Structures et leur évolution

Cours 2 : Le champ de densité et son évolution linéaire (théorie newtonienne)

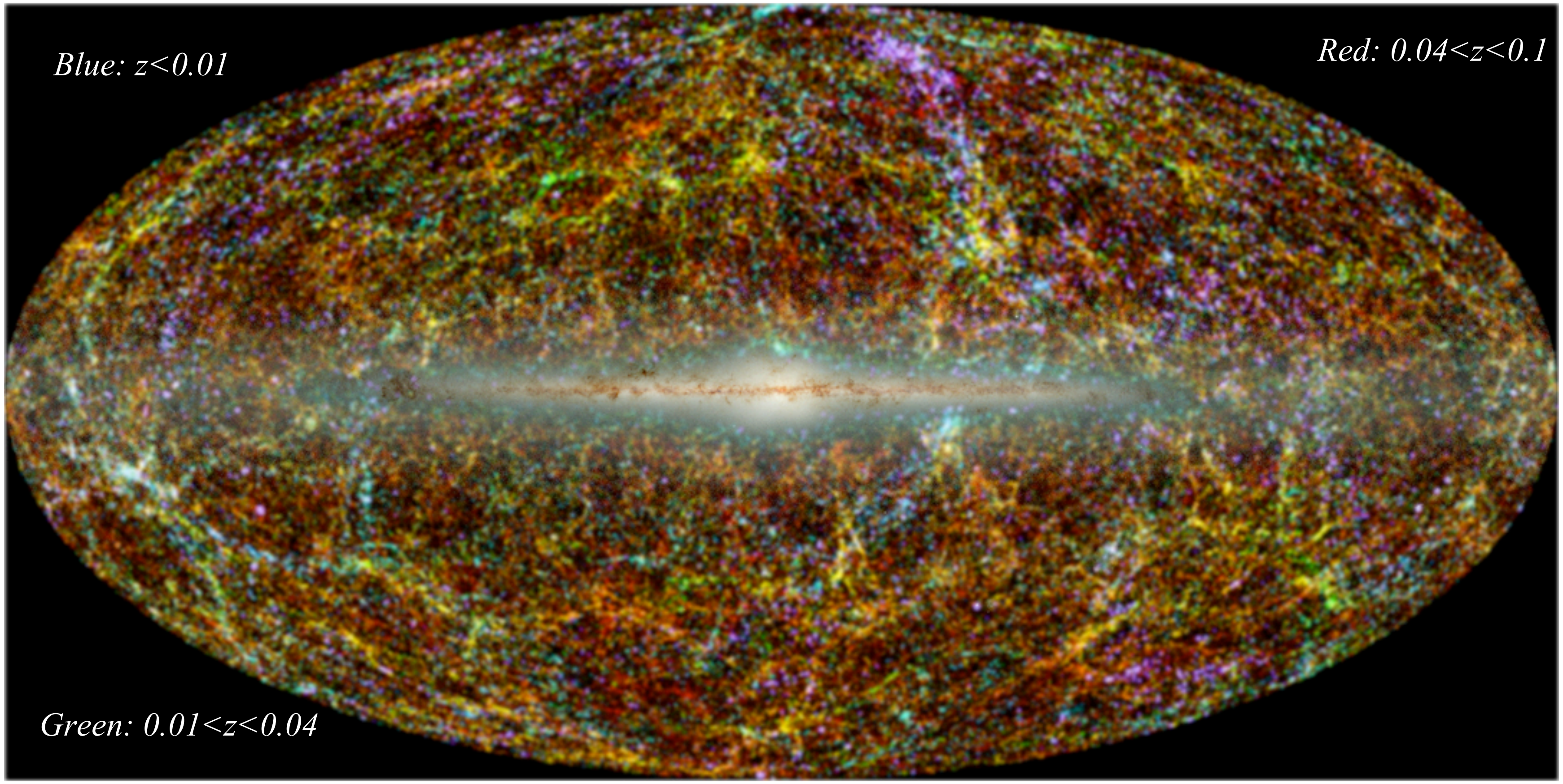
Pas de cours lundi prochain (28 janvier)
Mardi matin
Mercredi matin
Jeudi matin
Vendredi journee

Key Points for Today

- Modeling the density field as a (Gaussian) random field
  - Correlation function, power spectrum
- Biasing: relation galaxy-mass
- Linear evolution (linear regime)
  - The Harrison-Zel'dovich (HZ) spectrum
  - Why the measured power spectrum is a wonderful success of cosmology
  - How we use the power spectrum to constrain cosmology: $\boxed{\Omega_M h \sim 0.2}$

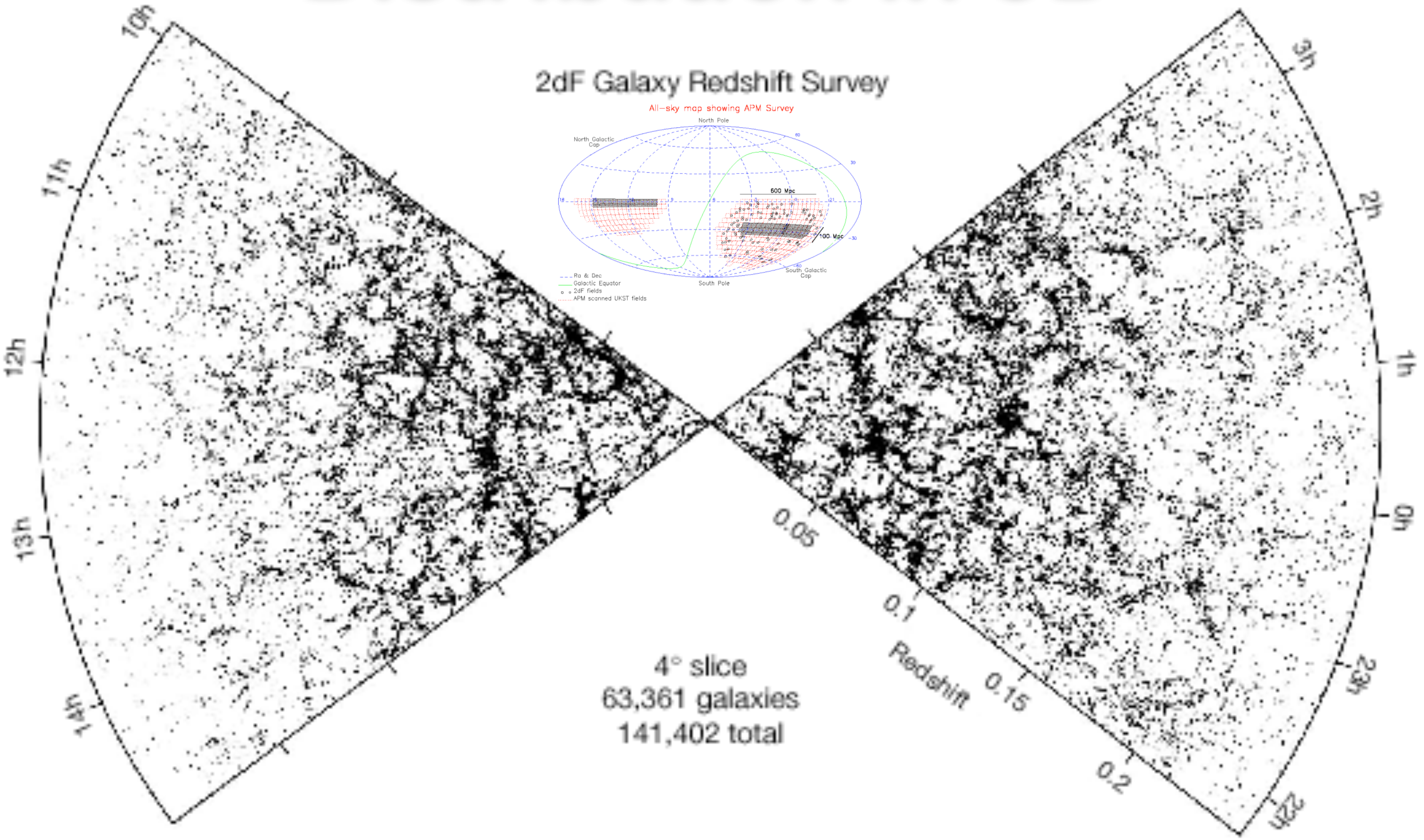
A Universe of Galaxies

2MASS = 2 Micron All Sky Survey; 3 bands J(1.25 microns) H (1.65) K_s (2.17)



Jarrett, T.H. 2004, PASA, 21, 396 http://spider.ipac.caltech.edu/staff/jarrett/papers/LSS/

Distribution in 3D



redshift space diagram ≠ real space … more later…

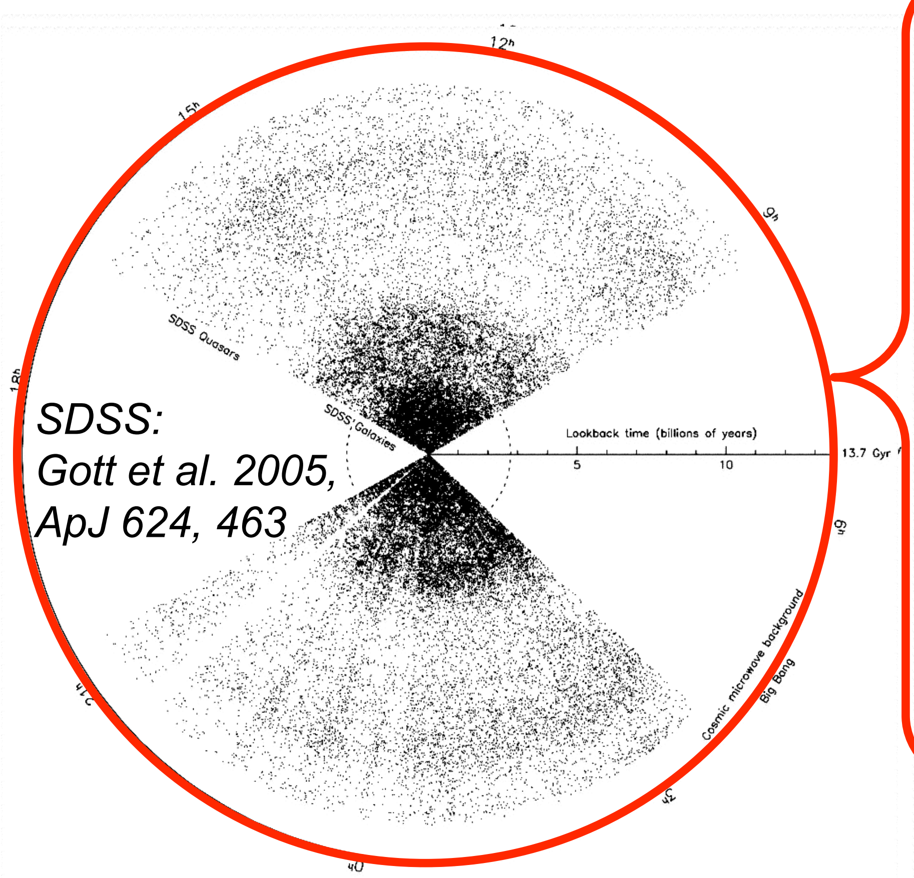
Cosmological Principle

The Universe is ***statistically*** isotropic and homogeneous

- There is no special place, ie, no center
- Extension of Copernican principle
- Simple
  - ⇒ Particularly when missing data (e.g., time of Einstein's proposal!)*
- Theoretical/philosophical motivation:
  - ⇒Mach's principle
- Observational basis…

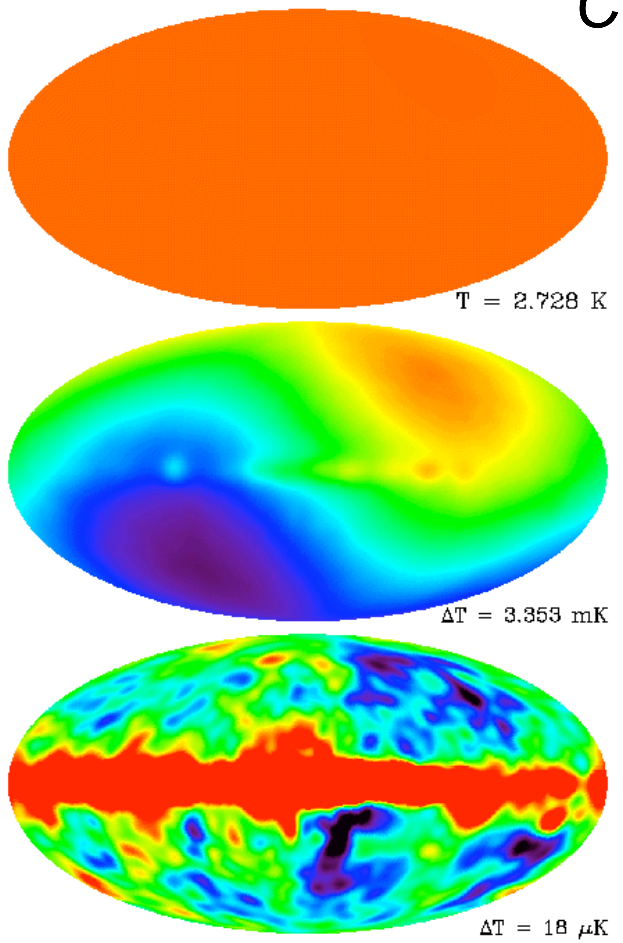
Cosmological Principle

Observational basis:



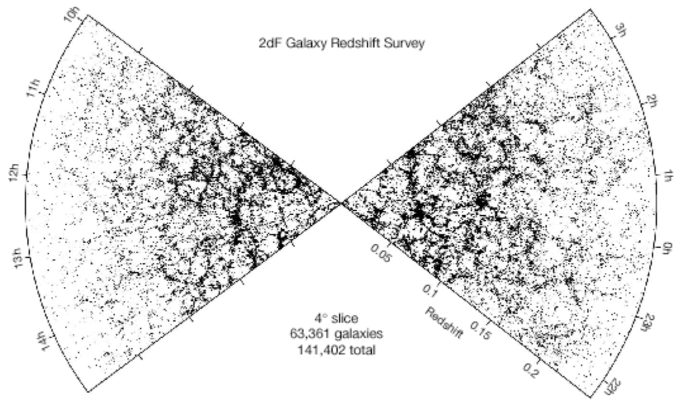
SDSS:
Gott et al. 2005,
ApJ 624, 463

CMB



COBE DMR

Modeling the Density Field



galaxy fluctuations

mass density fluctuations

perturbation theory

(early times, large scales)

- Cosmological Principle
  - Uniform clustering…?
    - Statistical process
    - Random fields
- Galaxy-mass relation
  - Biasing
- Evolution of perturbations
  - Primordial spectrum
  - Transfer function

Random Fields

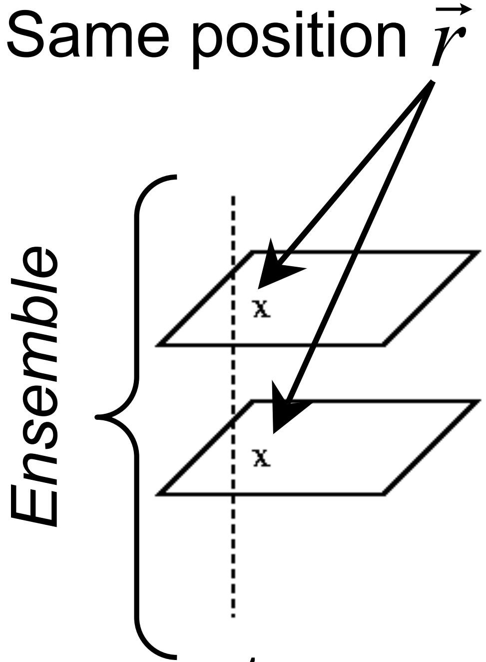
Idea: uniform distribution with deviations (*density contrast*)

$$\delta(\vec{r}) = \frac{\rho(\vec{r}) - \bar{\rho}}{\bar{\rho}} \qquad \bar{\rho} = \text{const.} \quad \text{(C.P.)}$$

The deviations are taken to be the result of a stochastic physical process:

$\delta(\vec{r})$ *Is a random variable (RV) at each point in space; it is a random field (RF)*

$$\boxed{\text{Description = statistics of the field } \delta(\vec{r})}$$



Ensemble averages:

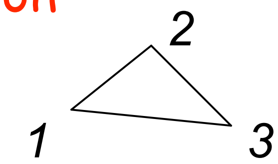
$$\langle \delta(\vec{r}) \rangle = const = 0$$

definition

Cosmo. Principle:
Homogeneous/isotropic clustering

$$\langle \delta(\vec{r}_1)\delta(\vec{r}_2) \rangle = \xi(|\vec{r}_1 - \vec{r}_2|) = \xi(r_{12})$$ = 2-pt function; (auto)correlation function

$$\langle \delta(\vec{r}_1)\delta(\vec{r}_2)\delta(\vec{r}_3) \rangle = \zeta(r_{12}, r_{13}, r_{23})$$ = 3-pt function



$$\vdots$$

N-pt functions; complete description of the RF

Fourier description:

$$\delta(\vec{r}) = \frac{1}{(2\pi)^3}\int d^3k\, e^{-i\vec{k}\cdot\vec{r}}\delta(\vec{k})$$

$\delta(\vec{k})$ Is a RV for each wavevector

$$\tilde{\delta}(\vec{k}) = \int d^3r\, e^{i\vec{k}\cdot\vec{r}}\delta(\vec{r})$$

$$\left\langle \delta(\vec{k}) \right\rangle = 0$$

$$\left\langle \delta(\vec{k}_1)\delta^*(\vec{k}_2) \right\rangle = (2\pi)^3 \overbrace{\delta_D(\vec{k}_1 - \vec{k}_2)}^{\text{Independent RV}} P(k_1)$$

⋮

Power spectrum:

$$\boxed{P(k) = \int d^3r\; e^{i\vec{k}\cdot\vec{r}}\;\; \xi(r)}$$

P is only a function of k (mag) & delta Dirac

$\Longleftrightarrow$ *ξ is only a function of r (mag).*

(The field represents a stationary process)

Gaussian Random Fields

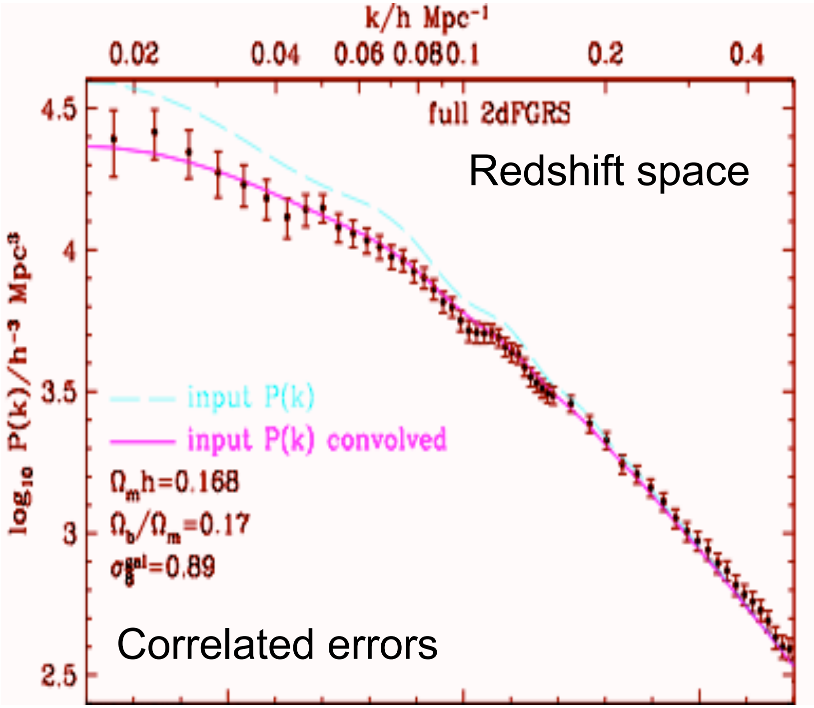
Field values δ follow a multivariate Gaussian (functional) distribution $\mathscr{F}\left\{\delta(\vec{r})\right\} = \mathrm{Gauss}\left\{\delta(\vec{r}_1),\delta(\vec{r}_2),\delta(r_3),\ldots\right\}$

- Statistics *uniquely* specified by mean and covariance
  - In real space
    - By definition the *mean* is zero: $\left\langle\delta(\vec{r})\right\rangle = 0$
    - *Covariance* given by: $\left\langle\delta(\vec{r}_1)\delta(\vec{r}_2)\right\rangle = \xi(r_{12})$
  - *Equivalently,* in Fourier space
    - By definition the *mean* is zero: $\left\langle\delta(\vec{k})\right\rangle = 0$
    - *Covariance* given by: $\left\langle\delta(\vec{k}_1)\delta^*(\vec{k}_2)\right\rangle = (2\pi)^3 \underbrace{\delta_D(\vec{k}_2-\vec{k}_1)}_{\textit{Independent RV}} P(k_1)$

...That's it!

…That's it!

- In a Gaussian theory, *P(k)* uniquely specifies the density pert. field
- This is adopted in the Standard Model
- Consistent to date with observations (see later)



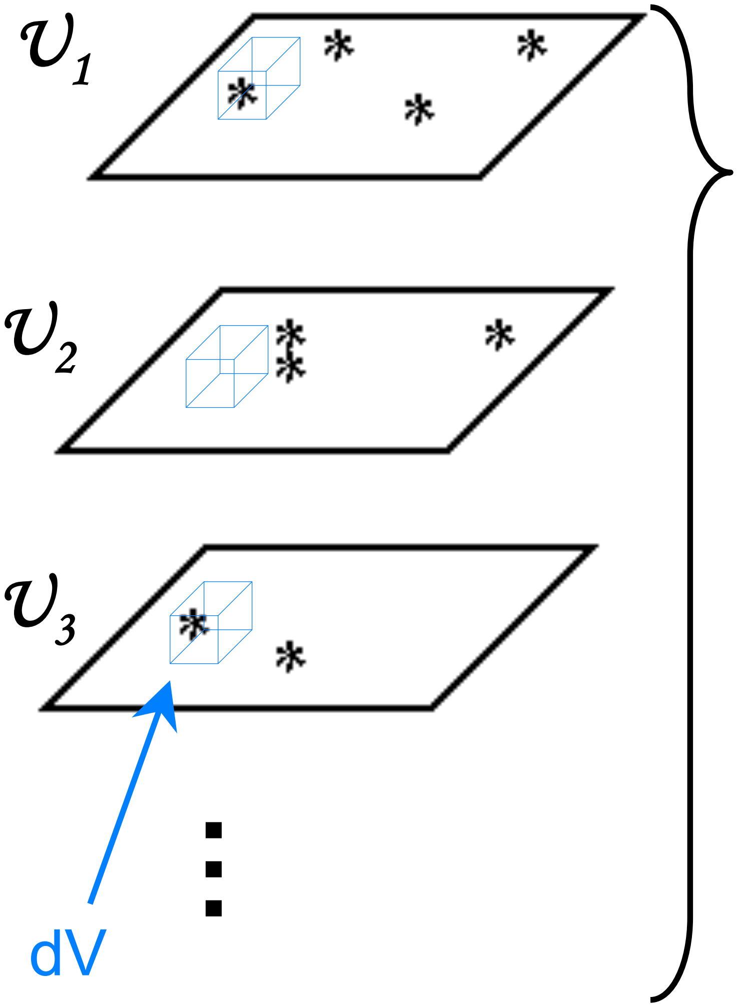
Measured *galaxy* power spectrum

e.g., 2dF with 221,414 galaxies

Cole et al. 2005, MNRAS 362, 505

- Important success of standard model
- Galaxy ↔ mass?
- Shape gives info. on cosmology

Poisson point process



Number of gals. *N* in volume dV is a Poisson ***Random Variable***:

ensemble of possible realizations

$$\langle N \rangle = \text{ensemble mean} = \bar{n}\, dV$$

$$\bar{n} = \text{ mean number density}$$

const. (uniformity)	⟺	$\mathcal{C.P.}$

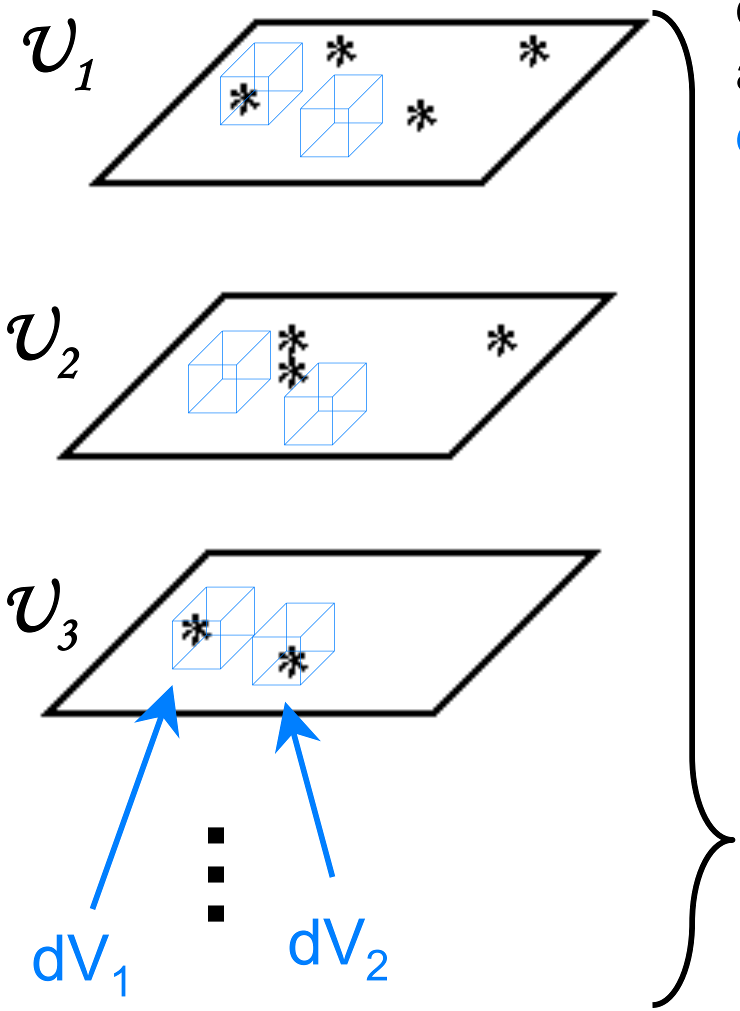
For *small* volumes:

$$P_1 \equiv \Pr(N=1) = \langle N \rangle + O(dV^2)$$

$$= \bar{n}\, dV$$

Think of space as divided into little volume elements, each with this probability of containing a galaxy.

Galaxy correlation function ξ



ensemble of possible **universes**

Probability of finding a gal. in dV_1 **and** dV_2

$\bar{n}$ = mean (uniform) gal. density

$$P_{12} = \bar{n}^2 [1 + \xi(r)] dV_1 dV_2$$

Poisson term

excess over Poisson

ξ(r) is a function of only the separation $r = |\vec{r}|$

$\Leftrightarrow$ *Cosmo. Prin.*

Volume elements separated by distance *r*

Galaxy-Mass Relation

Simplest: *galaxy density* $n = \dfrac{\rho}{m_{gal}}$

A bit more imagination: $\delta_{gal} \equiv \dfrac{(n-\bar{n})}{\bar{n}} = b\dfrac{(\rho-\bar{\rho})}{\bar{\rho}} = b(\delta)\delta$

Accounting for complicated galaxy formation process

The quantity *b* is known as the galaxy *bias factor*

For *linear bias*, ie $b=b_o=const.$

$$\Rightarrow \quad P_{12} = \langle n_1 n_2 \rangle = \bar{n}^2 dV^2 \left[1+\xi_{gal}(r_{12})\right] \quad \text{with} \quad \xi_{gal}(r_{12}) = b_0^2 \xi(r_{12})$$

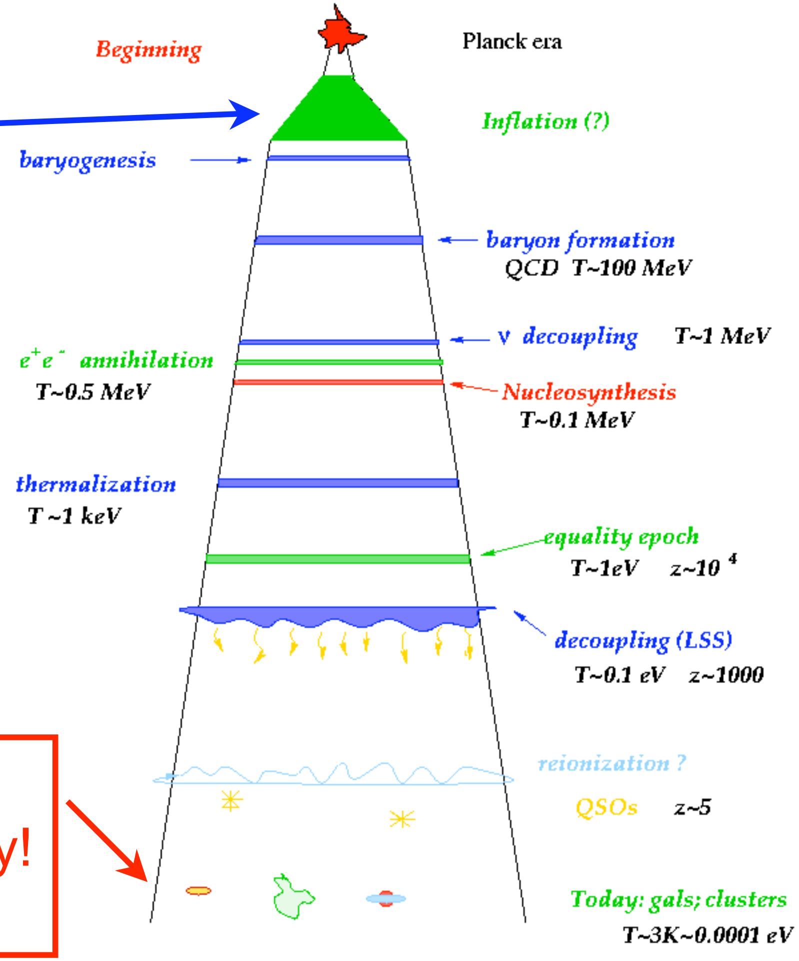
- $b\sim1$ on large scales, e.g., $\geq 10\ h^{-1}$ Mpc separations
- b is not linear on small scales, e.g., $\leq 5h^{-1}$ Mpc separations
- The bias can, and does, depend on galaxy type

Density Perturbations & Their Evolution

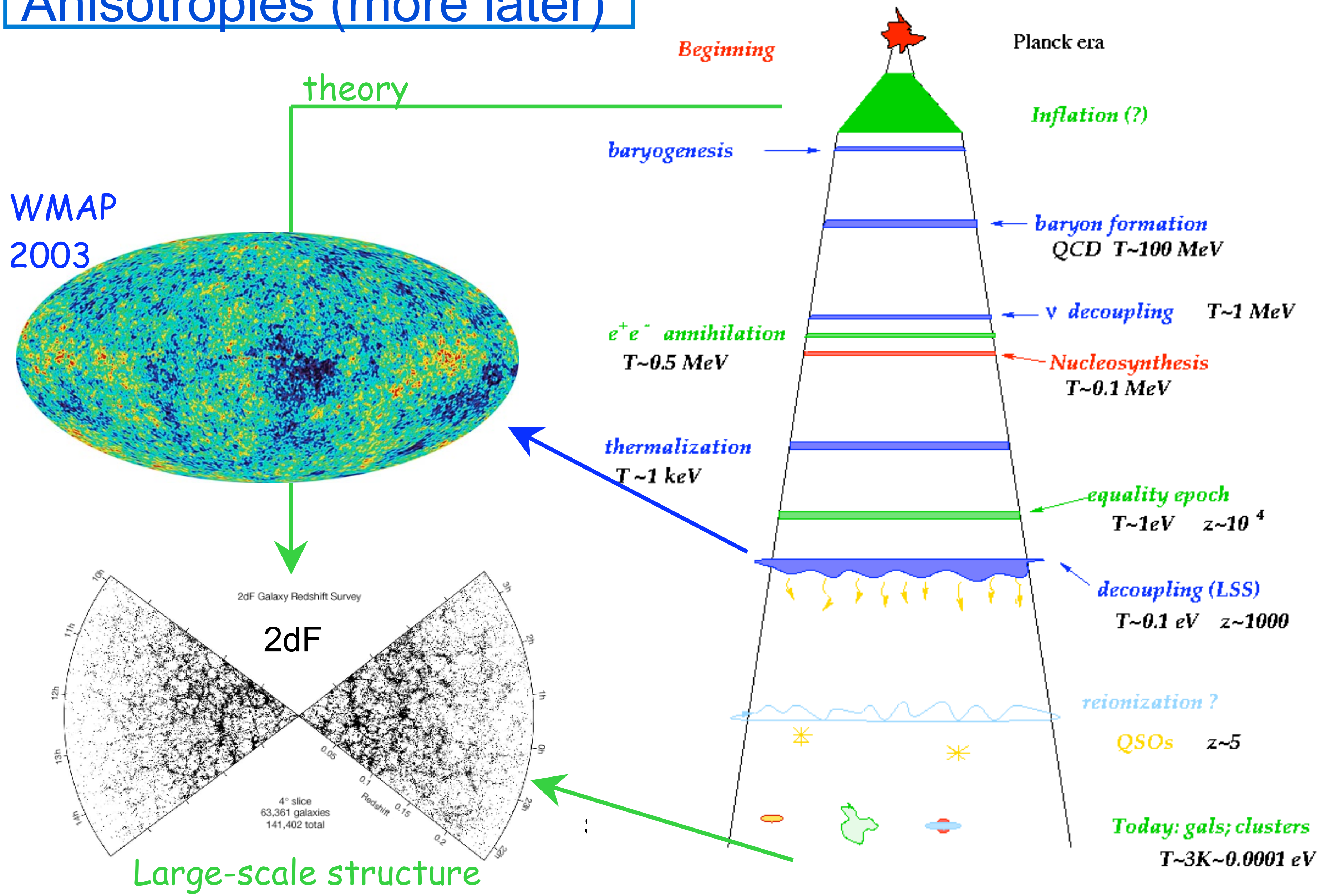
- *Primordial* spectrum: $P_i(k)$
  - Inflation physics
- Transfer function *T(k)*
  - Gravitational evolution ("passive")
- *Final* spectrum:

$$P(k) = T^2(k)P_i(k)$$

Observations
- *Success of theory!*
- Cosmo params.



Relation to CMB Anisotropies (more later)



Perturbation theory (Newtonian)

Observed structures

- inside horizon (λ < t)
- φ, v << 1

$\sim 10^{-5}$ $<10^{-2}$

Newtonian theory sufficient

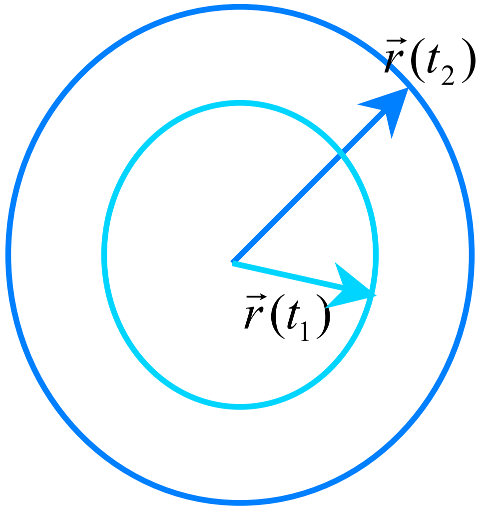
Model matter as a perfect fluid with pressure ***p***, energy density **ρ**, velocity $\vec{u}$, and an equation-of-state $p[\rho]$

$$\dot{\rho} + \nabla \cdot \rho\vec{u} = 0$$ continuity

$$\rho\dot{\vec{u}} + \rho(\vec{u}\cdot\nabla)\vec{u} = -\rho\nabla\Phi - \nabla p$$ Euler

$$\nabla^2\Phi = 4\pi G\rho$$ Poisson

Change from these proper coordinates to comoving coordinates;



express result in terms of perturbation variables:

$$\delta(\vec{r}) \equiv \frac{\rho(\vec{r}) - \bar{\rho}}{\bar{\rho}}, \quad \delta_p \equiv \frac{\delta p}{\bar{p}}, \quad \vec{v} \equiv \vec{u} - \vec{v}_H,$$

$$\varphi = \Phi - \bar{\Phi}$$

$$\vec{r}(t) = a(t)\vec{x}$$

proper coordinate

comoving coordinate

and separate out the 1st order part:

(0th order = Newtonian expansion model)

$$\boxed{\dot{\delta} + \frac{1}{a}\nabla \cdot \vec{v} = 0 \qquad \dot{\vec{v}} + \frac{\dot{a}}{a}\vec{v} = -\frac{1}{a}\nabla\varphi - \frac{1}{\bar{\rho}a}\nabla\delta p \qquad \nabla^2\varphi = 4\pi G\bar{\rho}a^2\delta}$$

Solutions may be divided into $\vec{v} = \vec{v}_\perp + \vec{v}_\parallel$

scalar perturbations: $\nabla \times \vec{v}_\parallel = 0$

(compression)

vector perturbations: $\nabla \cdot \vec{v}_\perp = 0$

(rotational)

useful later in Rel. pert. theory (+ tensor modes)

The **vector** mode **decays** with time:

$$\frac{d}{dt}\left[a\nabla \times \vec{v}\right] = 0 \;\Rightarrow\; \nabla \times \vec{v}_\perp \propto \frac{1}{a(t)}$$

The **scalar** mode satisfies

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} = 4\pi G\bar{\rho}\delta + \frac{1}{\bar{\rho}a^2}\nabla^2\delta p$$

Linear theory solutions

I Dust eq.-of-state: $p=0$

a/ Matter-dominated era (MD): a(t) $\propto t^{2/3}$

$$\delta(\vec{x},t) = A(\vec{x})t^{2/3} + B(\vec{x})t^{-1}$$

decaying mode $D_d(t)$

growing mode $D_g(t) \propto a(t)$

b/ Curvature-dominated era: a(t) $\propto t$

$$\delta(\vec{x},t) = A(\vec{x}) + B(\vec{x})t^{-1}$$

c/ Vacuum-dominated era: a(t) $\propto e^{Ht}$

$$\delta(\vec{x},t) = A(\vec{x}) + B(\vec{x})e^{-2Ht}$$

d/ Radiation-dominated era (RD): a(t) $\propto t^{1/2}$

$$\delta(\vec{x},t) = A(\vec{x})\ln t + B(\vec{x})$$

$\boxed{\text{II}}$ Collisional gas with pressure: *p>0*

For adiabatic perturbations: $c_s^2 = \dfrac{\delta p}{\delta \rho}$ **sound speed**

New associated scale:

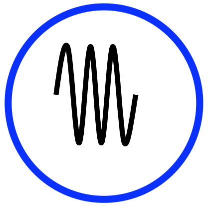
$$a\lambda_J = \sqrt{\frac{\pi c_s^2}{G\bar{\rho}}}$$ **the Jeans length**

(comoving Jeans length)

Perturbations with:

$\lambda >> \lambda_J$	**grav. growth**
$\lambda << \lambda_J$	**oscillations** (sound waves)

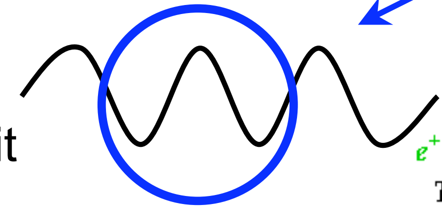
1. Quantum fluctuation
inside horizon
(Causal physics)



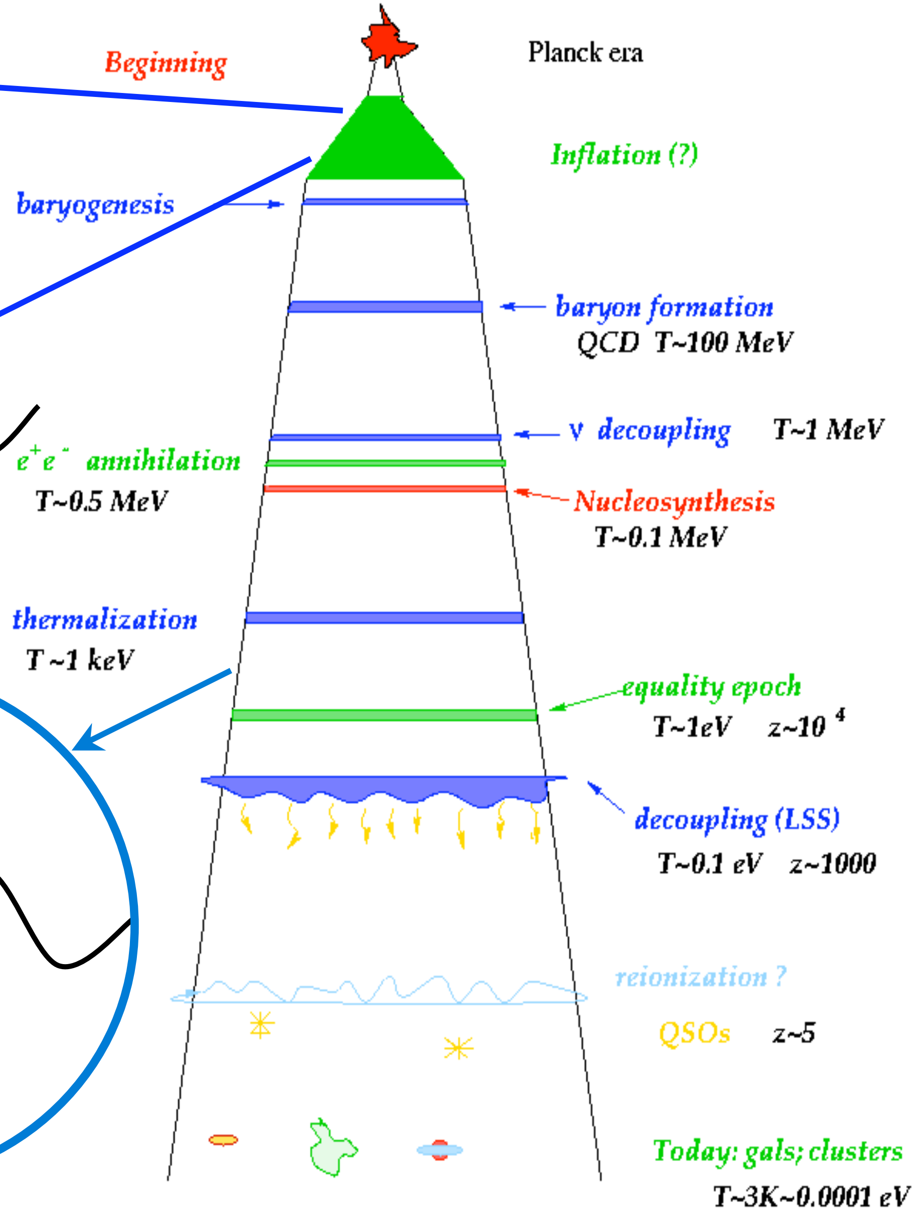
Horizon size fixed during inflation

$$d_{EH} \sim H^{-1} = c^{ste}$$

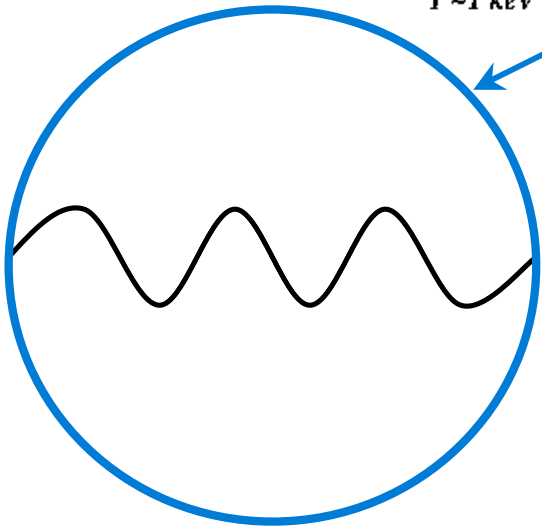
$$\lambda \propto a(t) \sim e^{Ht}$$



2. Expansion drives it
outside horizon
(No causal physics)



3. Horizon grows faster than λ
reenters horizon
(Causal physics)

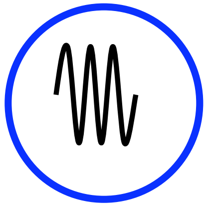


$$d_H \sim H^{-1} = t$$

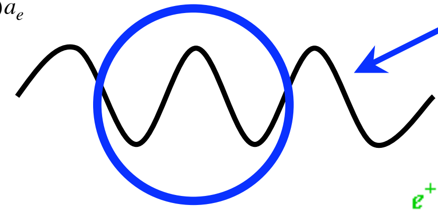
$$\lambda \propto a(t) \sim t^n \quad n < 1$$

At horizon exit

$$\Delta^2\varphi \equiv \frac{k^3\left|\delta\varphi\left(\vec{k}\right)\right|^2}{2\pi^2} \sim \left.\frac{H^2}{\dot{\varphi}}\right|_{k\sim H(t_e)a_e}$$



$$\lambda \propto a(t) \sim e^{Ht}$$



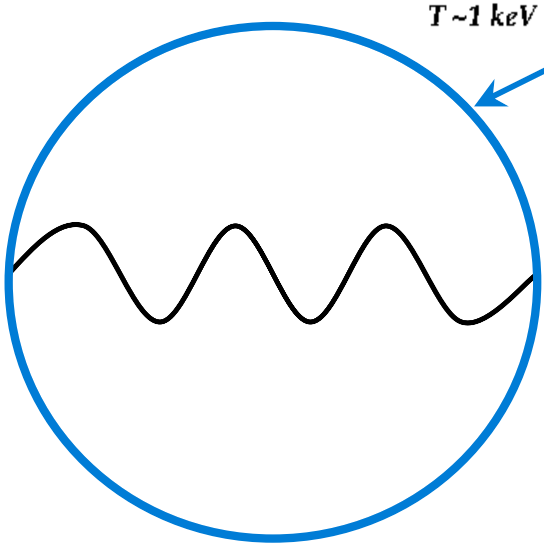
After reheating

$$\varphi \sim cste$$

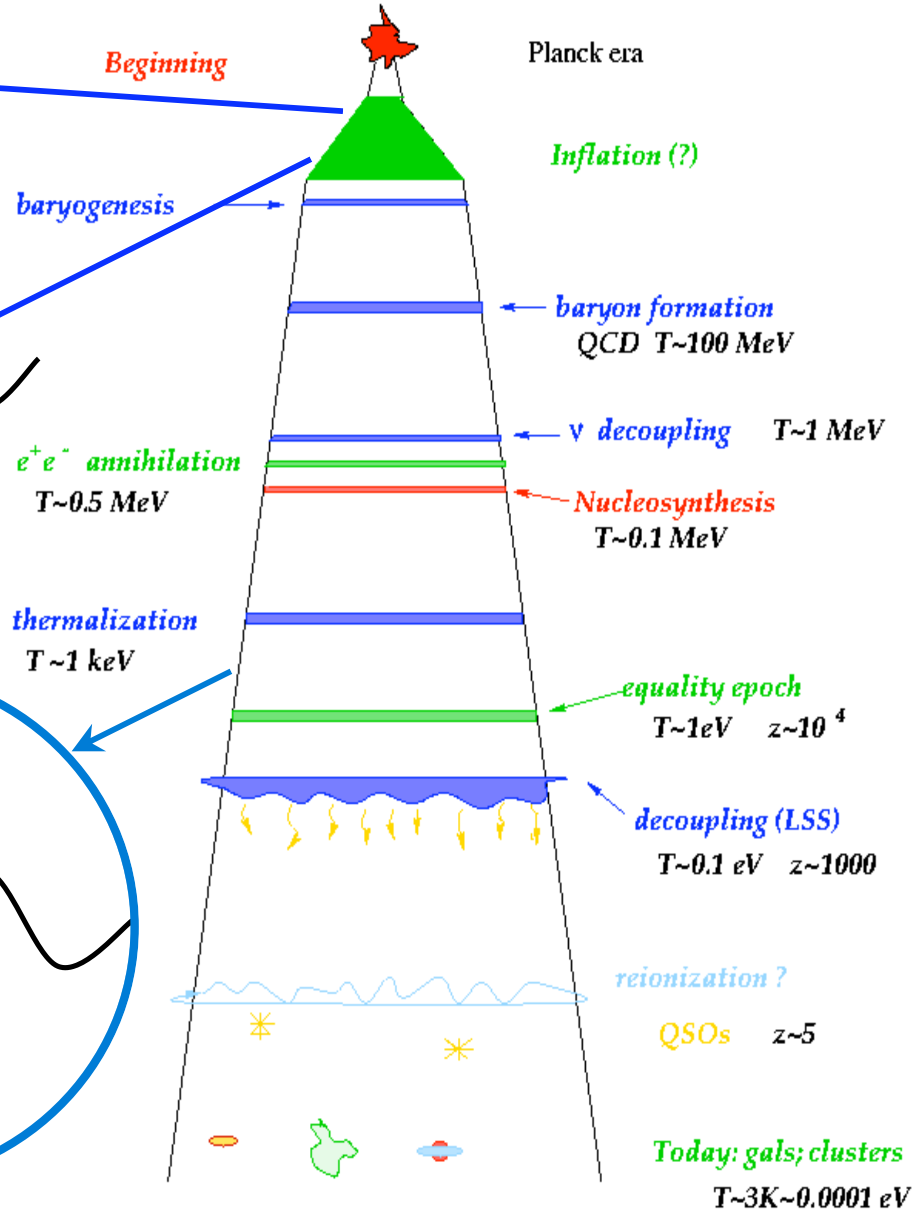
At horizon rentry

$$\left.\Delta^2\varphi\right|_{k\sim H(t_r)a_r} \sim A\left.\frac{H^2}{\dot{\varphi}}\right|_{k\sim H(t_e)a_e}$$

$C^{ste}(k)$



$$\lambda \propto a(t) \sim t^n \qquad n<1$$



Subhorizon Evolution

Radiation epoch

$\delta_{B-\gamma}(\vec{k})$ oscillates

$\delta_{DM}(\vec{k})$ grows logarithmically

Matter epoch

Before recombination

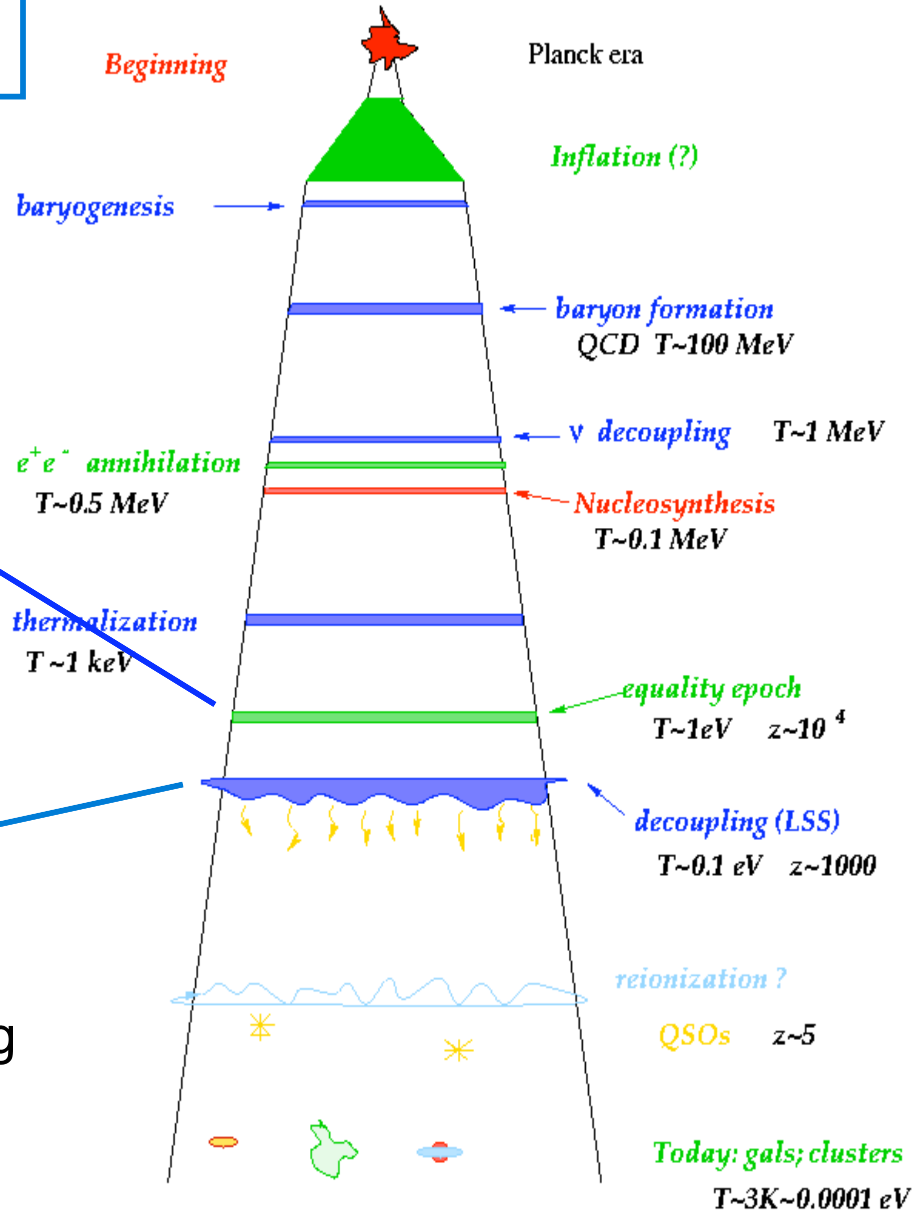
$\delta_{B-\gamma}(\vec{k})$ oscillates

$\delta_{DM}(\vec{k}) \propto a(t)$

After recombination (*z>few*)

$\delta_{\gamma}(\vec{k})$ decays by free-streaming

$\delta_{DM}(\vec{k}), \delta_B \propto a(t)$



Harrison-Zel'dovich (Peebles) Spectrum:

Long before the theory of inflation…

Simplicity (absence of motivated scale): $P(k) \propto k^{n_s}$ for $aH < k < k_{eq}$

HZ spectrum: $n_s = 1$ *scale invariant spectrum*

For *a>a$_{eq}$*:

$$\Delta^2(k,a) = \left(\frac{a}{a_r}\right)^2 \Delta^2(k,a_r)$$

$$k = a_r H(t_r) \sim a_r^{-1/2}$$

At reentry:

$$\boxed{\Delta^2(k,a_r) \propto k^{n_s - 1}}$$

$\therefore n_s = 1 \Rightarrow$ *cste* at reentry

and/or

In terms of potential

$$k^2 \Phi(\vec{k}) \sim \delta(\vec{k}) \Rightarrow \Delta^2_\Phi(k) \equiv \frac{k^3 P_\Phi(k)}{2\pi^2} \propto k^{n_s - 1}$$

$\therefore n_s = 1 \Rightarrow \Phi$ has a scale invariant spectrum, inside and outside horizon (and is *cste* in time in mat. era)

Inflation provides a *physical mechanism* that realizes a near HZ spectrum (*n_s~1).*

Formation of late-time CDM-like spectrum

Passive: Initial conditions, followed by pure grav. evolution

- Near scale-invariant (e.g., Inflation)
- Adiabatic: $n_i / n_j = const$
- Gaussian

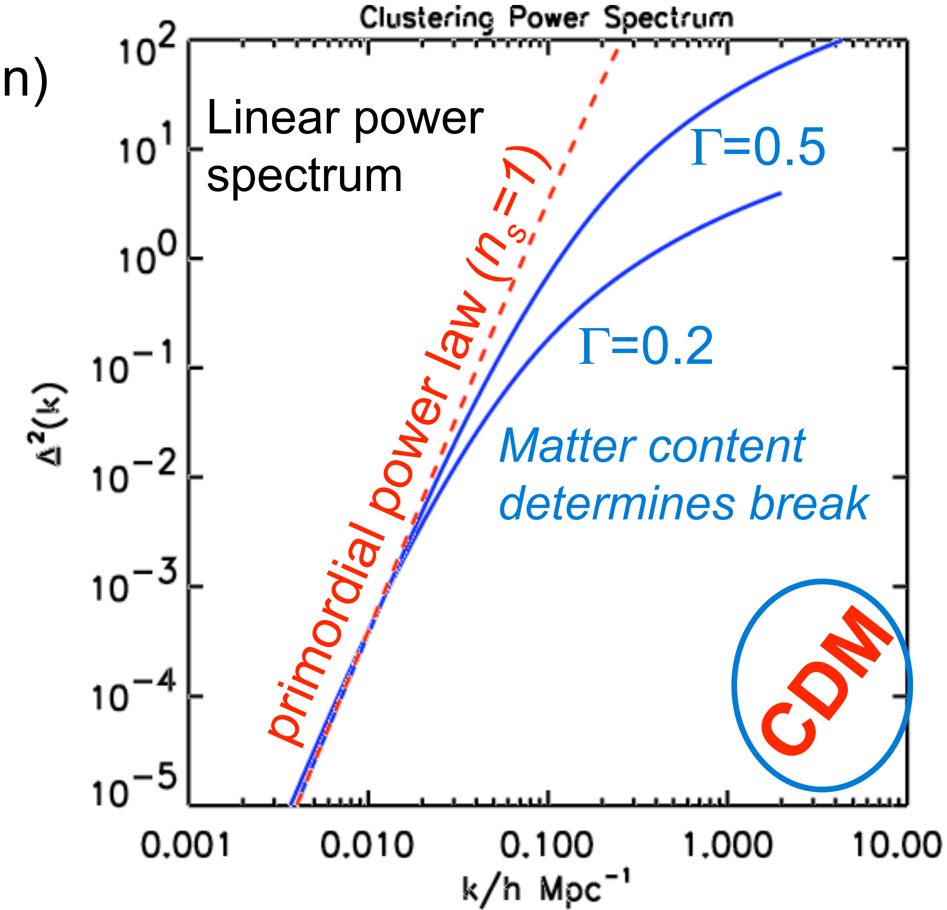
Late-time $(z>z_{eq})$ power spectrum for CDM $(p=0)$

$$P(k, z < z_{eq}) = T^2(k) A k^{n_s - 1}$$

T(k) = transfer function

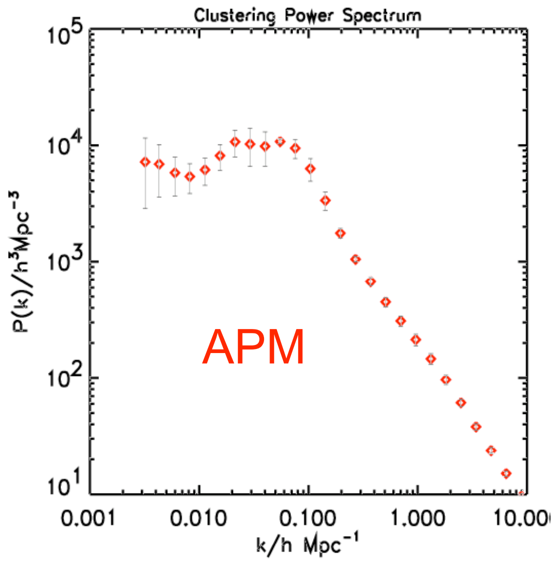
$$q \equiv \frac{k}{\Omega h^2 \ \mathrm{Mpc}^{-1}} \equiv \frac{k}{(h \ \mathrm{Mpc}^{-1})} \frac{1}{\Gamma}$$

Γ = *shape parameter*



$$T(k) = \frac{\ln(1+2.34q)}{2.34q} \left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right]^{-1/4} \quad \text{(BBKS)}$$

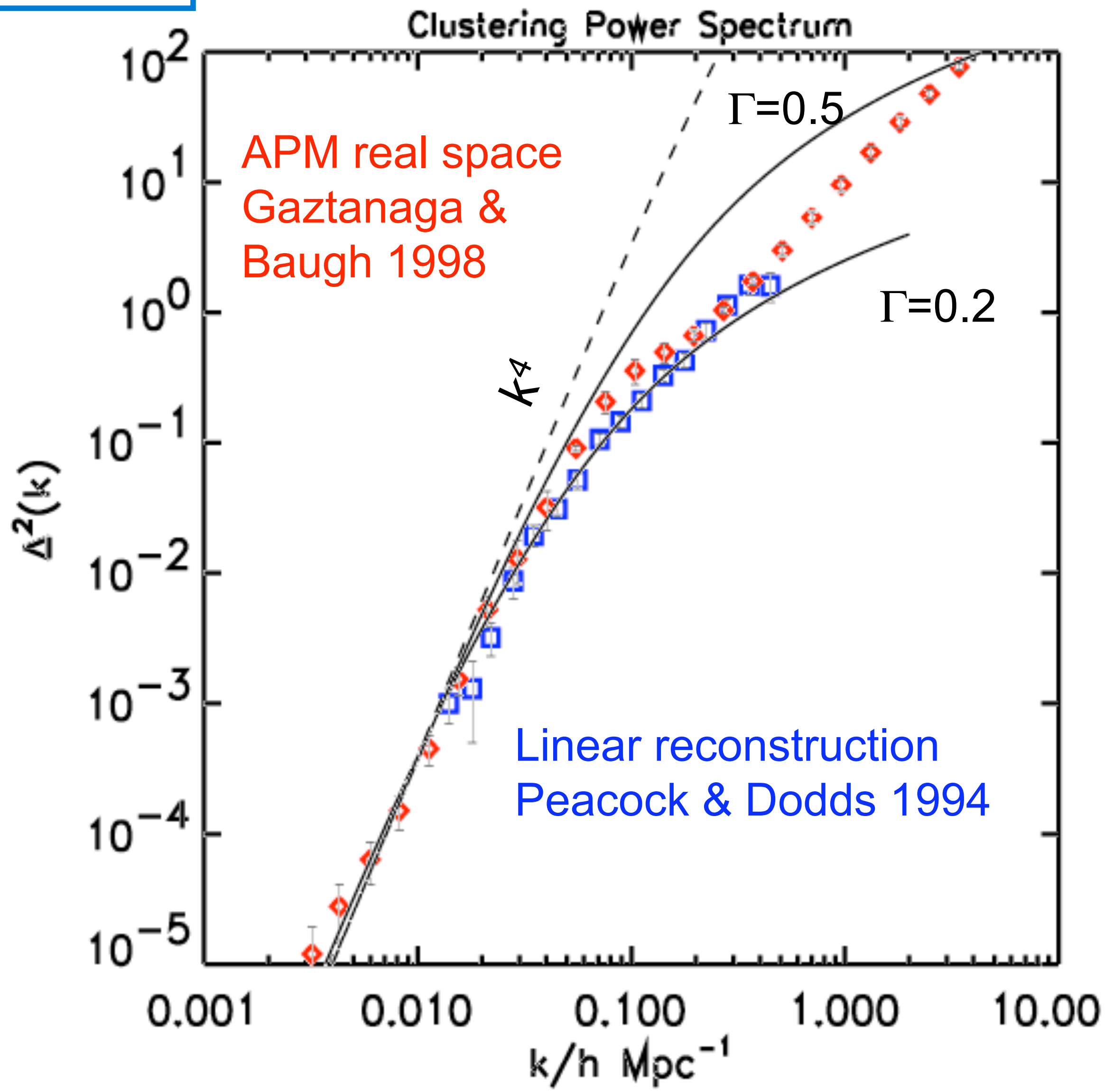
Some Observations



Different analysis
Indicate: $\Gamma \approx 0.2$

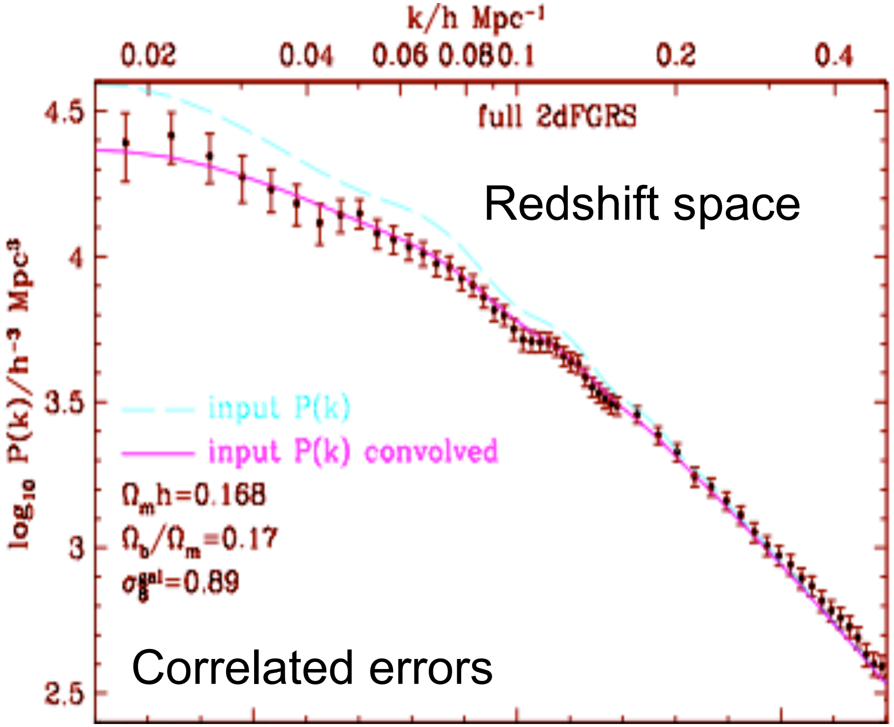
e.g., *Percival et al. 2001, MNRAS 327, 1297:*

$$\Gamma = \Omega h = 0.20 \pm 0.03$$



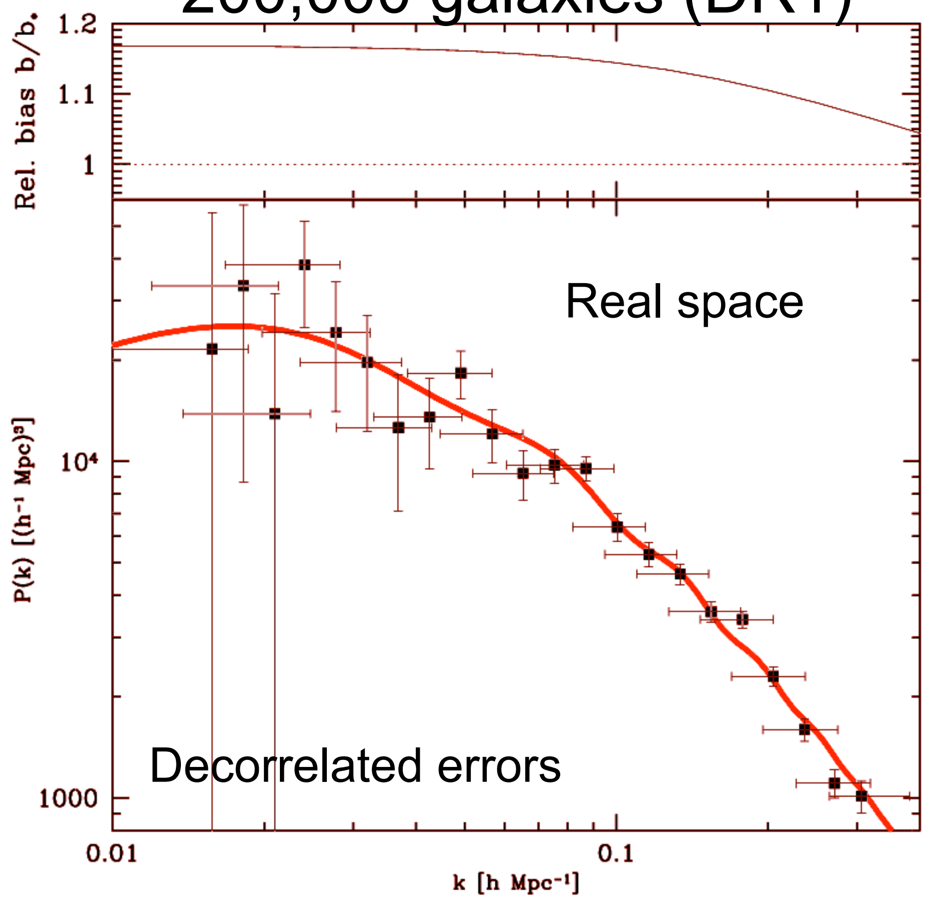
Measured *Galaxy* Power Spectrum: latest results

2dF
221,414 galaxies



Cole et al. 2005, MNRAS 362, 505

SDSS
200,000 galaxies (DR1)



Tegmark et al. 2004, ApJ 606, 702