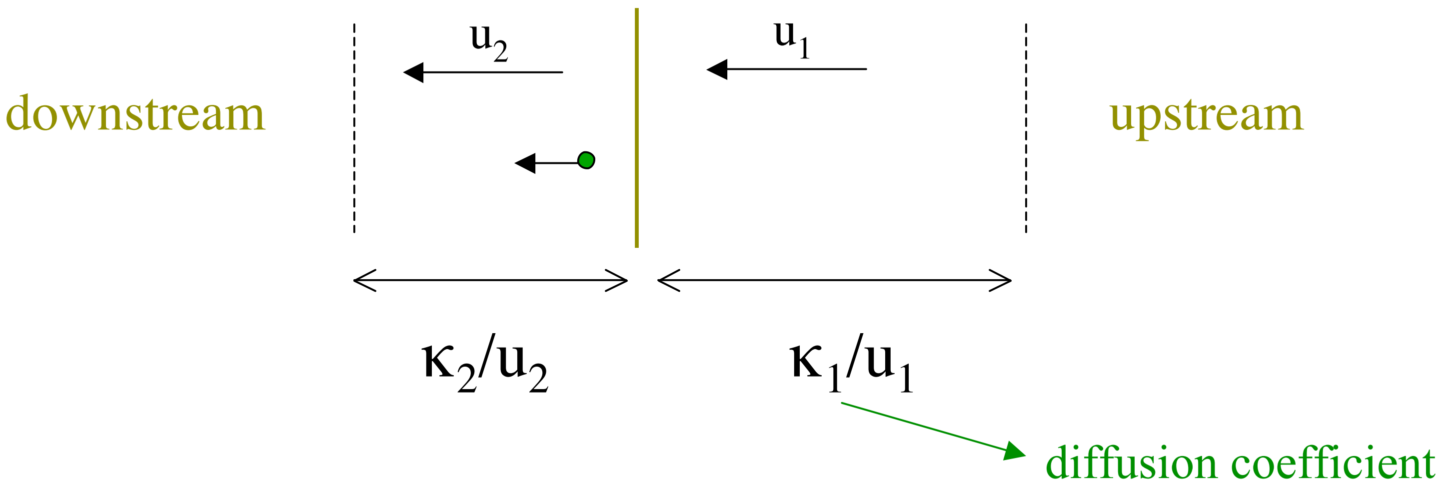
$$\Delta E = kE$$

- Cf. stochastic, second order Fermi acceleration process: exponential growth of the energy

- Acceleration rate: $\mathrm{d}E/\mathrm{d}t = E/\tau_{\text{acc}}$ with τ_{acc} independent of E

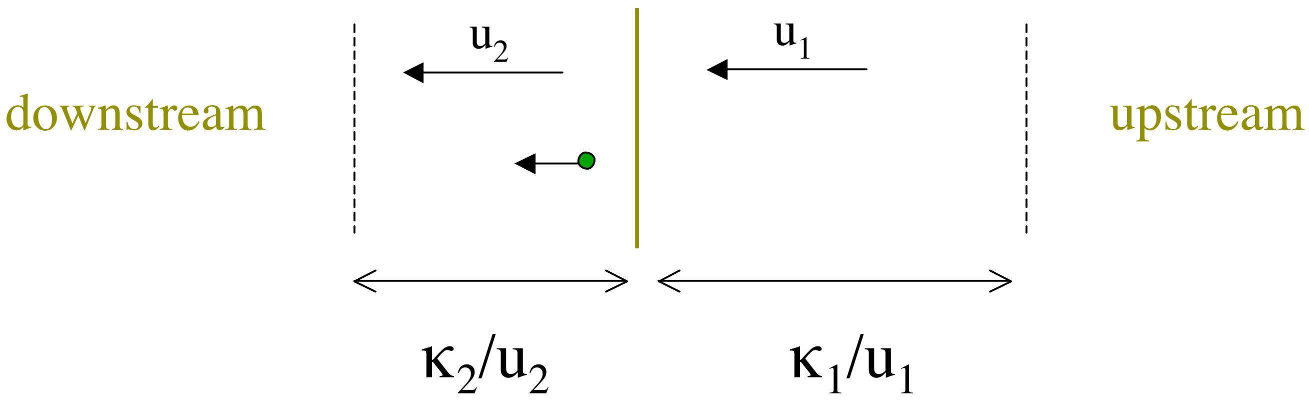
- Escape rate independent of E $\Rightarrow$ $\mathrm{d}P_{\text{esc}} = \mathrm{d}t/\tau_{\text{esc}}$

Acceleration rate



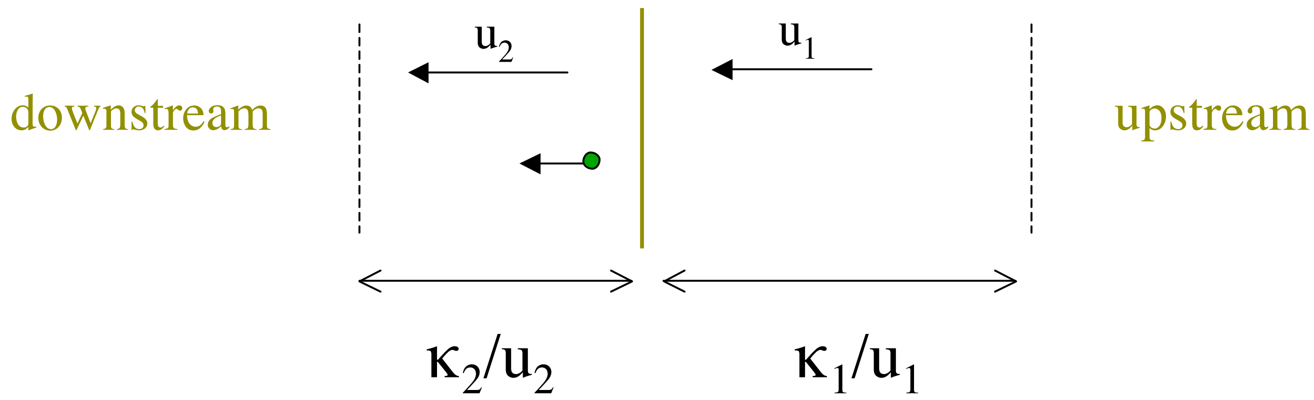
- Time to complete one cycle:
  - Confinement distance: κ/u
  - Average time spent upstream: $t_1 \approx 4\kappa / cu_1$
  - Average time spent upstream: $t_2 \approx 4\kappa / cu_2$

Acceleration rate



- Bohm limit: $\kappa = r_g v/3 \sim E\beta^2/3qB$
  - Proton at 10 GeV: $\kappa \sim 10^{22}$ cm^2/s
  - $\Rightarrow t_{cycle} \sim 10^4$ seconds !
- Finally, $\tau_{acc} \sim t_{cycle} \times V_s/c \sim$ 1 month !

Achtung !

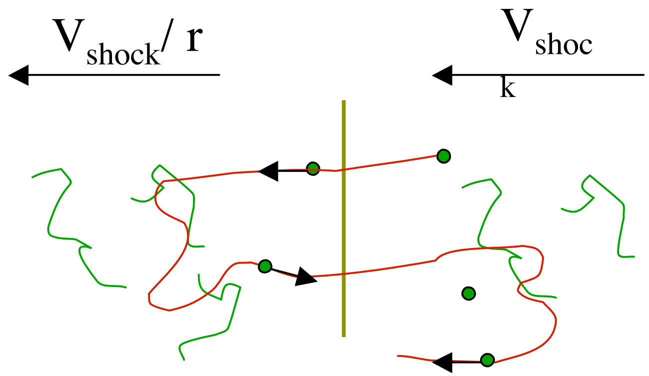


$$\tau_{acc} = \frac{E}{\Delta E / \Delta t} \quad \text{with} \quad \langle \Delta t \rangle = 4 \left(\frac{\langle \kappa_1 \rangle}{v_1 c} + \frac{\langle \kappa_2 \rangle}{v_2 c} \right)$$

and $\kappa \propto E^{\alpha}$

- The acceleration timescale depends on E
- But this was required to get a power-law spectrum (cf. Fermi miracle!)

Second ingredient: escape timescale



- Flux of particles crossing from upstream to downstream

$$\Phi_{\rm ud} = \frac{1}{4} n_0 v$$

- Flux of particles escaping downstream

$$\Phi_{\rm esc} = n_0 v_2 = \frac{n_0 V_{\rm choc}}{4}$$

- Escape probability

$$\mathcal{P}_{\rm esc} = \frac{\Phi_{\rm esc}}{\Phi_{\rm ud}} = \frac{4 v_2}{v} = \frac{4}{r}\frac{V_{\rm choc}}{v} = \frac{4}{r}\beta_{\rm choc}$$

- Escape timescale $\tau_{\rm esc} = \mathcal{P}_{\rm esc} \times \Delta t_{\rm cycle}$ also depends on E!

A power-law spectrum anyway!

- Both τ_{acc} and τ_{esc} depend on E, but τ_{acc}/τ_{esc} does not!

- So let's forget about time, and think in terms of cycles…
  - Both the relative energy gain and the escape probability **per cycle** are independent of E: this is enough!

$$\mathcal{P}_{\mathrm{esc}} = \frac{\Phi_{\mathrm{esc}}}{\Phi_{\mathrm{ud}}} = \frac{4v_2}{v} = \frac{4}{r}\frac{V_{\mathrm{choc}}}{v} = \frac{4}{r}\beta_{\mathrm{choc}}$$

$$\Delta E = kE \qquad \text{where} \qquad k = \frac{4}{3}\frac{r-1}{r}\beta_{\mathrm{choc}}$$

Resulting energy spectrum

- Return probability: $\mathcal{P}_{\rm ret} = 1 - \mathcal{P}_{\rm esc}$
- Remaining number of particles after n cycles:

$$N_n = N_0 \mathcal{P}_{\rm ret}^n = N_0 (1 - \mathcal{P}_{\rm esc})^n$$

- Energy after n cycles $E_n = (1+k)^n E_0$

$$n = \frac{\log(E/E_0)}{\log(1+k)} \quad \Longrightarrow \quad N(\geq E) = N_0 (1 - \mathcal{P}_{\rm esc})^{\frac{\log(E/E_0)}{\log(1+k)}}$$

$$\rightarrow \quad N(\geq E) = N_0 \left(\frac{E}{E_0}\right)^{\frac{\log(1-\mathcal{P}_{\rm esc})}{\log(1+k)}} \quad \rightarrow \quad N(E)$$

Resulting energy spectrum

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$$\Rightarrow \quad N(\geq E) = N_0 \left(\frac{E}{E_0}\right)^{\frac{\log(1-\mathcal{P}_{\rm esc})}{\log(1+k)}} \quad \Rightarrow \quad N(E) = \left|\frac{\mathrm{d}N(\geq E)}{\mathrm{d}E}\right|$$

Universal power-law index

- One obtains:

$$N(E) = (x-1)\frac{N_0}{E_0}\left(\frac{E}{E_0}\right)^{-x} \quad \text{with} \quad x = 1 - \frac{\ln(1-\mathcal{P}_{esc})}{\ln(1+k)}$$

- For a non-relativistic shock
  - $P_{esc} << 1$
  - $\Delta E/E << 1$

  ⇒ $$x = 1 + \frac{\mathcal{P}_{esc}}{k} = \frac{r+2}{r-1}$$

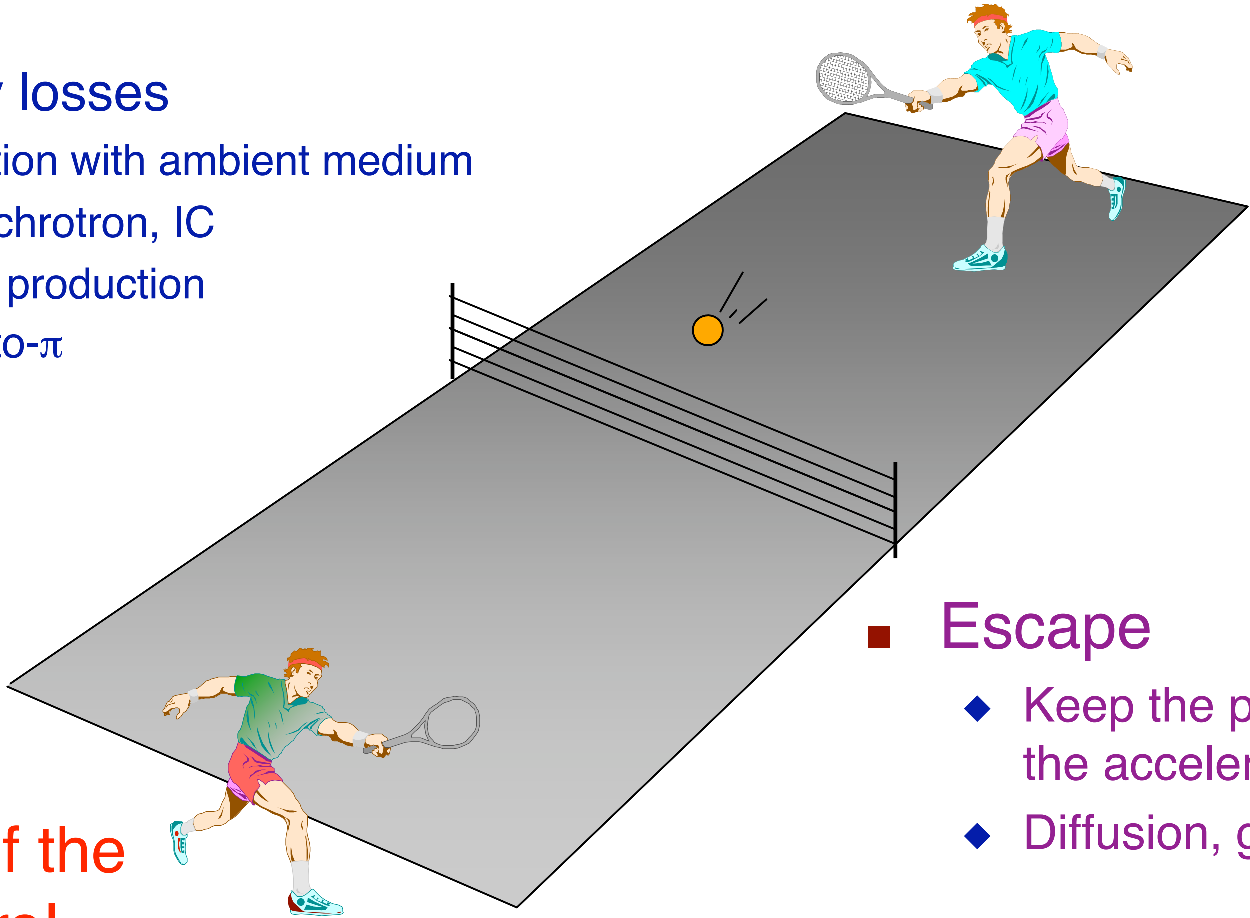
- … where 'r' is the shock compression ratio
  - $r = \gamma+1/\gamma-1$ for a strong shock

- For a monoatomic or fully ionised gas, $\gamma=5/3$

⇒ **x = 2, compatible with observations**

Maximum energy achievable?

- Energy losses
  - Friction with ambient medium
  - Synchrotron, IC
  - Pair production
  - photo-π



- Escape
  - Keep the particle in the accelerator
  - Diffusion, gyroradius
- Age of the players!
- Destruction
  - Photo-disintegration

Fermi 1 vs Fermi 2

- Acceleration timescale $\tau_{\rm acc} = \dfrac{\Delta t_{\rm coll}}{\langle \Delta E/E \rangle_{\rm coll}}$

- Second order (stochastic) Fermi acceleration

$\Delta t_{\rm coll} \sim \tau_{\rm s}$ and $<\Delta E/E>_{\rm coll} \sim \beta_{\rm A}^2$ with $\beta_{\rm A} \equiv v_{\rm A}/c$

so $\boxed{\tau_{\rm acc,II} \sim \tau_{\rm s}/\beta_{\rm A}^2}$

- First order (shock) Fermi acceleration

$\Delta t_{\rm coll} \sim \kappa/V_{\rm choc}c$ and $<\Delta E/E>_{\rm coll} \sim \beta_{\rm choc}$ with $\beta_{\rm choc} = V_{\rm choc}/c$

so $\boxed{\tau_{\rm acc,I} \sim \tau_{\rm s}/\beta_{\rm choc}^2}$