Putting it Together

Cosmological Constraints & Future Research Directions

Cosmological Parameters

- Homogeneous background parameters
  - Hubble parameter: overall scale
  - Density parameters
  - Dark energy equation-of-state
- Perturbation parameters
  - Density perturbations
  - Gravity wave perturbations

Background Parameters

Find most in the Friedmann Equation

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_M(1+z)^3 + \Omega_K(1+z)^2 + \Omega_W\right] = H_0^2 E^2(z)$$

Hubble parameter today: $H_0 = h \times 100 km/s/Mpc$

Density parameters:

$$\Omega_M \equiv \frac{8\pi G\rho_M}{3H_0^2} = \Omega_X + \Omega_B$$ Total matter density

$$\Omega_K \equiv -\frac{\mathrm{K}}{H_0^2}$$ Curvature term

$$\Omega_W \equiv \frac{8\pi G\rho_W}{3H_0^2}$$ Dark energy

1 Constraint:

$$\boxed{\Omega_M + \Omega_W + \Omega_{\mathrm{K}} = \Omega_{TOT} + \Omega_{\mathrm{K}} = 1}$$

Dark Energy

Equation-of-state $p = w\rho$

$$w(a) = w_0 + (1-a)w_a$$

Cosmological constant: $w = -1$

Evolution: $\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0$

$\rho(a) = \rho_0 a^{-3(1+w)}$ w=const < -1/3

$\rho(a) = \rho_0 a^{-3} e^{-3\int d\ln a\, w(a)}$ In general

Perturbation Parameters

- Simple single field inflation
  - Density (scalar) perturbations
  - Tensor (gravity wave) perturbations
- Slow-roll approximation

E.g., real scalar field: $\Im = \frac{1}{2}\partial^{\mu}\varphi\partial_{\mu}\varphi - V(\varphi)$

$$\varepsilon_V = \frac{M_{pl}^2}{2}\left(\frac{V'}{V}\right)^2 \qquad \eta_V = M_{pl}^2\left(\frac{V''}{V}\right) \qquad \xi_V = M_{pl}^4\left(\frac{V'V'''}{V^2}\right)$$

Scalar Perturbations

Density perturbations:
(see Peiris et al. 2003 - WMAP)

$$\Delta_R^2(k) = \frac{k^3 P_R(k)}{2\pi^2} = \Delta_R^2(k_0)\left(\frac{k}{k_0}\right)^{n_s - 1 + \alpha_s \ln\left(\frac{k}{k_0}\right)}$$

$$\Delta_R^2(k_0) = \frac{V}{24\pi^2 M_{pl}^2}\left(\frac{1}{\varepsilon_V}\right)\Bigg|_{k=k_0}$$

Scalar amplitude (curvature R)

$$n_s = n_s(k_0) = 1 - 6\varepsilon_V + 2\eta_V\Big|_{k=k_0}$$

Scalar spectral index

$$\alpha_s = \frac{1}{2}\frac{dn_s}{d\ln k}\Bigg|_{k=k_0} = 8\varepsilon_V\eta_V - 12\varepsilon_V^2 - \xi_V\Big|_{k=k_0}$$

Scalar running index

$$k_0 = 0.002\, Mpc^{-1}$$

Pivot point

Tensor Perturbations

Gravity waves:

$$\Delta_t^2(k) = r\Delta_t^2(k_0)\left(\frac{k}{k_0}\right)^{n_T + \alpha_t \ln\left(\frac{k}{k_0}\right)}$$

$$r = 16\varepsilon_V\Big|_{k=k_0}$$ Tensor amplitude

$$n_t = n_t(k_0) = -2\varepsilon_V\Big|_{k=k_0}$$ Tensor spectral index

$$\alpha_t = \frac{1}{2}\frac{dn_t}{d\ln k}\Bigg|_{k=k_0} = 2\varepsilon_V\eta_V - 4\varepsilon_V^2\Big|_{k=k_0}$$ Tensor running

Cosmological Parameters

Minimal model

Parameter	Description	Definition
H_0	Hubble expansion factor	$H_0 = 100h$ Mpc^{-1}km s^{-1}
ω_b	Baryon density	$\omega_b = \Omega_b h^2 = \rho_b/1.88 \times 10^{-26}$ kg m^{-3}
ω_c	Cold dark matter density	$\omega_c = \Omega_c h^2 = \rho_c/18.8$ yoctograms/ m^{-3}
f_ν	Massive neutrino fraction	$f_\nu = \Omega_\nu/\Omega_c$
$\sum m_\nu$	Total neutrino mass (eV)	$\sum m_\nu = 94\Omega_\nu h^2$
N_ν	Effective number of relativistic neutrino species	
Ω_k	Spatial curvature	
Ω_{DE}	Dark energy density	For $w = -1$, $\Omega_\Lambda = \Omega_{DE}$
Ω_m	Matter energy density	$\Omega_m = \Omega_b + \Omega_c + \Omega_\nu$
w	Dark energy equation of state	$w = p_{DE}/\rho_{DE}$
$\Delta^2_\mathcal{R}$	Amplitude of curvature perturbations $\mathcal{R}$	$\Delta^2_\mathcal{R}(k = 0.002/Mpc) \approx 29.5 \times 10^{-10} A$
A	Amplitude of density fluctuations ($k = 0.002$/Mpc)	See Spergel et al. (2003)
n_s	Scalar spectral index at 0.002/Mpc	
α	Running in scalar spectral index	$\alpha = dn_s/d\ln k$ (assume constant)
r	Ratio of the amplitude of tensor fluctuations to scalar potential fluctuations at k=0.002/Mpc	
n_t	Tensor spectral index	Assume $n_t = -r/8$
τ	Reionization optical depth	
σ_8	Linear theory amplitude of matter fluctuations on 8 h^{-1} Mpc	
Θ_s	Acoustic peak scale (degrees)	see Kosowsky et al. (2002)
A_{SZ}	SZ marginalization factor	see appendix A
b_{sdss}	Galaxy bias factor for SDSS sample	$b = [P_{sdss}(k, z = 0)/P(k)]^{1/2}$ (constant)
C_{220}^{TT}	Amplitude of the TT temperature power spectrum at $\ell = 220$	
z_s	Weak lensing source redshift	

Spergel et al. 2007

Observational Techniques

- The Classics
- Cosmic Microwave Background
- The Quartet
- Others

Metric(s)

$$ds^2 = dt^2 - a^2(t)R^2\left[d\chi^2 + S_k^2(\chi)\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]$$

R = curvature radius

$S_{-1} = \sinh(x)$ ($R^2<0$) $S_0 = x$ ($R^2=0$) $S_1 = \sin(x)$ ($R^2>0$)

$$d\chi = \frac{dz}{H(z)}$$ Comoving distance

$$D_{ang}(z) = (1+z)^{-1} R S_k\left(\frac{\chi}{R}\right)$$ Angular diameter distance

$$D_{lum}(z) = (1+z)^2 D_{ang}(z)$$ Luminosity distance

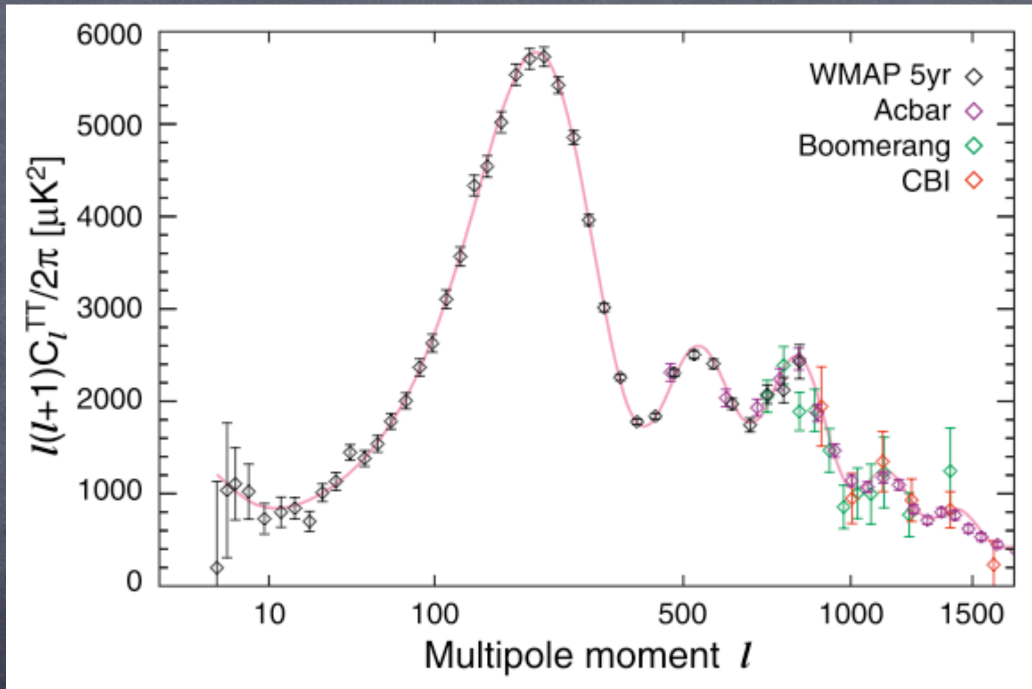
The Classics

- General shape of $P_{gal}(k)$
  - Equality epoch: $\Omega_M h^2$
- Redshift-space distortions
  - Linear growth factor: $D_{grow}(\Omega_M$, D.E., $\gamma_{gravity})$
- Galaxy cluster M/L & $f_{gas}(z)$
  - Ω_M, dark energy

CMB Anisotropies

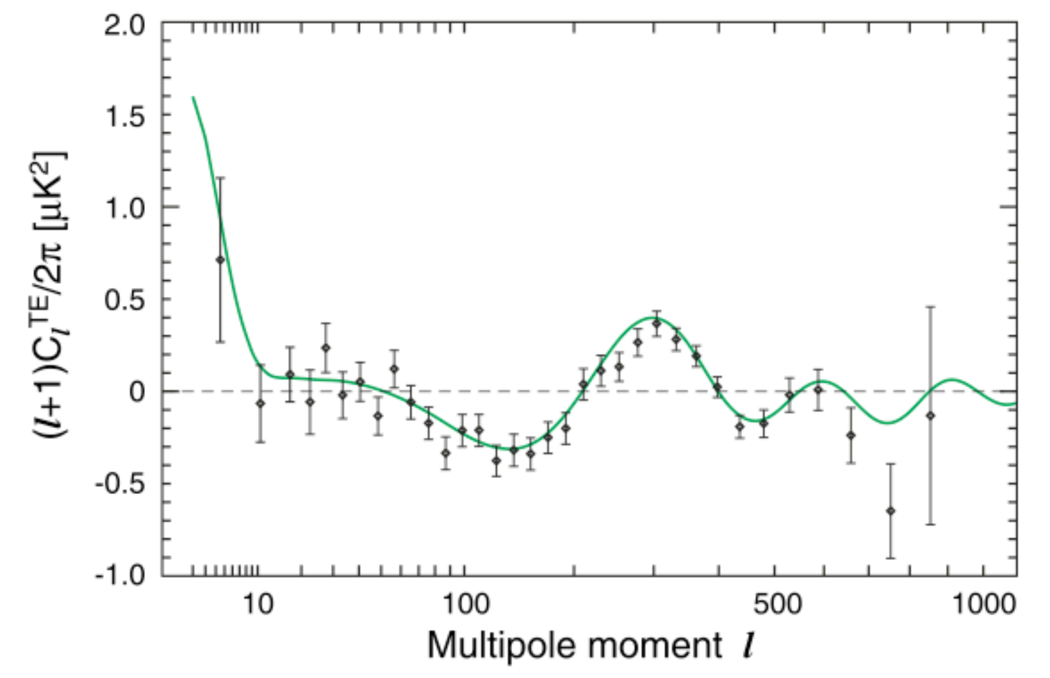
- Temperature anisotropies: TT
- Polarization anisotropies
  - TE, EE, BB
- Important points
  - Geometric degeneracy
  - Insensitive to dark energy (z=1000!)
- WMAP 5-year data: Komatsu et al. 2008

WMAP 5-Year Data



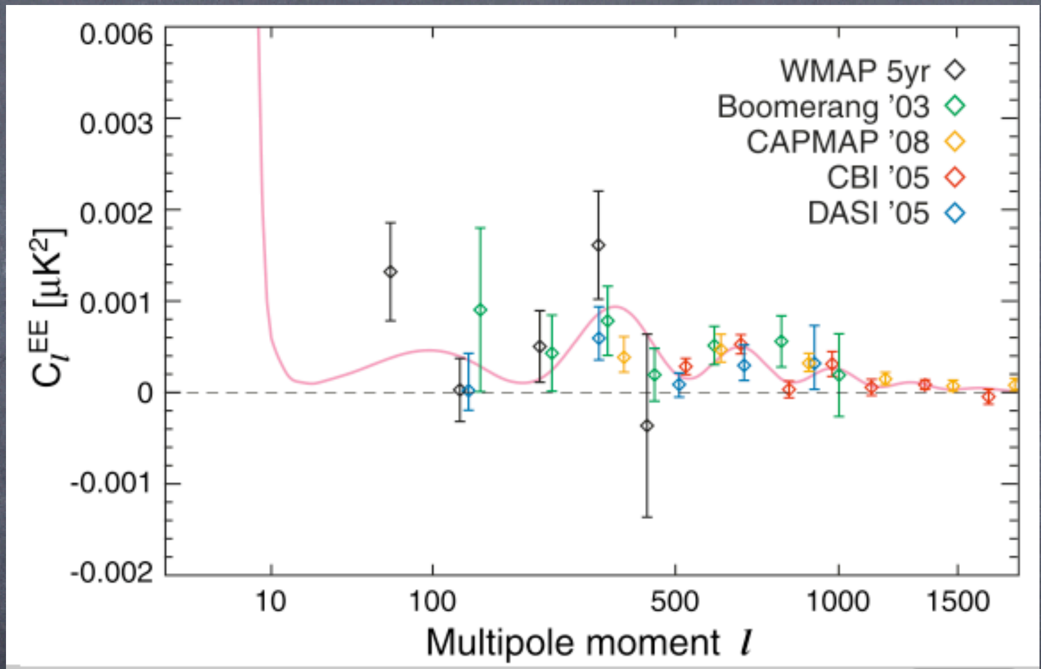
Nolta et al. 2008

WMAP 5-Year Data



Nolta et al. 2008

WMAP 5-Year Data



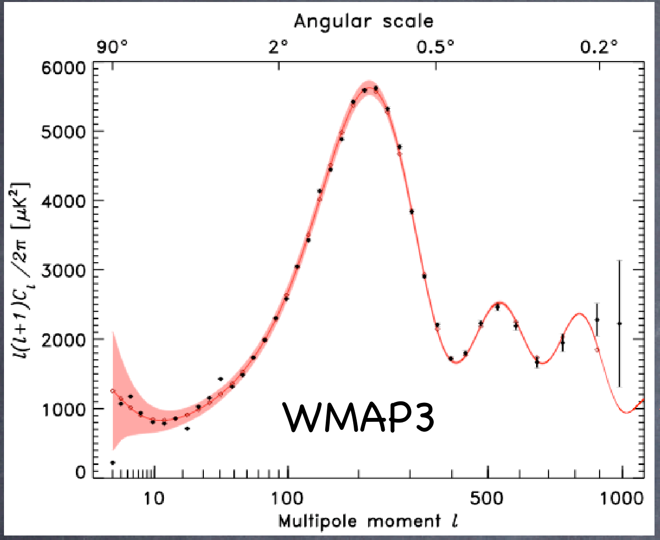
Nolta et al. 2008

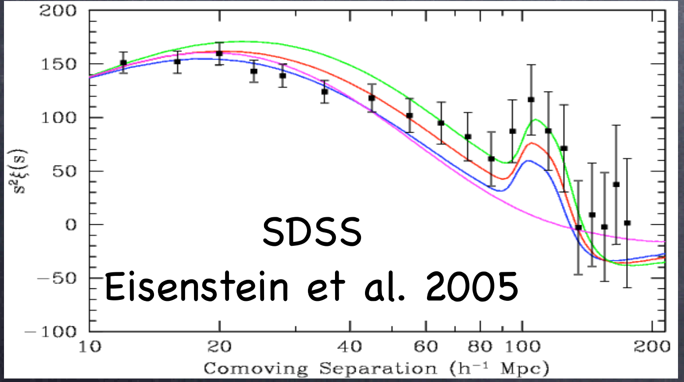
The Quartet

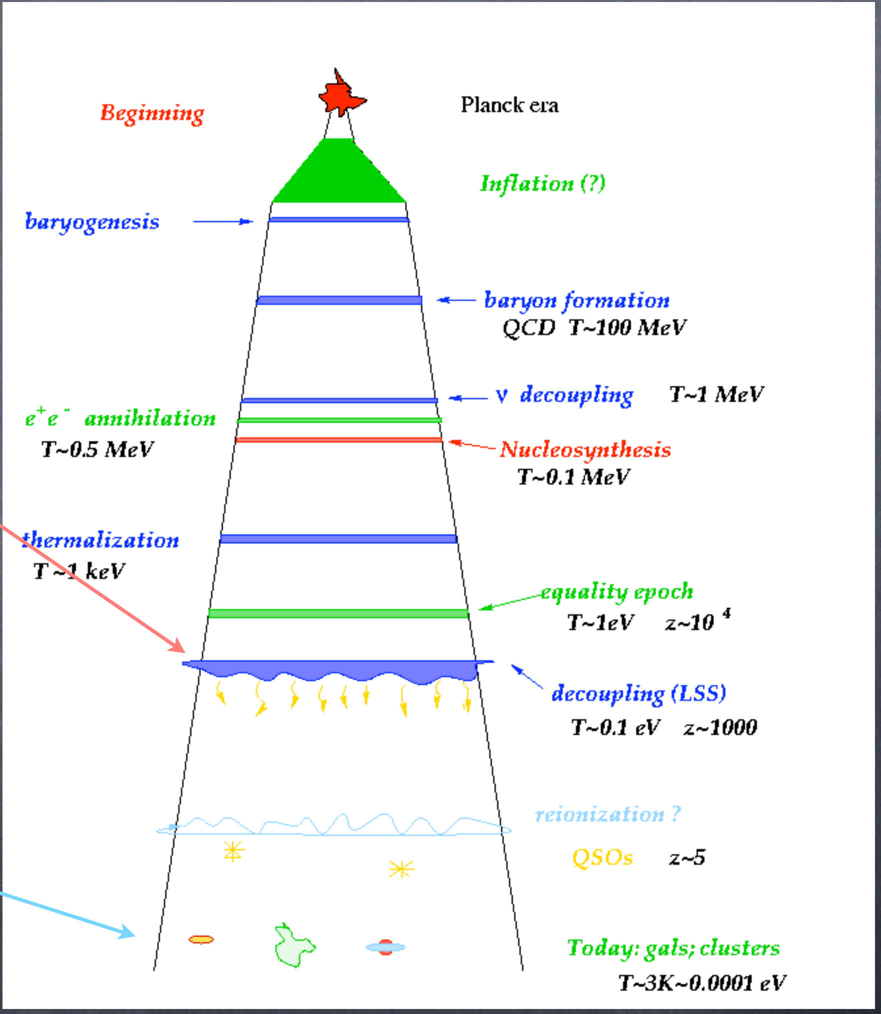
Low redshift - Dark Energy Probes

- SNIa distance measurements
- Baryon acoustic oscillations (BAO) in galaxy distribution
- Cosmic shear
- Galaxy cluster evolution

Baryon Acoustic Oscillations







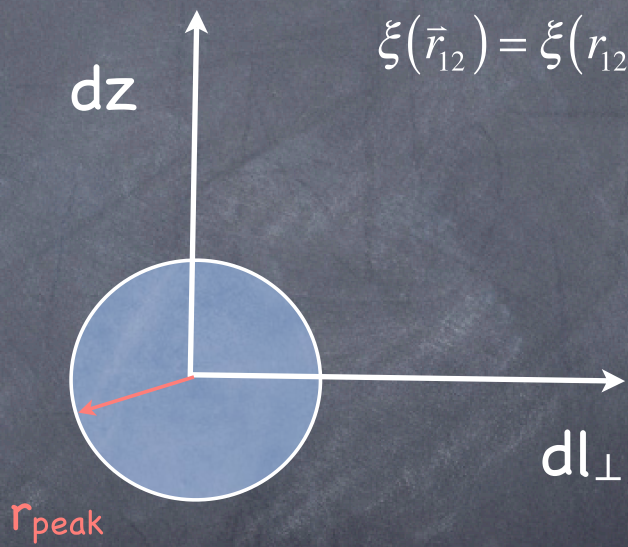
Using the BAO Scale

Cosmological Principle!

$$d\chi = \frac{dz}{H(z)}$$

$$D_{ang}(z) = (1+z)^{-1} R S_k\left(\frac{\chi}{R}\right)$$

$$\xi(\vec{r}_{12}) = \xi(r_{12})$$



$$H(z) = \frac{dz}{r_{peak}}$$

$$D_{ang}(z) = \frac{r_{peak}}{d\theta_{peak}}$$

Others

- Linear growth factor in redshift (e.g., DEEP2, VIMOS, ADEPT, SPACE)
- Late-time Sachs-Wolfe effect
  - Dark energy

State-of-the-Art

Following Komatsu et al. 2008

- WMAP 5-year analysis
- SNIa
- Baryon acoustic oscillations

FLAT Power-Law ΛCDM

WMAP5+SNIa+BAO

$\Omega_C h^2 = 0.1143 \pm 0.0034$

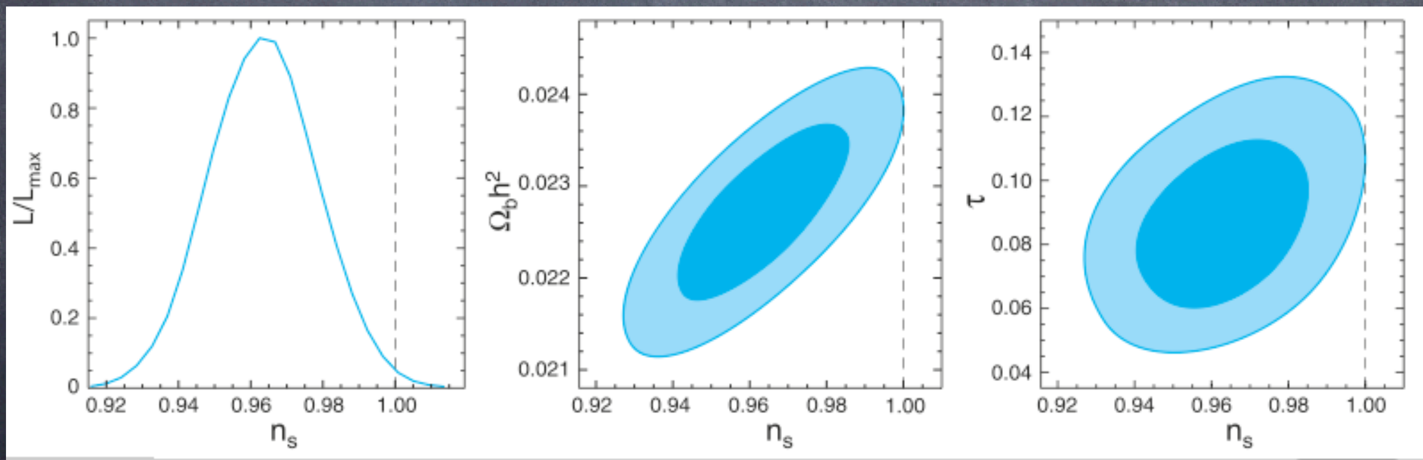
$\Omega_B h^2 = 0.02265 \pm 0.00059$

$\Omega_\Lambda = 0.721 \pm 0.015$

$\Delta_R^2(k_0) = \left(2.457 \pm^{0.092}_{0.093}\right) \times 10^{-9}$

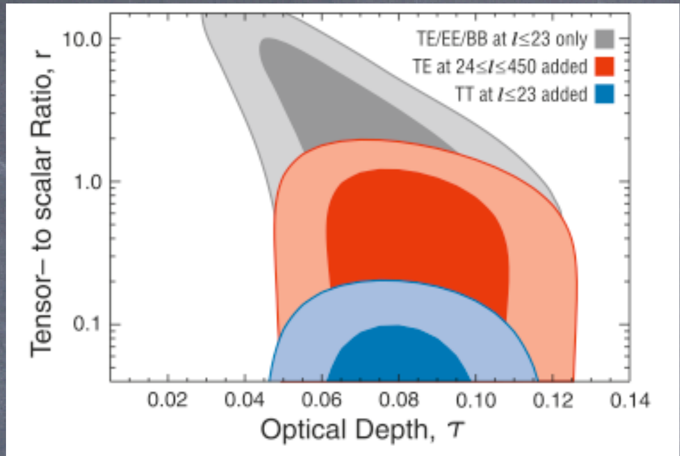
$n_s = 0.960 \pm^{0.014}_{0.013}$

$\tau = 0.817 \pm 0.026$

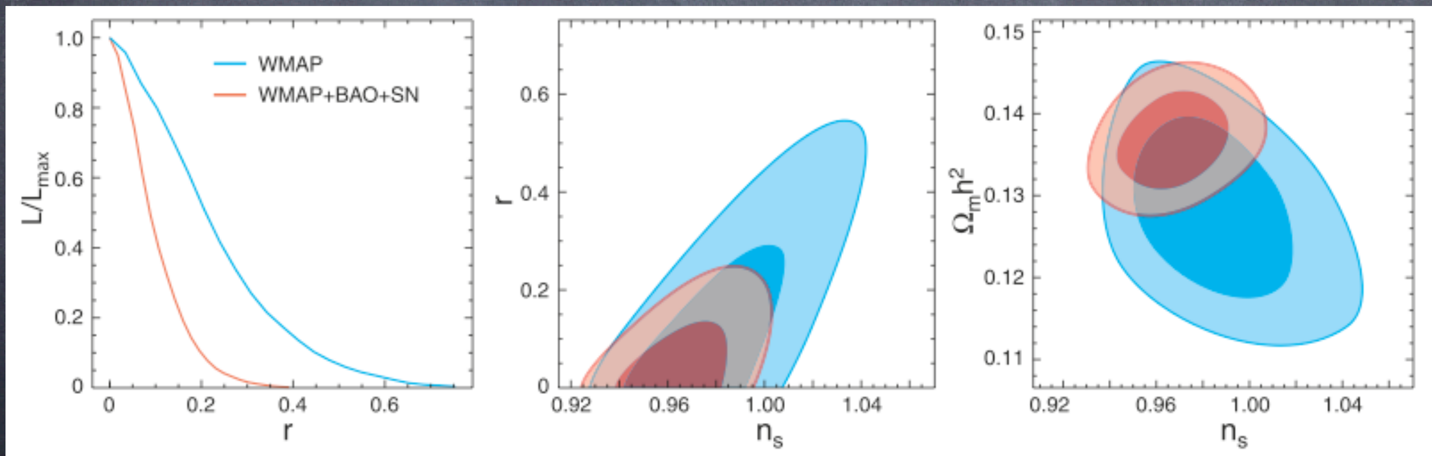


Komatsu et al. 2008

Tensor Perturbations

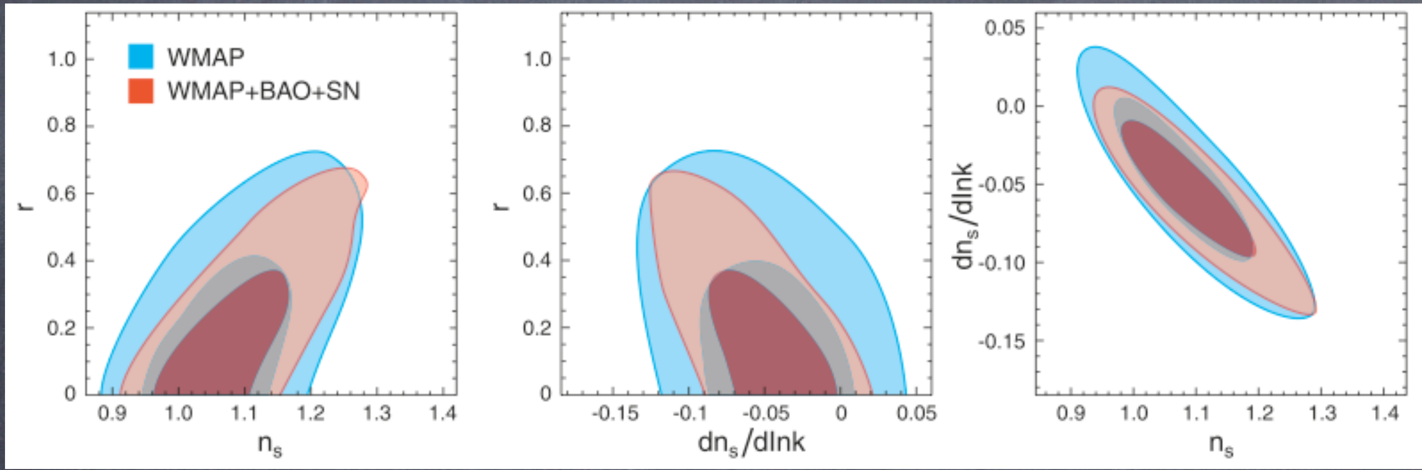


$r < 0.20$ (95%)



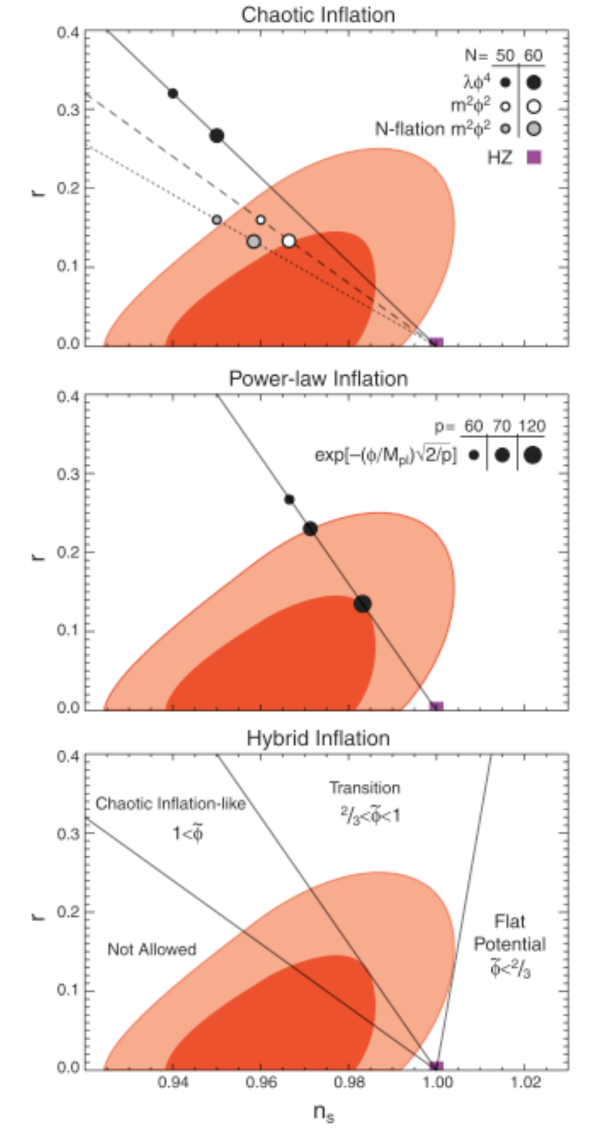
Komatsu et al. 2008

Scalar Running



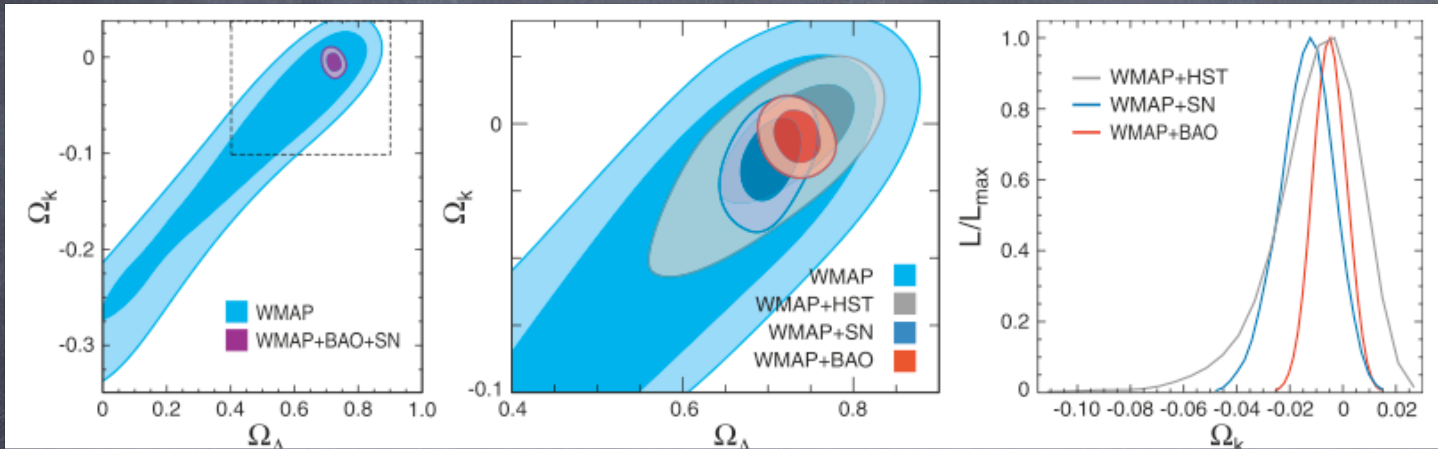
Komatsu et al. 2008

Constraints on Inflation



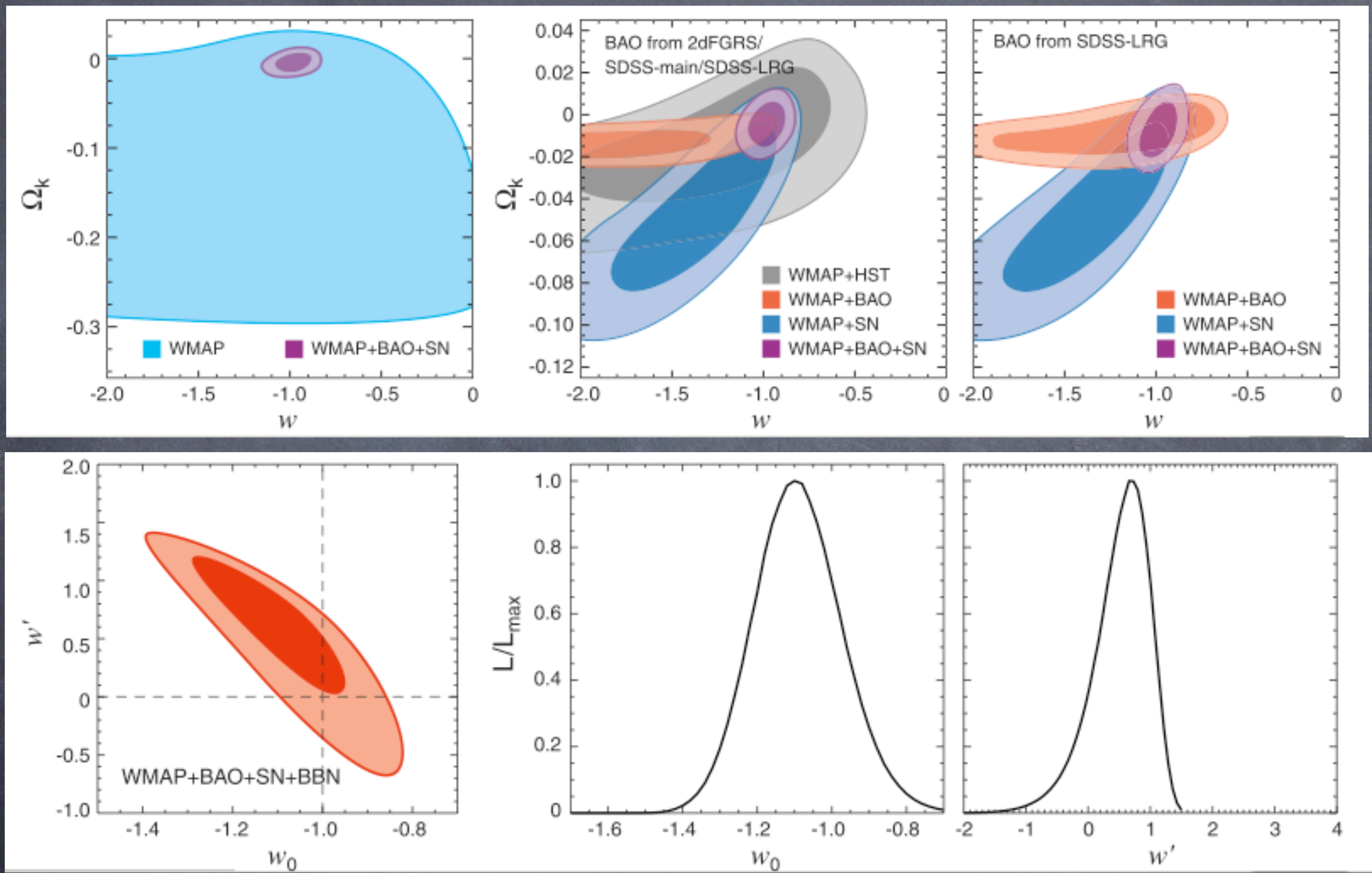
WMAP5+SNIa+BAO

Curvature



Komatsu et al. 2008

Dark Energy



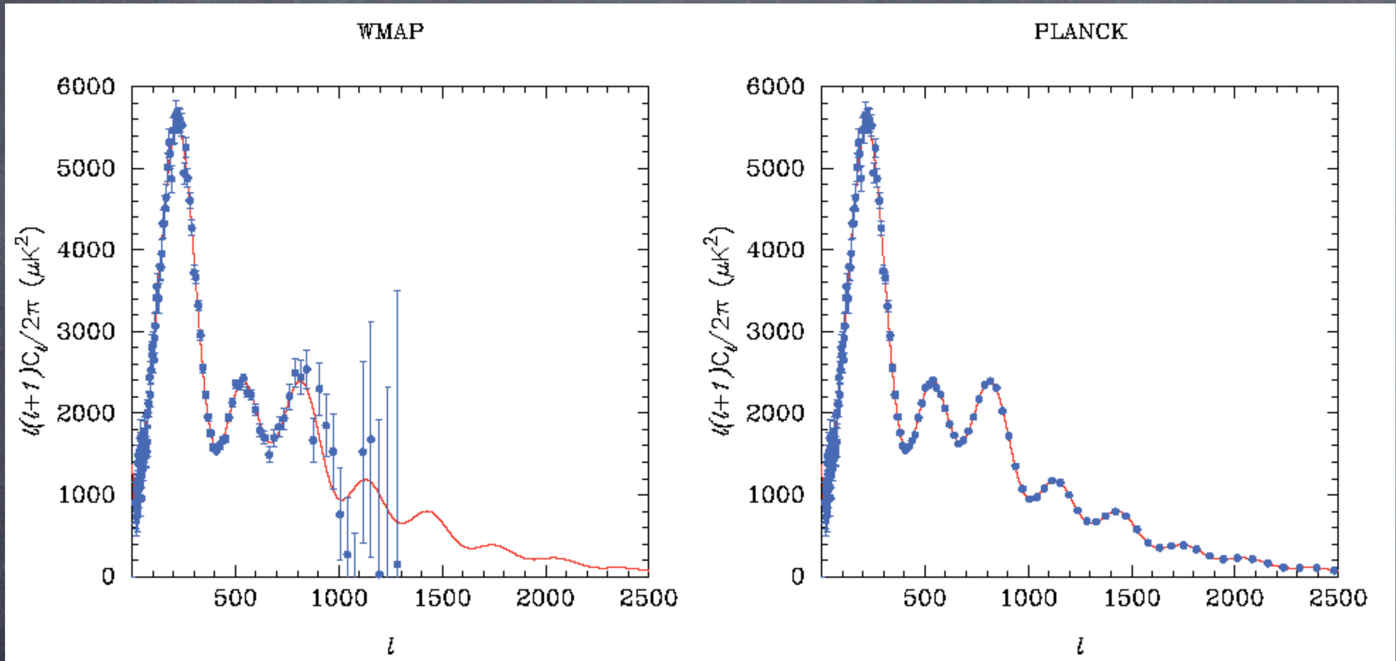
$w' = w_a$

Komatsu et al. 2008

Future Directions

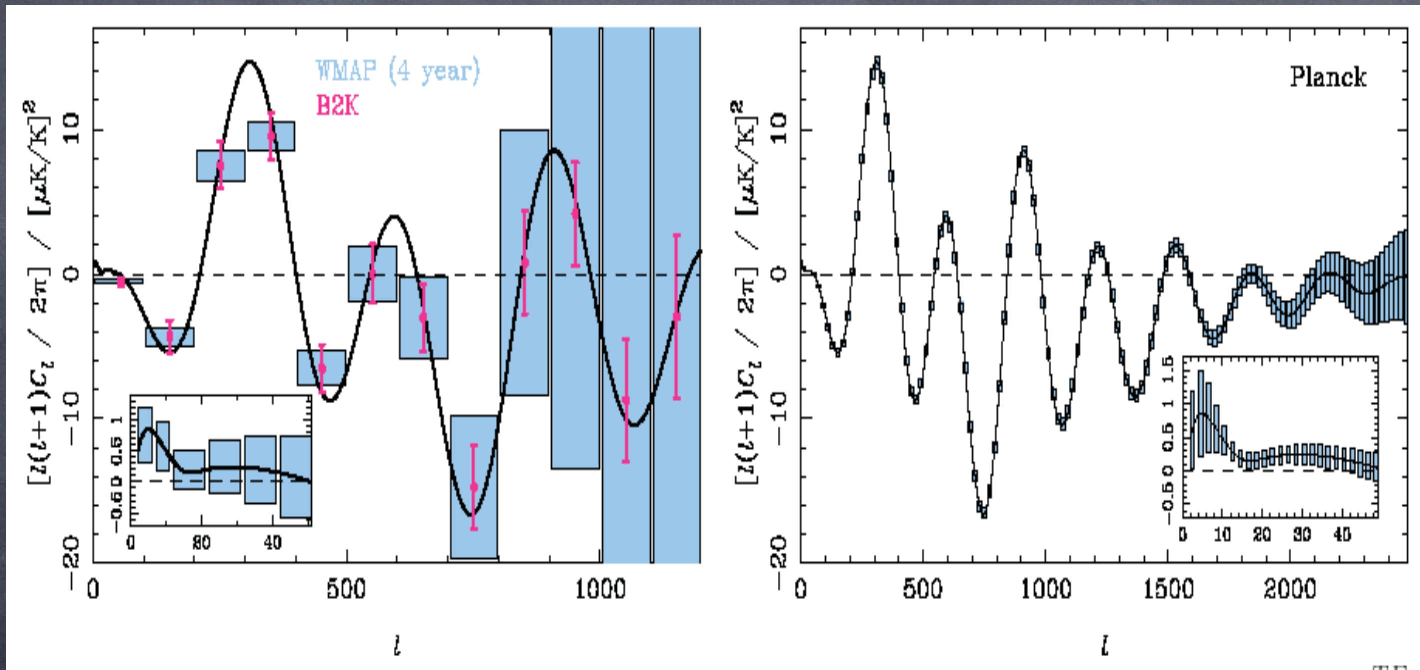
- CMB anisotropies
  - Planck mission: launch 31/10/2008
  - Polarization
- Dark energy

Planck Mission



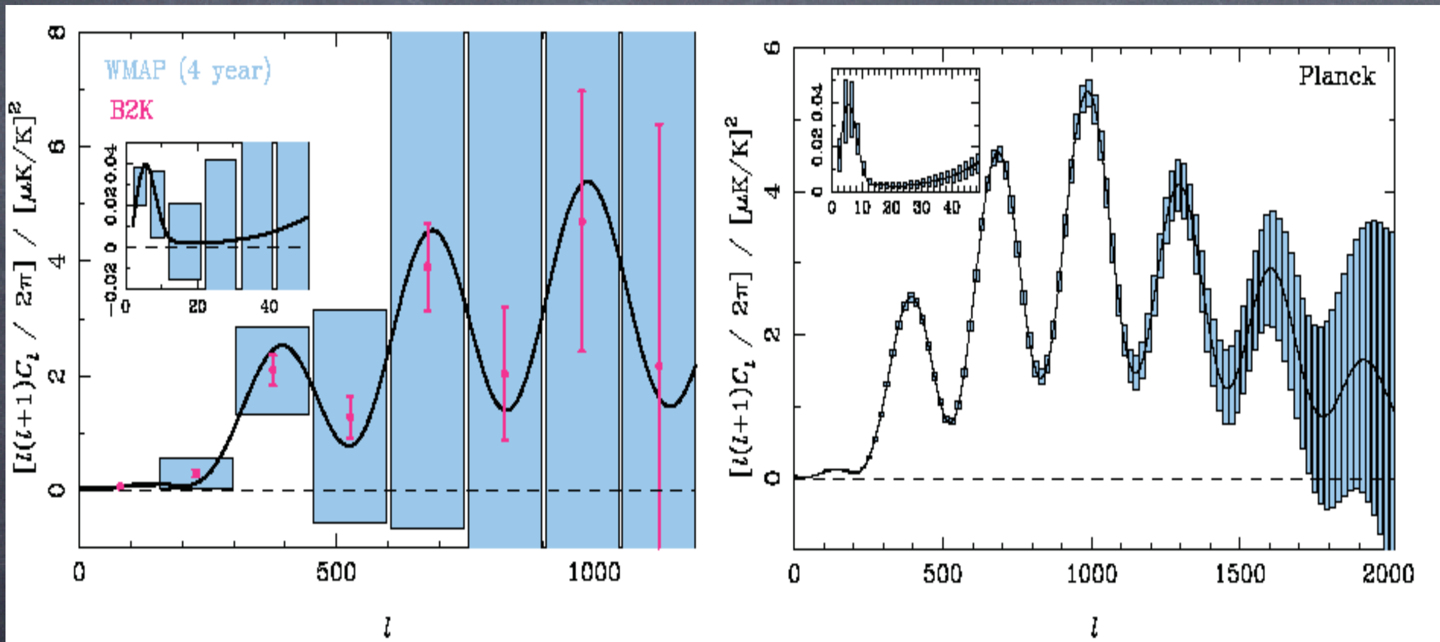
Planck Blue Book

Planck Mission



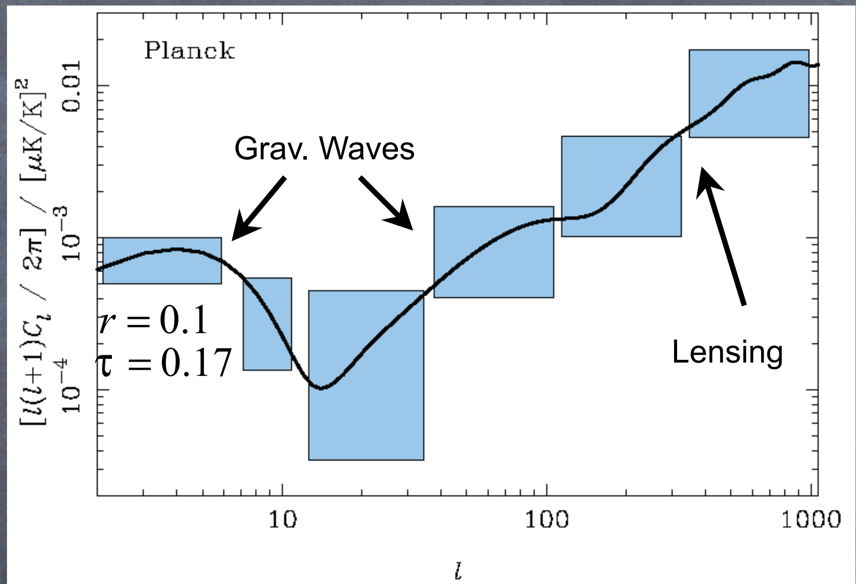
Planck Blue Book

Planck Mission



Planck Blue Book

Planck Mission



B spectra

Planck Blue Book

CMB Polarization

- B-mode: gravity waves from Inflation
- Lensing
  - Dark energy
  - Dark matter & neutrino mass
- Ground-suborbital campaign (r<0.01 in ~5 years!)
- Future dedicated satellite: 2020?

Dark Energy RoadMap

- Dark Energy Task Force (DETF): Interagency (AAAC, HEPAP) reporting to DOE, NASA and NSF - June 2006
- BEPAC: NRC committee formed at request of DOE & NASA - 2007
  - JDEM mission given priority for 2009 start
- ESA Cosmic Vision Programme:
  - Astrophysics WG: dark energy priority

Dark Energy Stage IV
The Big Surveys

- Ground: LSST(2015-2025), SKA
- Space mission
  - NASA JDEM: Selection 2/2009, launch 2015
    - ADEPT, DESTINY, SNAP
  - ESA Cosmic Vision Programme: launch 2017
    - DUNE, SPACE

DETF Stage IV