Total foreground maps from Planck by Wiener filtering in the needlet domain.

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We present work on the decomposition of Planck channel maps into three contributions: CMB, foreground emission and noise. Two key features are the use of a flexible non parametric model of the foreground correlation structure and the use of needlets (spherical wavelet) to capture the distribution of emissions across the sky and across multipoles. The model is fitted in the maximum likelihood sense using the SMICA framework. From the resulting fit, one then builds Wiener filters yielding components with minimum mean square error. In particular we obtain maps of the total foreground emission at each Planck frequency with minimum contamination by CMB and noise.

Needlets (Spherical wavelets).

Needlets are spherical wavelets. A spherical map is represented on a needlet frame by a set of coefficients β_{jk} where j indexes the needlet scale (corresponding to a range of multipoles) and k indexes a direction in the sky. Needlet coefficients are thus localized both in real and harmonic spaces. Needlet statistics are covariance matrices $[\widehat{R}_{jz}]_{ab} = \langle \beta^a_{jk} \beta^b_{jk} \rangle_z$ where a and b are channel indices, j is the needlet scale and $\langle \cdot \rangle_z$ is the average over a region of the sky indexed by z.

Non parametric correlation model.

The covariance matrix R_{jz} of the observations for wavelet scale j and in zone z is modeled as sum of contributions from CMB, foregrounds and noise:

$R_{jz} = R^{cmb}_{jz} + R^{fg}_{jz} + N_{jz}$ with the following constraints:

- $R^{cmb}_{jz} = aa^T c_j$: the CMB has a know emission law a but unknown position-independent power c_j at scale j.
- $R^{fg}_{jz} = AP_{jz}A^T$ where matrix A has N columns and P_{jz} is a positive matrix depending on scale j and zone z. This 'N-dimensional component' capture foreground emission 'non parametrically'.
- N_{jz} is only constrained to be diagonal. Its diagonal elements capture the average noise level in each channel, in each scale j and in each zone z.

That model is fitted by maximum likelihood using the SMICA framework.

Needlet filtering.

Once the model $R_{jz} = aa^T c_j + AP_{jz}A^T + N_{jz}$ has been fitted to $\widehat{R}_{jz}$, one can build localized Wiener filters, adapted to the local SNR. Three variations are:

$$W^1_{jz} = (A^T N^{-1}_{jz} A + P^{-1}_{jz})^{-1} A^T N^{-1}_{jz}$$
$$W^2_{jz} = (A^T N^{-1}_{jz} A + P^{-1}_{j})^{-1} A^T N^{-1}_{jz}$$
$$W^3_{jz} = (A^T N^{-1}_{jz} A)^{-1} A^T N^{-1}_{jz}$$

The usual Wiener filter solution (W^1_{jz}) uses the fitted galactic power and its response to the galactic component differs in each scale and zone. For the sake of processing characterization, one can think of some variation using a mean (over the sky) galactic power value (W^2_{jz}), or adding the constraint of a filter that keeps the component unchanged which gives the high-SNR-limit version (W^3_{jz}).

Minimum contamination.

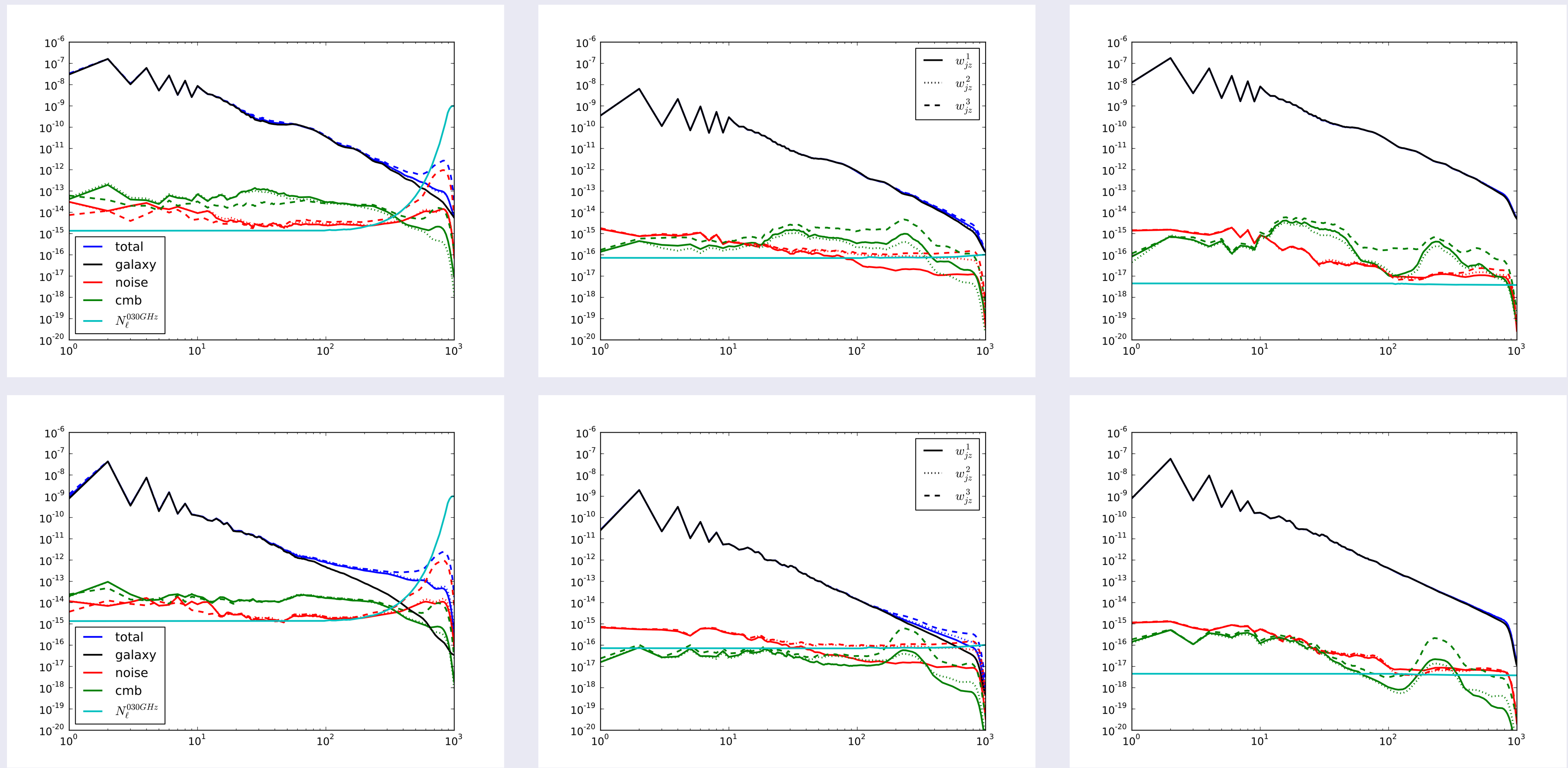

Figure: CMB and noise contamination estimated using a thin galactic mask (99% of the sky) (top) or a large galactic mask (75% of the sky) (bottom) for 3 Planck channels: 30GHz, 143GHz and 857GHz (from left to right). The input noise power spectrum is also plotted for each frequency.

By looking at the power spectra of the residual in each case, one can assess the performance of the different Wiener filters. The two first candidates show similar results, indicating that the best performance do not mean loss of characterisation. On the other hand, the constrained candidate shows higher level of contamination in poor SNR regime.

Galactic maps.

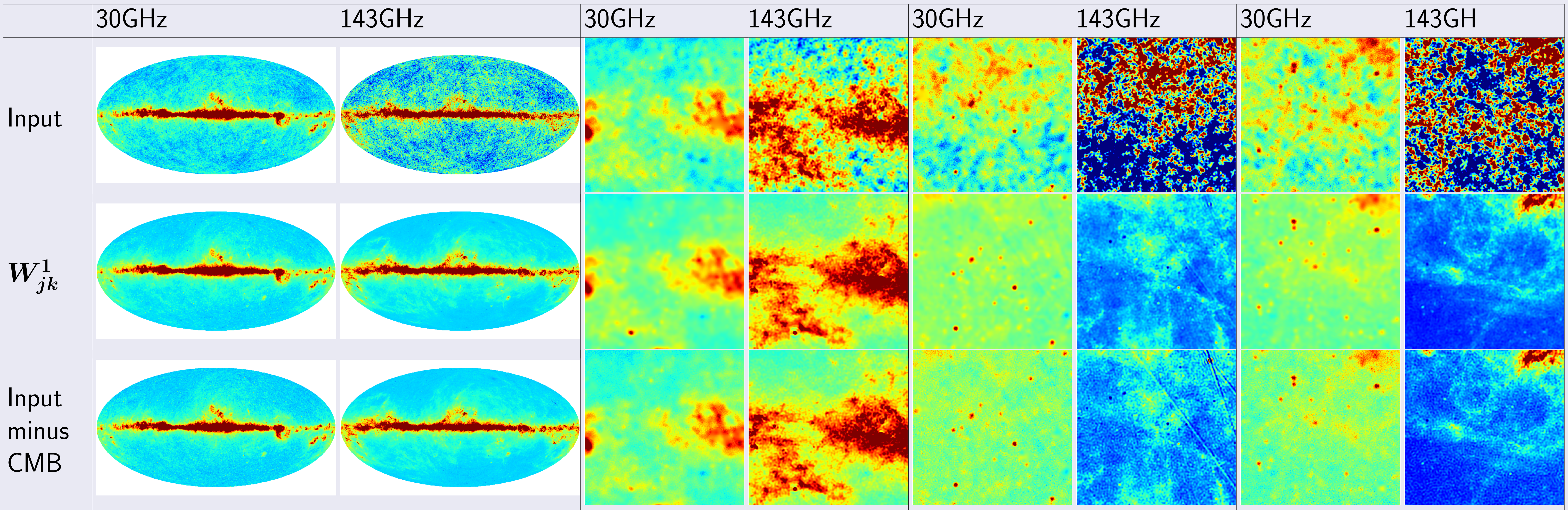

Figure: Full sky maps at 30 GHz and 143 Ghz and patches of 25 square degrees centered at [160,0.5], [70,-30] and [200,-50]. Color scales are constant in each column.

Some results with simulations based on the Planck Sky Model. All Planck channels are used but we show only results in in the 30 GHz and 143 GHz channels. First row: input maps. Second row: foreground estimates using the W^1_{jk} filter. Last row: foreground estimation by CMB subtraction (the CMB is estimated in the same framework). CMB subtraction leaves more noise than direct foreground estimation.