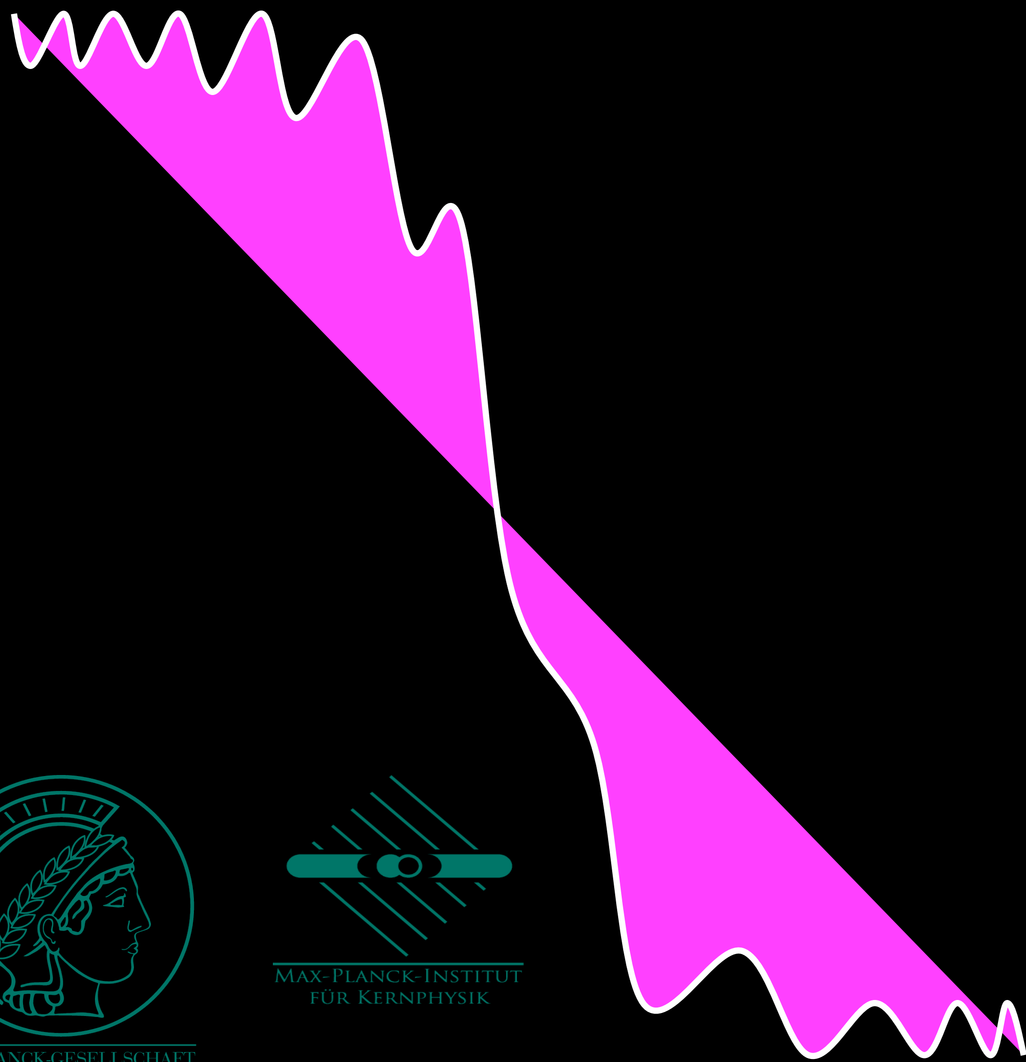
The Mikheyev- Smirnov-Wolfenstein (MSW) effect



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"History of Neutrino" Paris, September 6, 2018

MAX-PLANCK-GESELLSCHAFT

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Outline

adiabatic

The MSW effect is the flavor transformations of neutrinos in matter with varying density driven by change of mixing in the course of propagation

Developments of main notions and concepts

1. Wolfenstein papers and follow-up, 1978 - 1984
2. Mikheyev and Smirnov mechanism, 1984 - 1985
3. Further developments 1985 - 1986

Moriond 86

1. Wolfenstein

Lincoln Wolfenstein

1923 - 2015

Rare case for theorist: he was 55 in 1978 when major results have been obtained

"Neutrino oscillations in matter"

L. Wolfenstein, Phys. Rev. D17 (1978) 2369

motivation: Oscillations of massless neutrinos in models with non-diagonal neutral currents (FCNC)

L Wolfenstein, Nucl. Phys B91, 95, (1975)

Hamiltonian:

$$H = \frac{G}{\sqrt{2}} L_\lambda J^\lambda + \text{h.c.}$$

$$L_\lambda = \cos^2\alpha\,[\nu_a \gamma_\lambda(1 + \gamma_5)\,\nu_a + \nu_b \gamma_\lambda(1 + \gamma_5)\,\nu_b\,] + \sin^2\alpha\,[\nu_a \gamma_\lambda(1 + \gamma_5)\,\nu_b + \nu_b \gamma_\lambda(1 + \gamma_5)\,\nu_a\,]$$

Extreme case: off-diagonal NC

tests: Detection of neutrinos in Quebec, Canada 1000 km from their source at Fermilab

A K Mann and H Primakoff , Phys. Rev. D15 (1977) 655

Refraction

the key "Coherent forward scattering of neutrinos must be taken into account when considering oscillations in matter"

analogy of regeneration of K_S from the K_L beam, Optics (without discussion of validity and applicability)

Effect is described by refraction indices:

$$n_i = 1 + \frac{2\pi N_i}{k^2} f_i(0)$$

N - number density of scatterers
amplitude of scattering
k - neutrino momentum

→ modifies the phase of propagating state $e^{ikn_i x} \nu_i$

The phase difference (which affects oscillations):

$$k \Delta n\, x = 2\pi N k \Delta f(0)/k\, x$$

Direct calculations (no details)

$$k \Delta n = 2 G_F \sin^2\alpha \ \Sigma_\iota g_i N_i$$ $i = p, n, e$

$\sim G_F N_i$

Refraction length, scale of the effect

Effective oscillation length (= refraction length)
- the length over which the phase difference equals 2π

$$l_0 = 2\pi / k\,\Delta n$$

For massless neutrino case (the only source of phase difference) it equals the oscillation length

$$l_m = l_0$$

Estimation $l_0 \sim 10^9$ cm

- comparable with the radius of the Earth;
effect can be seen in experiments with baseline 10^8 cm

The refraction length does not depend on neutrino energy

$$l_0 \sim 1 / G_F N$$

At low energies $l_{inel} \gg l_0$ - inelastic interactions can be neglected

Oscillations

Notion

The eigenstates for propagation in matter -
the states which diagonalize the Lagrangian of NC interactions

These states have definite refraction indices n_i and therefore acquire definite phases

These states differ from the neutrino states produced in the charged current interactions → mixing

Evolution of neutrino states:

$$\nu_a (x) = \sin\theta_m \nu_{1m}\, e^{ikn_1x} + \sin\theta_m \nu_{2m}\, e^{ikn_2x}$$

Expression for probability → straghtforward

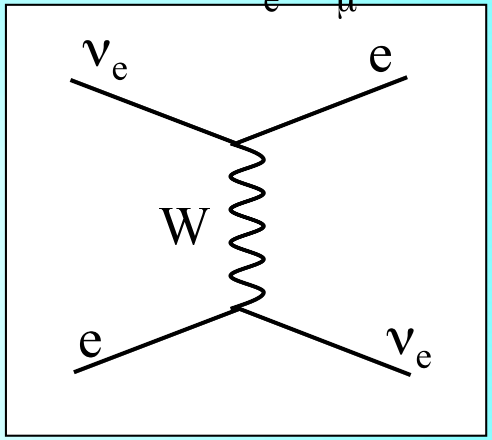
oscillation length = l_0

W: maximal mixing

Charged current contribution

footnote: I am indebted to Dr. Daniel Wyler for pointing out the importance of the charged-current term

for ν_e ν_μ



$$V = V_e - V_\mu = \sqrt{2}\, G_F N_e$$

⇨ Fierz transformation – as NC contribution

If one of the oscillating neutrinos is ν_e contributes to the phase difference

Changes mixing angle and oscillation length of massless neutrinos

Modifies the vacuum oscillations

Even when NC are diagonal and symmetric as in the SM

Modification of vacuum oscillations

For massive neutrinos another source of phase difference apart from phase factor arising from coherent scattering:

$$\nu_i(t) = e^{-i\, m_i^2\, t/2k}\, \nu_i$$

definite in the mass basis (matter effect - in the interaction basis)

To accommodate both factors → differential equation

$$i \frac{d}{dt}\begin{pmatrix}\nu_1 \\ \nu_2\end{pmatrix} = \begin{pmatrix} m_1^2/2k - G N_e c^2 & - G N_e sc \\ - G N_e sc & m_2^2/2k - G N_e s^2 \end{pmatrix}\begin{pmatrix}\nu_1 \\ \nu_2\end{pmatrix}$$

$s = \sin\theta$, $c = \cos\theta$ vacuum mixing angle

Master equation

Later – in the flavor basis

Follow-up

L Wolfenstein, AIP Conference proceedings 52, 108 (1979)

Refining discussion

Oscillations of massless neutrinos is analogous to the phenomenon of optical birefringence in which case two planes of polarization are eigenvectors and beams with other states of polarization are transformed as they pass through the crystal

Applications to the atmospheric neutrinos

Master equation

Evolution equation in the flavor basis

$$i \frac{d}{dt}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix} = -\frac{\pi}{l_\nu}\begin{pmatrix}\cos 2\theta - 2(l_\nu / l_0) & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{pmatrix}\begin{pmatrix}\nu_e \\ \nu_\mu\end{pmatrix}$$

In the standard case, the CC interactions of ν_e with electrons change the phase of ν_e relative to ν_μ. This differs from the case of ν_μ and ν_τ

L. Wolfenstein, Neutrino 78 Adiabaticity (for massless neutrinos)

Parameters of oscillations

Constant density

Mixing angle in matter relates the eigenstates for propagation in matter and the flavor states

$$\tan 2\theta_m = \tan 2\theta \left(1 - \frac{l_\nu}{l_0} \cos^{-1} 2\theta \right)^{-1}$$

The oscillation length in matter

$$l_m = l_\nu \left(1 + \left(\frac{l_\nu}{l_0} \right)^2 - 2\cos 2\theta \frac{l_\nu}{l_0} \right)^{-1/2}$$

Transition probability

$$P = \tfrac{1}{2} \sin^2 2\theta \, (l_m / l_\nu)^2 \, [\, 1 - \cos(2\pi x / l_m)]$$

Three cases

1. $l_\nu \ll l_0$ - nearly vacuum $\quad l_m \sim l_\nu \quad \theta_m \sim \theta$
2. $L_\nu \gg l_0$ - matter dominance $\quad l_m \sim l_0 \quad \sin 2\theta_m \to 0$

   suppression of oscillations
3. $l_\nu \sim l_0$ - intermediate case

   the quantitative results in matter are quite different from in vacuum

Table for transition probabilities for $l_\nu = l_0$

$\theta = 15^0 \quad x/l_0 = 0.5 \quad P = \begin{cases} 0.492 \text{ matter} \\ 0.250 \text{ vacuum} \end{cases}$

enhancement

Vacuum mimicking

independent of the value l_ν / l_0 , as long as oscillation phase is small , $2\pi x/l_m < 1$, the oscillation probability in the medium is approximately the same as in vacuum

Applications

Focused on suppression of oscillations (for constant density)

LBL experiment

searches for oscillations detection of neutrinos 1000 km distant from their source at Fermilab

Solar neutrinos

"If l_ν is large the oscillation should be calculated for actual vacuum path ignoring passage through matter. There are no significant oscillations inside the Sun or in transversals through the earth."

True – no oscillations! But the adiabatic conversion is completely missed

Supernova neutrinos

Vacuum oscillation are effectively inhibited from occurring … because of high density

$$\sin^2 2\theta_m \sim \sin^2 2\theta \, (l_0 / l_\nu)^2$$

very small

L. Wolfenstein, Phys. Rev. D20 (1979) 2634

Comments and remarks

1. Refraction of neutrinos has been considered before Wolfenstein

R. Opher, Coherent scattering of Cosmic Neutrinos, Astron. & Astroph. 37, (1974) 135

... to detect relic neutrinos

Refraction index n has been computed

2. Wolfenstein thanks E. Zavattini for asking the right question.
What is this? Zavattini was working on birefringence .

3. Discussed limits and not much the most interesting case $l_\nu \sim l_0$
Pole in $\tan 2\theta_m$ dependence - ignored

Matter effects on three-neutrino oscillations

V. D. Barger, K. Whisnant, S. Pakvasa, R.J.N. Phillips, Phys.Rev. D22 (1980) 2718

Standard case: vacuum mixing, CC on electrons, constant density

Correct expression for refraction $V = \sqrt{2}\, G_F N_e$

General expression for the probabilities in terms of level splitting ΔM_{ij}

Explicit expressions for ΔM_{ij} in the 3ν case

Matter effect resolves the VO ambiguity in sign of Δm_{ij}^2

The effect is different for neutrinos and antineutrinos

Enhancement of oscillations

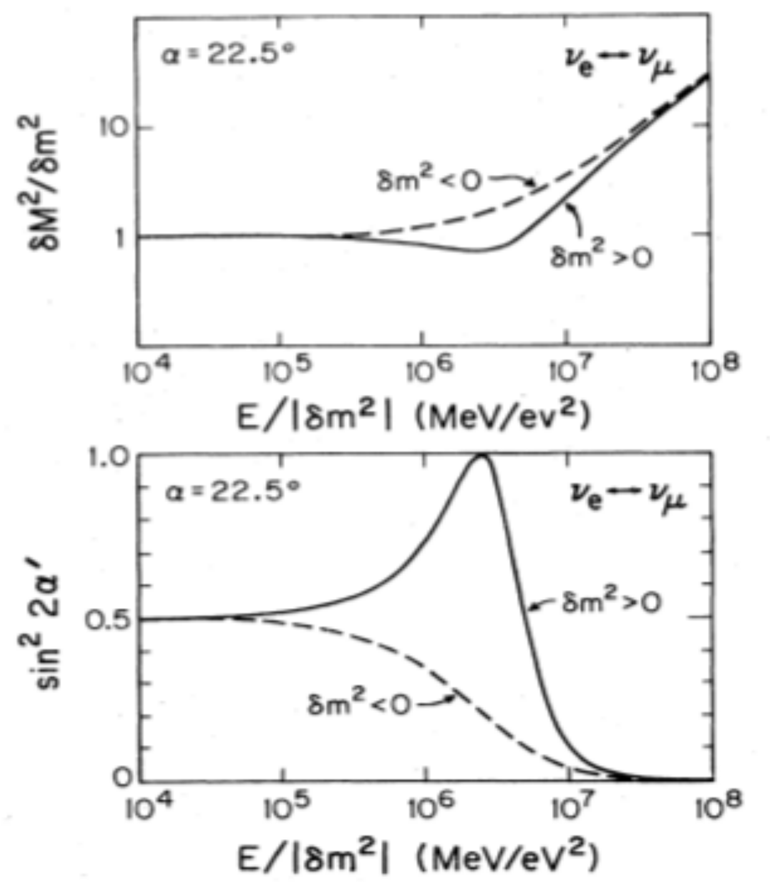


FIG. 1. Matter-to-vacuum eigenmass-squared difference ratio and matter amplitude $\sin^2 2\alpha'$ for oscillations of two neutrinos with vacuum amplitude $\sin^2 2\alpha = 0.5$ ($\alpha = 22.5°$).

There is always some energy, where

$$l_\nu / l_0 = \cos 2\theta \qquad (*)$$

and hence $\theta_m = 45^0$ either for ν or $\bar{\nu}$ depending on the sign of Δm^2

At this energy the survival probability vanishes at a distance

$$L = \frac{1}{2}\, l_0 \cot 2\theta$$

(*) is nothing but the resonance condition introduced later by MS

Enhancement of oscillations

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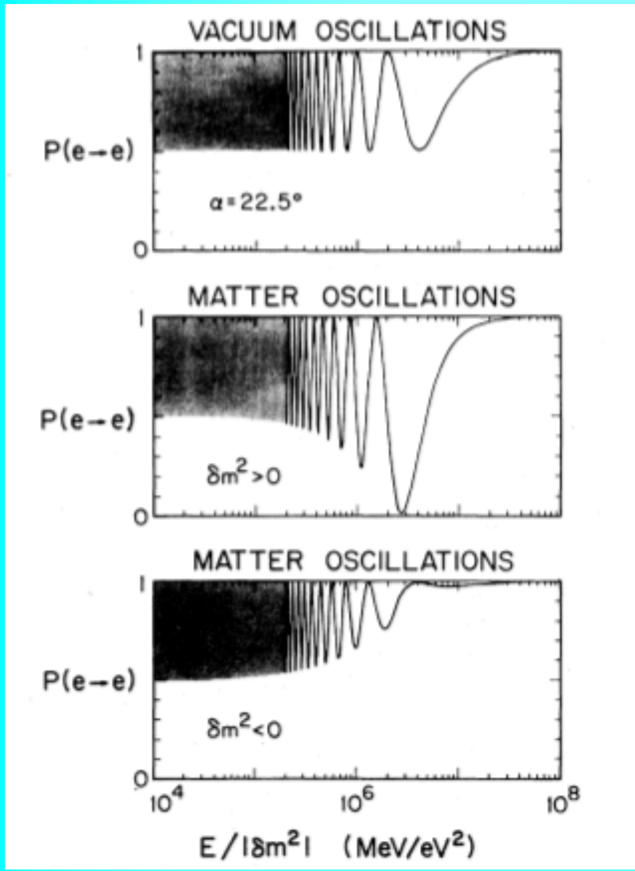

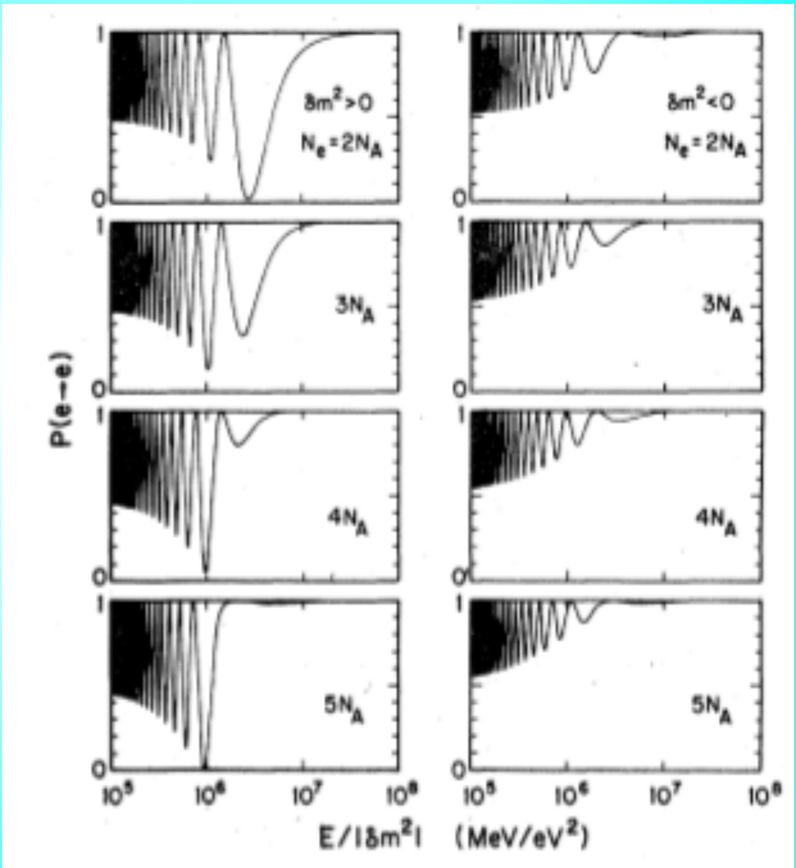

2. Mikheyev and Smirnov

Background

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OLVENA:

Department of Leptons of High Energies and Neutrino Astrophysics, INR Academy of Sciences

G.T. Zatsepin

- Solar neutrino spectroscopy, Gallium, Clorine, Li exp.
- Supernova neutrinos, Artemovsk, LSD
- Cosmic rays, neutrinos, (Pamir..)

A.E.Chudakov

- Baksan Neutrino telescope
- Atmospheric neutrinos

Mikheyev

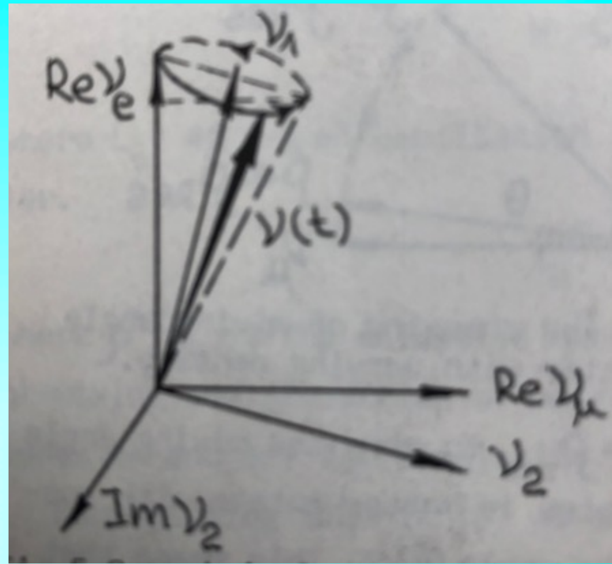
Experimentalist, Baksan telescope
Later – MACRO, K2K, Baikal neutrino telescope

Analysis of the atmospheric neutrino data, searches for oscillations

Smirnov

Cosmic neutrino with Berezinsky
Neutrino decay etc

Oscillations



Geometrical representation of oscillations

Qualitative results and in some cases – quantitative. That played crucial role in developments of MSW

Improvents of sensitivity to oscillations: instead of long distance, use long time of neutrino emission (use long lived isotopes) *A. Linde*

At Moriond 1980 – presented the first bounds on oscillations of atmospheric neutrinos obtained with Baksan telescope

Starting point

February -March 1984

Stas Mikheyev

Do you know the Wolfenstein paper? Is it correct? Should we take into account his effect in the oscillation analysis of the atmospheric neutrinos ?

I did not know about Wolfenstein's paper, Stas gave me reference

I started to read it

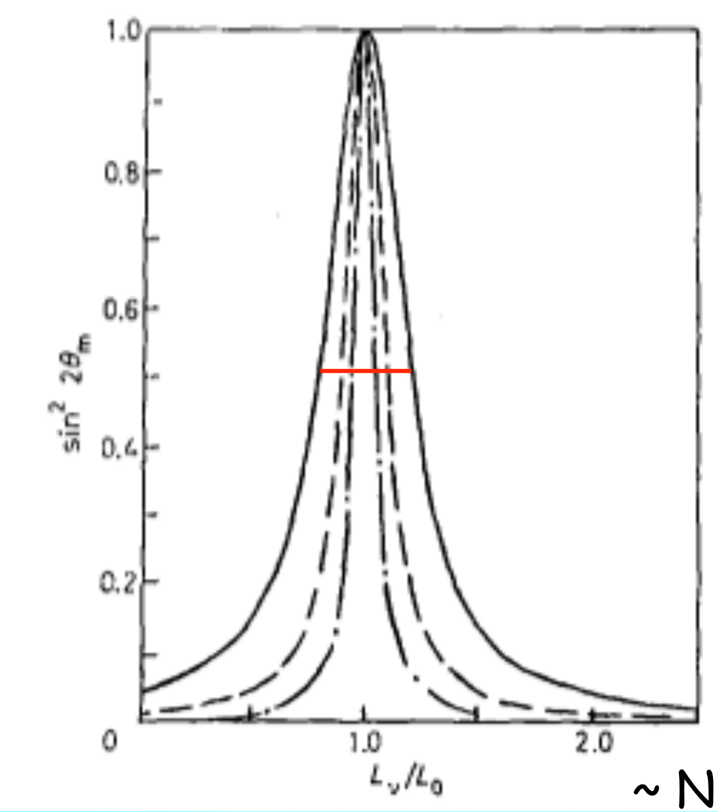
One of the first things I did I have drawn dependence of the mixing parameter $\sin^2 2\theta_m$ as function of l/l_0 for different vacuum mixing angles using formula from Wolfenstein's paper

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{1 - 2\,(l_\nu / l_0) \cos 2\theta + (l_\nu / l_0)^2}$$

The result was astonishing

Resonance

… dependence of $\sin^2 2\theta_m$ on (l_ν / l_0) has a resonance behavior for small values of $\sin^2 2\theta$



~ N E

$\sin^2 2\theta = 0.04, 0.01, 2.5\ 10^{-3}$

At $l_\nu / l_0 = \cos 2\theta$ resonance condition

the amplitude of oscillations reaches maximum: $\sin^2 2\theta_m = 1$

physics

for small mixing $l_\nu \sim l_0$

the eigenfrequency of the system equals the eigenfrequency of surrounding medium

Width of the resonance

$$\Delta(l_\nu/l_0) = (l_\nu/l_0)_{res} \tan 2\theta = \sin 2\theta$$

Oscillation length in resonance

$$l_m = l_\nu/\sin 2\theta$$

Resonance

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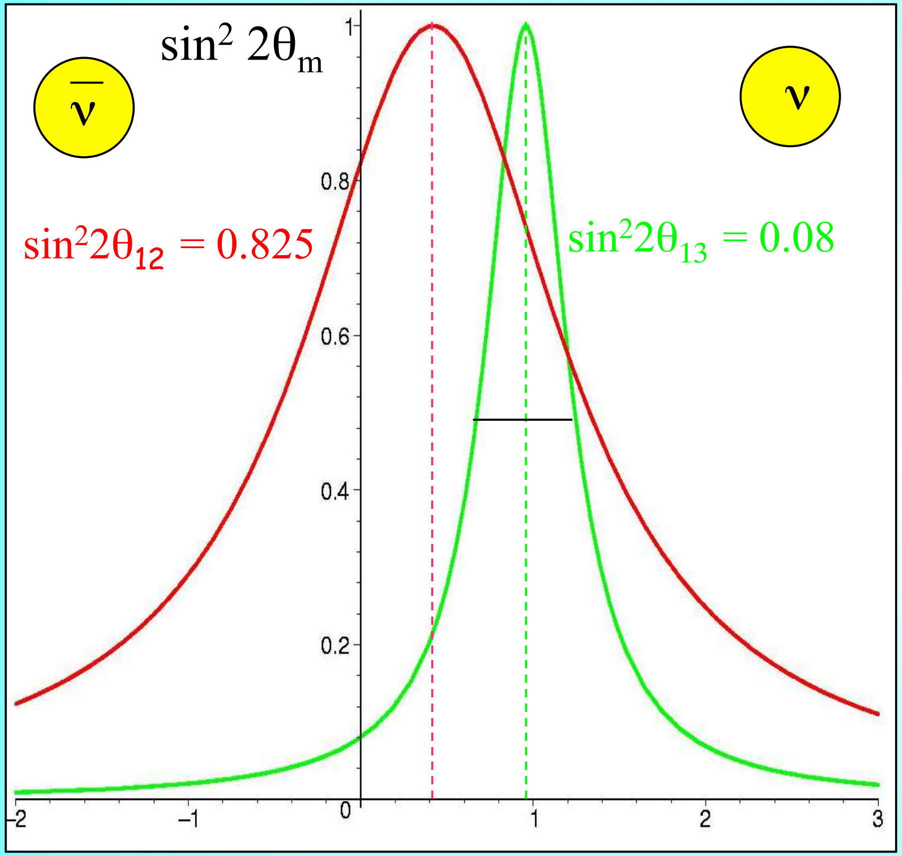


$l_\nu / l_0 \sim n\,E$

Resonance condition

$\sin^2 2\theta_m = 1$

Flavor mixing is maximal

$l_\nu = l_0 \cos 2\theta$

Vacuum oscillation length $\approx$ Refraction length

$V = \frac{\Delta m^2}{2E} \cos 2\theta$

Resonance width: $\Delta n_R = 2 n_R \tan 2\theta$

Resonance layer: $n = n_R +/- \Delta n_R$

Two different realizations of resonance

"Two different manifestations of the resonance enhancement can be distinguished"

Constant density, Continuous neutrino spectrum

Resonance enhancement of oscillations

Varying density, Monoenergetic neutrinos

Adiabatic conversion of neutrinos

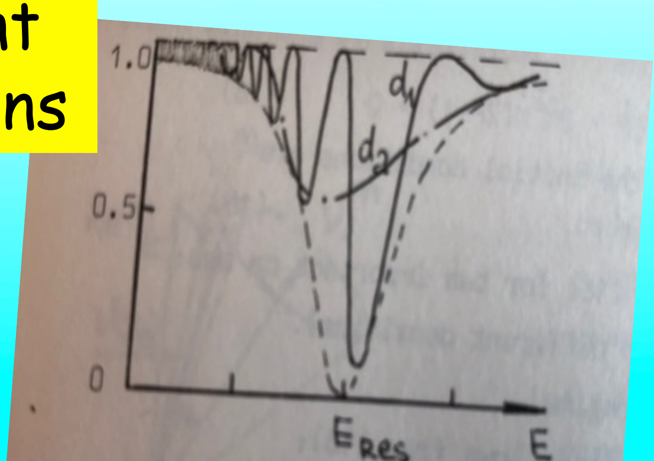


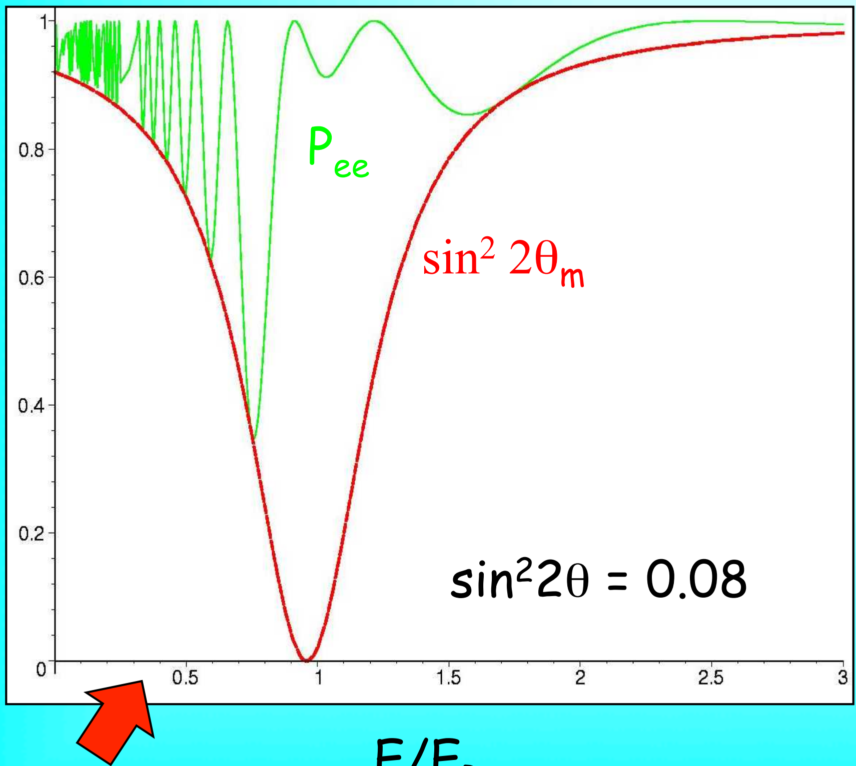
Fig.4 Resonant amplification of ν-os-

Resonance enhancement of oscillations

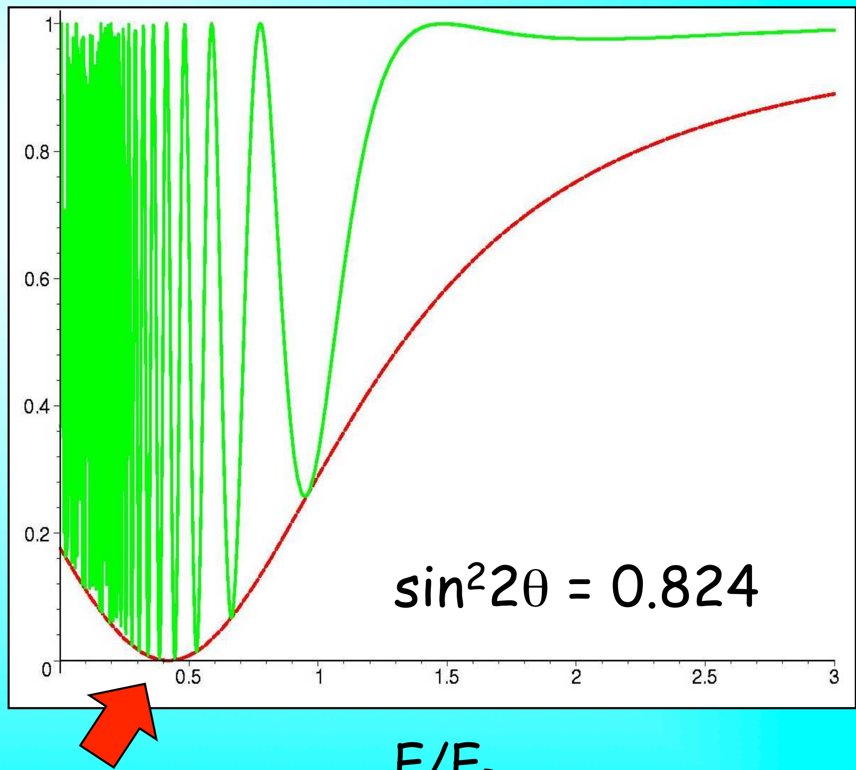
Resonance energy $E_R = a \dfrac{\Delta m^2 \cos 2\theta}{\rho}$ $a = m_N / 2\sqrt{2}\, G_F Y_e$

Width of the resonance (enhancement region) $\Delta E_R = E_R \tan 2\theta$

$\dfrac{F(E)}{F_0(E)}$



E/E$_R$



E/E$_R$

$k = \pi L / l_0 = 10$ L – layer length

Comments

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Wolfenstein obtained enhancement of probability due to matter effect - no discussion

Barger et al., have written condition for maximal oscillation depth and shown enhancement of oscillations. But resonance nature was not uncovered (used large mixing where it is not very clear)

MS: realized the resonance nature of the matter effect introduced notion of resonance, studied nature and properties of the resonance. In particular,

The smaller the vacuum mixing (strength of coupling) the narrower resonance

Shown that it has the same features as resonances in other systems

Explored possible manifestations of the resonance

Varying density

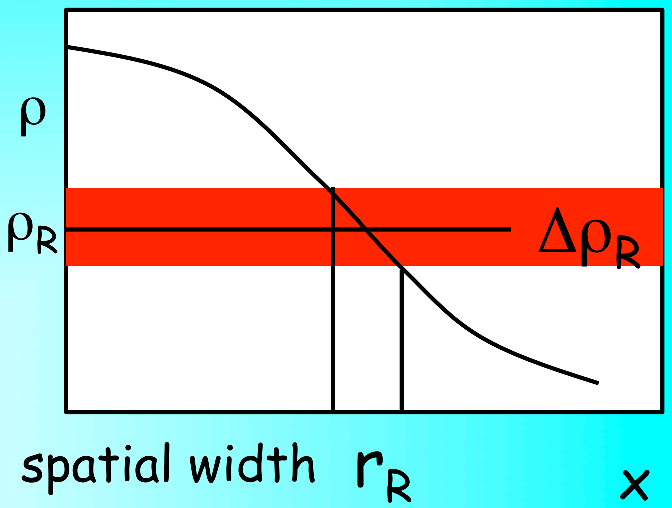
Monoenergetic neutrinos, E, varying density

Significant enhancement of oscillations occurs in the resonance layer with density

$$\rho_R = a \frac{\Delta m^2 \cos 2\theta}{E}$$

and width

$$\Delta\rho_R = \rho_R \tan 2\theta$$



Resonance enhancement will be sizable if the resonance layer is sufficiently thick

$$r_R > l_m^R = l_\nu / \sin 2\theta$$

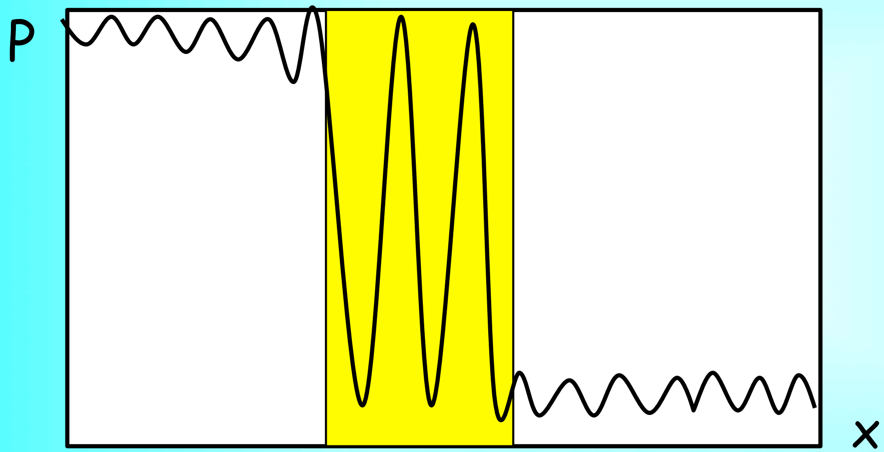
$$r_R = \left(\frac{d\rho_R}{dr}\right)^{-1} \Delta\rho_R$$

Relations for constant density in general, inapplicable in this case, but concepts of resonance layer its, density and width are useful for qualitative analysis

Furthermore, condition for strong transformation gives correct adiabaticity condition

Puzzle

In layer with varying density (Sun) both the resonance condition and condition for strong transformation are satisfied in wide energy range, so one would expect strong transitions in wide energy range



Resonance layer

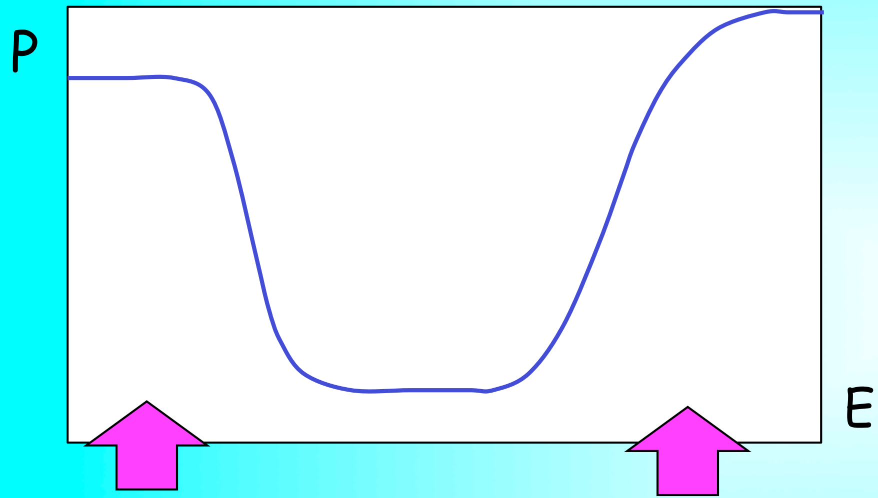
in a spirit of the slab model by Rosen and Gelb

If oscillations with large amplitude occur in the resonance layer, why the phase of oscillations at the end does not change with energy?

Since $l_m^R \sim E$ and r_R = const. (as for the Sun)
the condition $r_R > l_m^R$ is broken at high energies

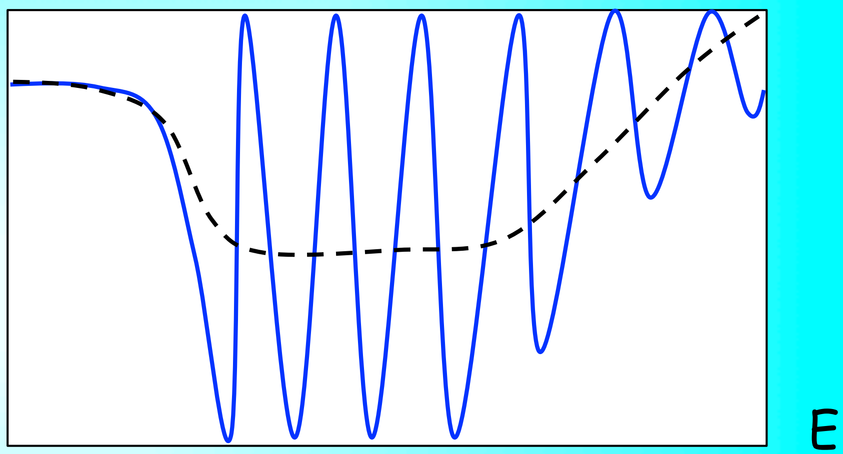
If both resonance condition and condition for strong transition (width of the layer) are satisfied

Our first guess



No resonance

Why not this?



Since $l_m^R \sim E$ and r_R = const. (the Sun) the condition $r_R > l_m^R$ is broken at high energies

Numerical computations confirmed this

Numerical solution

From Wolfenstein's evolution equation equations for probabilities

$$dP/dt = -\,2M\,I$$
$$dI/dt = -\,m\,R + M(2P - 1)$$
$$dR/dr = mI$$

$P = \nu_e^{*}\nu_e$

$R + iI = \nu_\mu^{*}\nu_e$

elements of density matrix, or components of neutrino polarization vector

$$2M = \frac{2\pi}{l_\nu}\sin 2\theta$$

$$m = \frac{2\pi}{l_\nu}\left(\cos 2\theta - \frac{l_\nu}{l_0}\right)$$

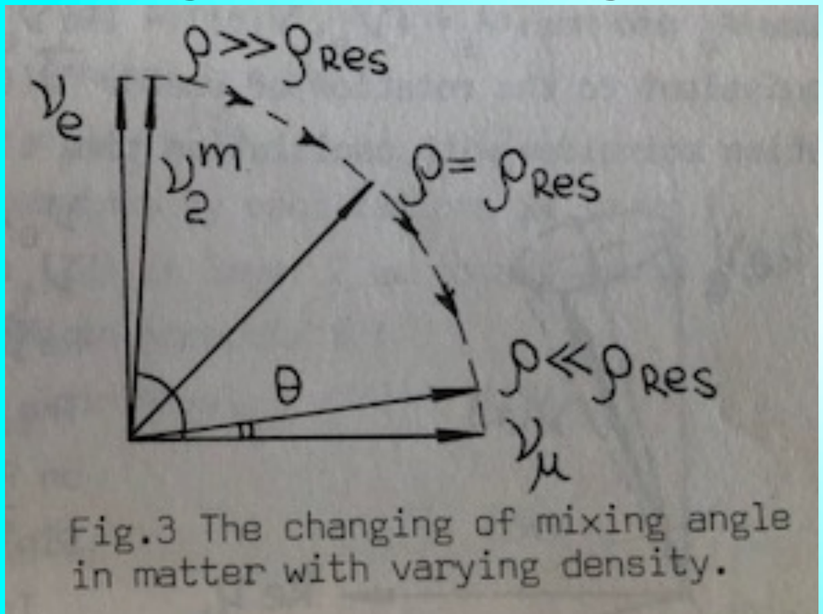
If ν_e is produced, the initial conditions

$$P(0) = 1,\quad I(0) = R(0) = 0$$

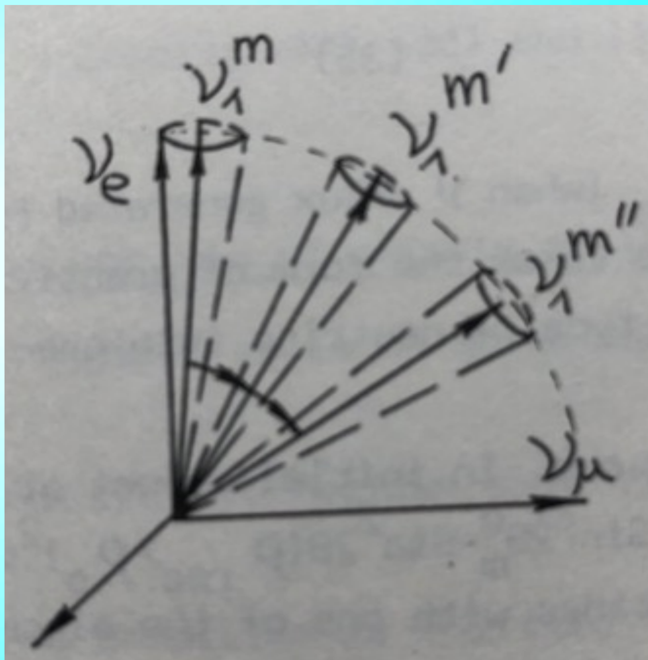
Towards adiabatic solution

In attempt to understand results used graphic representation

With changing density the mixing in matter changes



This mixing angle determines direction of the cone axis



From Moriond 86, used also in WIN 85

If density (and therefore the mixing angle in matter) changes slowly, the system (neutrino vector) has time to adjust these changes

This allowed to explain numerical results

Wolfenstein's letter and adiabaticity

We have sent to Wolfenstein one of preliminary versions of our paper

He had replied few months later. In short letter (unfortunately lost) he said essentially that

it should be no strong transitions inside the Sun due to adiabaticity

and gave reference:

L. Wolfenstein, in ``Neutrino-78'', Purdue Univ. C3, 1978.

We could not find this paper but cite it in our paper.

Also we started to call effect of adjustment of the system to the density change the adiabatic transition and the condition of strong transition, $r_R > l_m^R$, the adiabatic condition.

Our reply was

it is due to the adiabaticity strong transformation can occur.

We introduced this terminology in proofs of YF paper

Comments

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Wolfenstein's reply probably explains why he did not proceeded with further developments of his ideas

Bruno Pontecorvo told me that he had discussion with Wolfenstein and they concluded that it seems there is no practical outcome …

One can guess why Wolfenstein thought that adiabaticity prevents from strong transitions:

If initial density is large and vacuum mixing is small, then both in initial and final states the mixing is strongly suppressed. So, the adiabaticity which ensures that result of transtions depends on initial and final conditions only and does not depend on what happens in between.

May be he missed that although mixing is suppressed in the initial and final states, the states are different: in initial state $\nu_e \sim \nu_{2m}$, while in final state $\nu_e \sim \nu_{1m}$ (level crossing) .

MS 1985

We generalized our adiabaticity condition as

$$\left(\frac{d\rho}{dr}\right)^{-1} \rho > l_\nu / \tan^2 2\theta$$

which is reduced in resonance to $r_R > l_m^R$

Adiabaticity parameter

$$\kappa_R = r_R \,/\, l_m^R$$

It can be rewritten in the form

$$\kappa_R = \left(\frac{d\rho}{dr}\right)^{-1} \rho \tan^2 2\theta / l_\nu$$

$$\kappa_R = \left(\frac{d\rho}{dr}\right)^{-1} \rho \frac{\sin^2 2\theta}{l_0 \cos 2\theta} \sim \left(\frac{d\rho}{dr}\right)^{-1} \rho^2$$

Adiabaticity condition $\kappa_R > 1$

Adiabatic conversion

MS 1985

If neutrinos are produced at ρ_{max} and passes the layer with

$$0 \sim \rho_{min} \ll \rho_R(E) \ll \rho_{max}$$

then the initial mixing angle $\theta_m \sim \pi/2$

$$\nu_{init} = \nu_e \sim \nu_{2m}(\rho_{max})$$

ν_e coincides with the eigenstste ν_{2m}

Since ν_{2m} is the eigenstate in matter, in the course of propagation

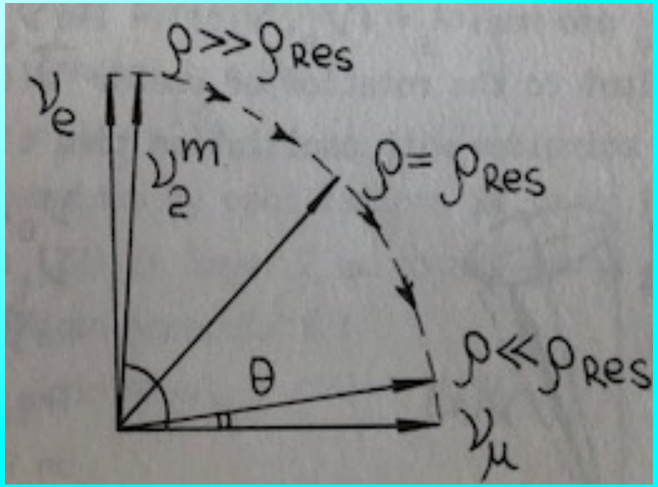
$$\nu_{2m}(\rho_{max}) \rightarrow \nu_{2m}(\rho_{min}) = \nu_{2m}(0) = \nu_2$$

In final state $\rho_{min} = 0$, $\theta_m = \theta$

Mixing angle changes by $\sim \pi/2$

$$\nu_{final} \sim \nu_{2m}(0) = \nu_2 \sim \nu_\mu$$

adiabaticity



Precisely: $< \nu_e | \nu_{final} > = < \nu_e | \nu_2 > = \sin\theta$

the survival probability $P = \sin^2\theta$

Other limits

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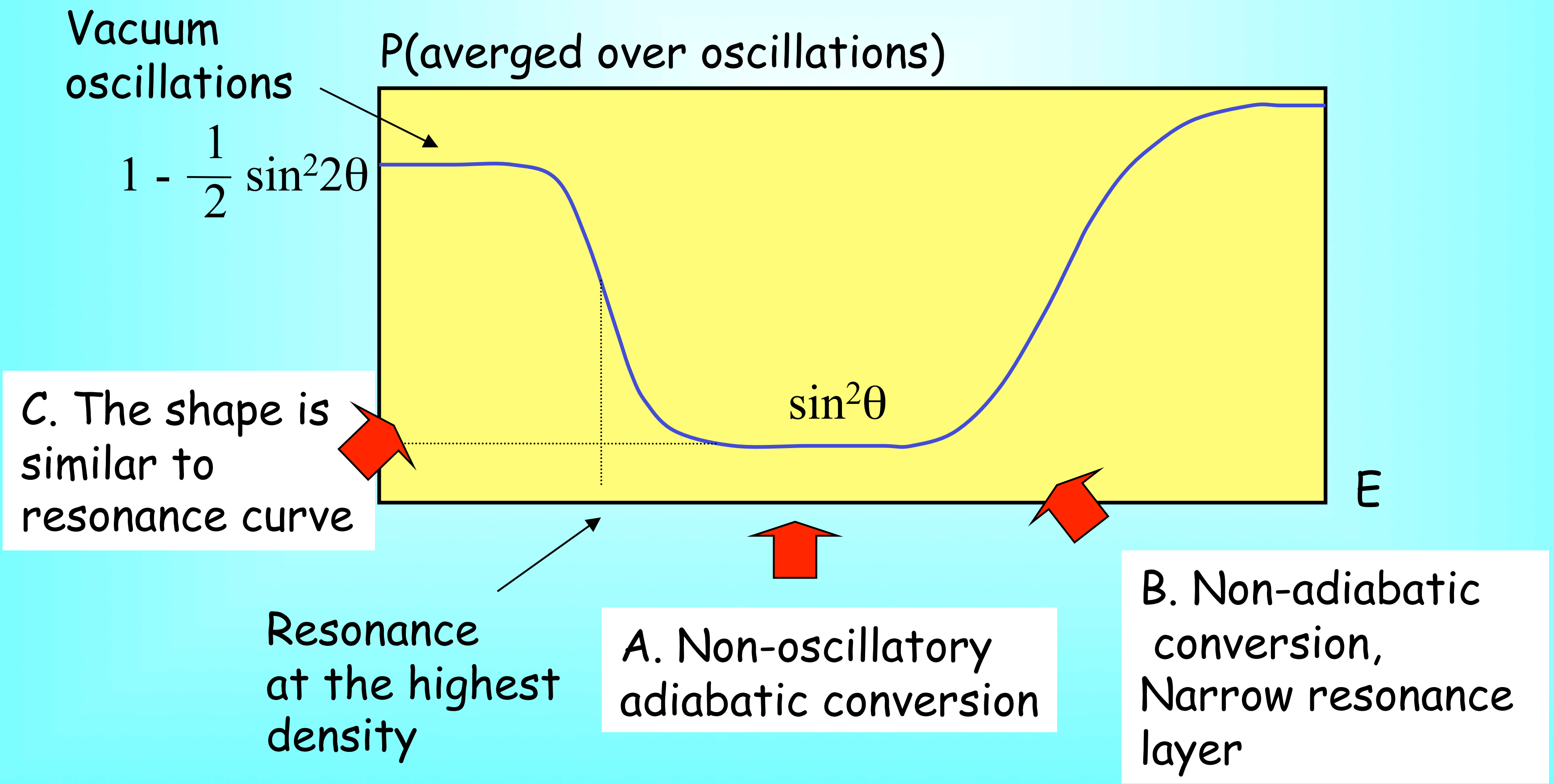
"Suppression bath"



Theory of adiabatic conversion

Spring, summer 1985 complete understanding adiabatic conversion.

S.P. Mikheev, A.Yu. Smirnov, Neutrino oscillations in a variable/density medium and ν - burst due to the gravitational collapse of stars,
Sov. Phys. JETP 64 (1986) 4-7, Zh.Eksp.Teor.Fiz. 91 (1986) 7-13, arXiv:0706.0454 [hep-ph]

To avoid problems with publications, we
- tried to hide the term resonance
- did not discussed solar neutrinos and even
- did not include reference to our paper on resonance enhancement

Did not helped….

Paper submitted in the fall 1985 to JETP letter, rejected (no reason for quick publication), resubmitted to JETP in December 1985

Results have been reported at 6th Moriond workshop (January 1986), Reprinted in the Solar neutrinos : the first Thirty Years.

Theory of Adiabatic conversion

From equations for P, R, I, derived in Yad. Fiz. - equation of the third order for P

$$M\frac{d^3P}{dt^3} - \frac{dM}{dt}\frac{d^2P}{dt^2} + M(M^2 + 4\bar{M}^2)\frac{dP}{dt} - 2\bar{M}^2\frac{dM}{dt}(2P-1) = 0, \quad (1)$$

adiabatic part

where

$$M = (2\pi/l_\nu)(\cos 2\theta - l_\nu/l_0), \quad 2\bar{M} = (2\pi/l_\nu)\sin 2\theta,$$

Initial conditions:

$$P(0) = 1, \quad \frac{dP(0)}{dt} = 0, \quad \frac{d^2P}{dt^2} = -2\bar{M}^2.$$

Adiabaticity - neglect highest (third and second) derivatives

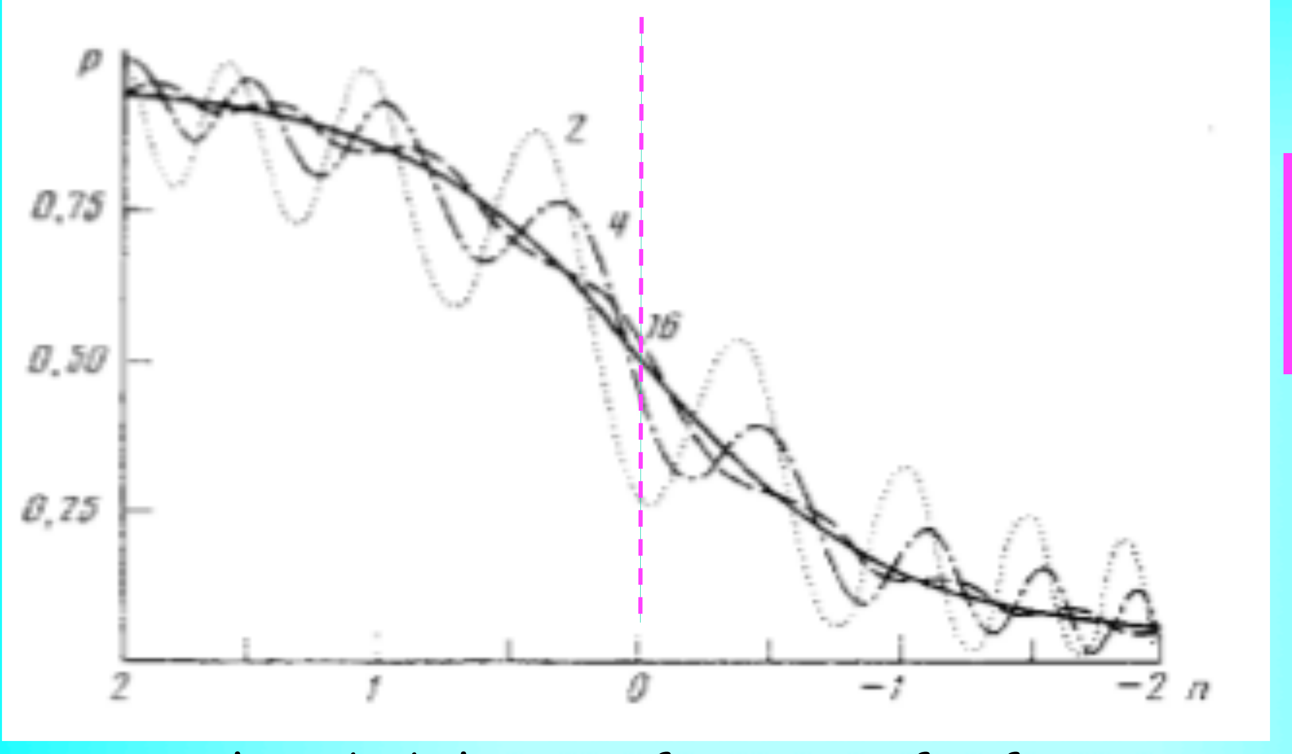
Solution for averaged P

$$\bar{P}(n, n_0) = [1 + n_0(n_0^2+1)^{-1/2}n(n^2+1)^{-1/2}]/2;$$

n, n_0 - distance from the resonance layer in units of width of the resonance layer

$$n = (\rho - \rho_R)/\Delta\rho_R$$

Adiabatic solution



P

Universal form

No oscillations with maximal depth in resonance

n (distance in density scale)

Survival probability as function of n for different values of n_0 (numbers at the curves)

With increase of initial density n_0 the amplitude of oscillations decreases. P converges to asymptotics non-oscillatory form

$$P(n)_{non-osc} = \frac{1}{2} [1 + n (n^2 + 1)^{-1/2}]$$

$n \to$ - inf

$P \to \sin^2\theta$

Since

$$n\,(n^2 + 1)^{-1/2} = \cos 2\theta_m$$

$$n_0\,(n_0^2 + 1)^{-1/2} = \cos 2\theta_m^0$$

One obtain standard form for the probability

$$P = \tfrac{1}{2}\,[1 + \cos 2\theta_m \cos 2\theta_m^0]$$

Wolfenstein's uknown paper

L. Wolfenstein, Effect of matter on Neutrino oscillations in ``Neutrino-78'', Purdue Univ. C3, 1978.

Contribution (not even a talk)

Practically no citations, no impact…
I saw the paper for the first time in 2003, when E. Lisi asked me to check if proceedings are available in the ICTP library

The case of Massless neutrinos

In the Sun the mixing in matter varies due to change of the chemical composition: y = neutron/proton changes from 0.41 to 0.13 (in original paper he just averaged this and considered constant density)

The percentage change in θ_m per oscillation is small (…1000 oscillations on the way out the sun) so that we can apply the adiabatic approximation

$$|\langle \nu_e | \nu_e \rangle|^2 = \cos^2\theta_0 \cos^2\theta_m(x) + \sin^2\theta_0 \sin^2\theta_m(x) + \tfrac{1}{2} \sin 2\theta_0 \sin 2\theta_m(x) \cos \Phi(x)$$

θ_0 and θ_m - mixings in matter in initial and in a given point x

... continued

xx

... in this case (varying density) neutrinos are transformed not only by virtue of the oscillating phase but also by adiabatic change in propagating eigenvectors.

For example, if $\theta_0 = 0$, the oscillating term vanishes but there is transformation ν_e into ν_μ since neutrino is propagating in eigenstate which originally ν_e but adiabatically transforms into a mixture of ν_e and ν_μ given by $\theta_m(x)$

Publications

What, when and how things were published
In contrast to Wolfenstein I can explain our case

The first paper had been submitted to Phys. Lett. Bin 1984 and was rejected (no reason for quick publication)

Updated version as been submitted to Yadernaya Fizika
Soviet Journal of Nuclear physics

Spring 1985 the paper got negative report and was almost rejected from Yad. Fiz.

Sceptical reaction from Pontecorvo

G.T. Zatsepin brought paper to Italy and asked Castagnoli (collaborator in LSD experiment) to publish it in Nuovo Cimento

G.T. ? If paper is wrong people will forget it, if correct / it is very important

Paper (slightly modified) was soon accepted to Nuovo Cim.

Suddenly it was accepted by Yad. Fiz. (Editor Kobsarev)
We made some corrections at proofs.

Talk at WIN-85, Savonlinna, Finland, June 16 - 22, 1985

The paper on adiabatic solution and supernova neutrinos has been submitted in the fall of 1985 to ZhETP Letter,
It was rejected (do not required of quick publication) ,
Resubmitted to ZHETP, published in 1986

Results have been presented at Moriond workshop, Jan. 1986

Perestroyka time a paper can be submitted to journal abroad only after it is published in Soviet Journal

We decided to preent our results at conferences and then we put this in review Uspechi Fiz. Nauk 1987

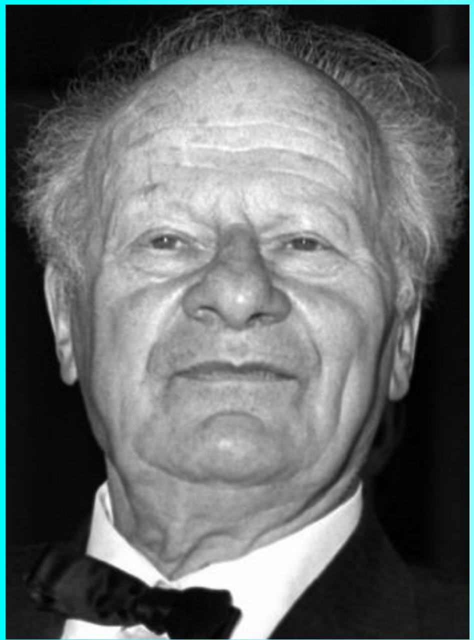
3. Further developments

Level crossing

Adiabatic condition
As condition that there is no transitions between eigenststes

Adiabaticity violation

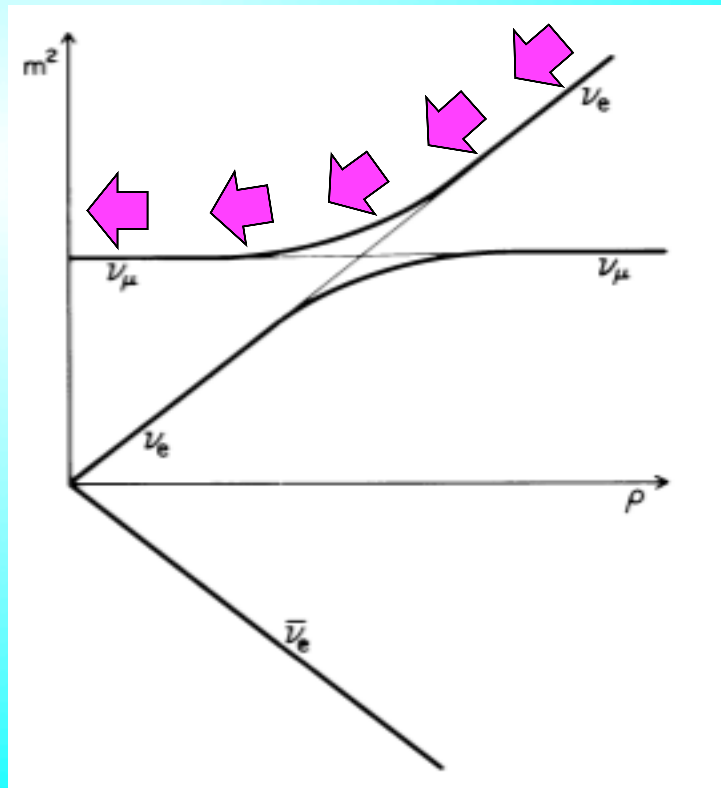
MSW as Level crossing phenomenon



H.A. Bethe, Phys.Rev.Lett. 56 (1986) 1305

Dependence of the eigenvalues of Hamiltonian in matter (effective masses) on density

Minimal splitting - in resonance



Adiabatic evolution as motion along fixed level without jump to another level.
ν_e produced at high density follows the upper curve

= No transition between the eigenstates

Important for further developments

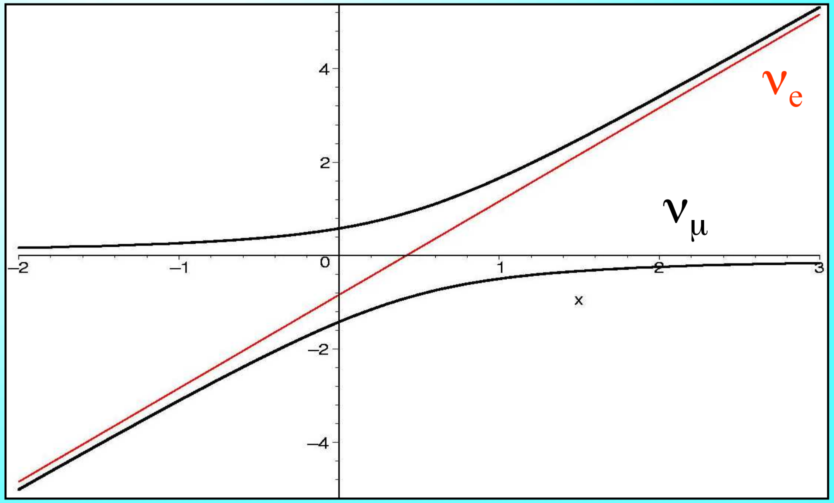
Level crossing phenomenon

Concluding talk at WIN 85

During excursion

Nicola Cabibbo
1935 - 2010

Serguey Petcov told me about your paper and I would like to include your result in my talk. I think the effects can be understood as the level crossing processes



Do you agree with this?

Yes

I missed Cabibbo talk – we left Savonlinna one day before

... even before

Valery Rubakov

Superheavy Magnetic Monopoles and Proton Decay, 1981

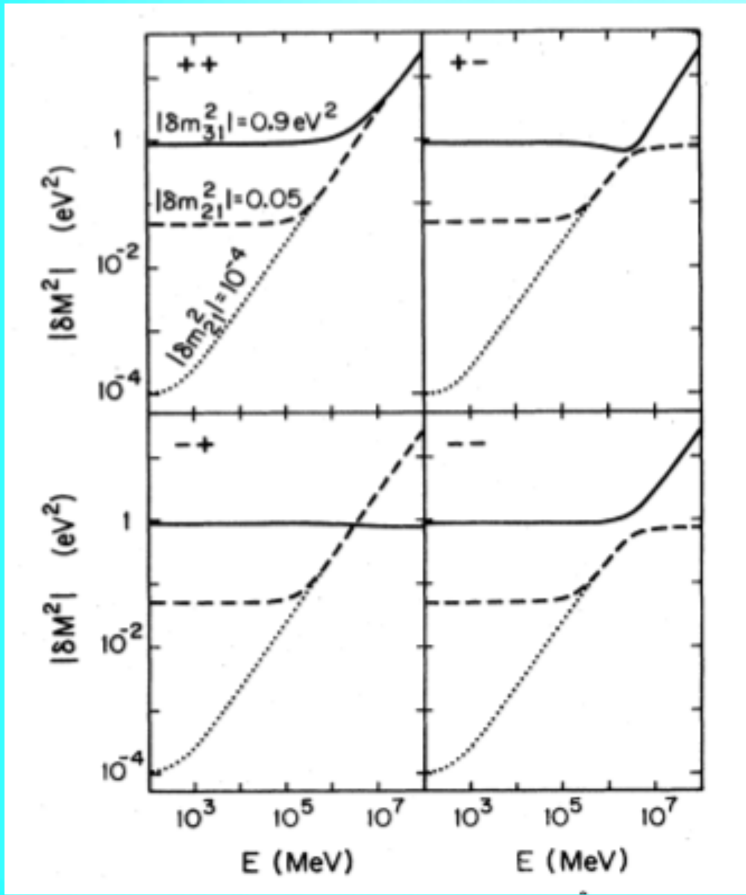
In spring 1985:

"Your transition has some similarity with catalysis of proton decay when monopole propagates near nucleon . This has interpretation as the level crossing phenomenon"

Complementary description in terms of the eigenvalues of the system.
I was happy with description in terms of the eigenstates

Almost the level crossing scheme ...

V. D. Barger, K. Whisnant, S. Pakvasa, R.J.N. Phillips, Phys.Rev. D22 (1980) 2718



Here dependence on energy and not density

Constant density

Completing theory of the adiabatic conversion

A. Messiah, Treatment of ν_{sun} -oscillations in solar matter. The MSW effect. 6th Moriond workshop, Tignes Jan. 1986 p.373

MS call it the "resonant amplification effect" a somewhat misleading denomination.

He did not liked/used the term "resonance" , claiming that effect can be readily deduced from the adiabatic solution of the equation of flavor evolution.

used complicated notations, operator form, etc.

Derived evolution equation for the eigenstates in matter ν_{im} or equivalently, the corresponding evolution matrix $U_H(x, x')$

$$i \frac{dU_H}{dx}(x, x') = \left[W(x)\, \sigma \mathbf{w}(x') + \frac{d\theta_m}{dx} \sigma \mathbf{k} \right] U_H(x, x')$$

$W(x)$ → level spacing; $\mathbf{w}(x')$ → σ_3; $\mathbf{k}$ → σ_2; U_H → ν_m

Adiabatic solution when second term can be neglected

...continued

Adiabatic condition $\omega = \frac{d\theta_m /dx}{2W} \ll 1$

$$\omega = \frac{\text{Rotation velocity of eigenvector}}{\text{level spacing}}$$

Adiabaticity parameter

Eigenvectors of Hamiltonian in matter rotate slowly

Components of vector of the neutrino state along the rotating eigenvectors stay constant

Corrections to the adiabatic solution ~ ω^2

Introduced amplitude of transition between eigenstates

$\beta = A(\nu_{2m} \to \nu_1)$

$$P_{ee} = \frac{1}{2}\left[1 + (1 - 2|\beta|^2)\cos 2\theta \cos 2\theta_m(x')\right]$$

known as Parke formula

$|\beta|^2 = P_c$ jump, flop probability

Adiabatic solution: $\beta = 0$

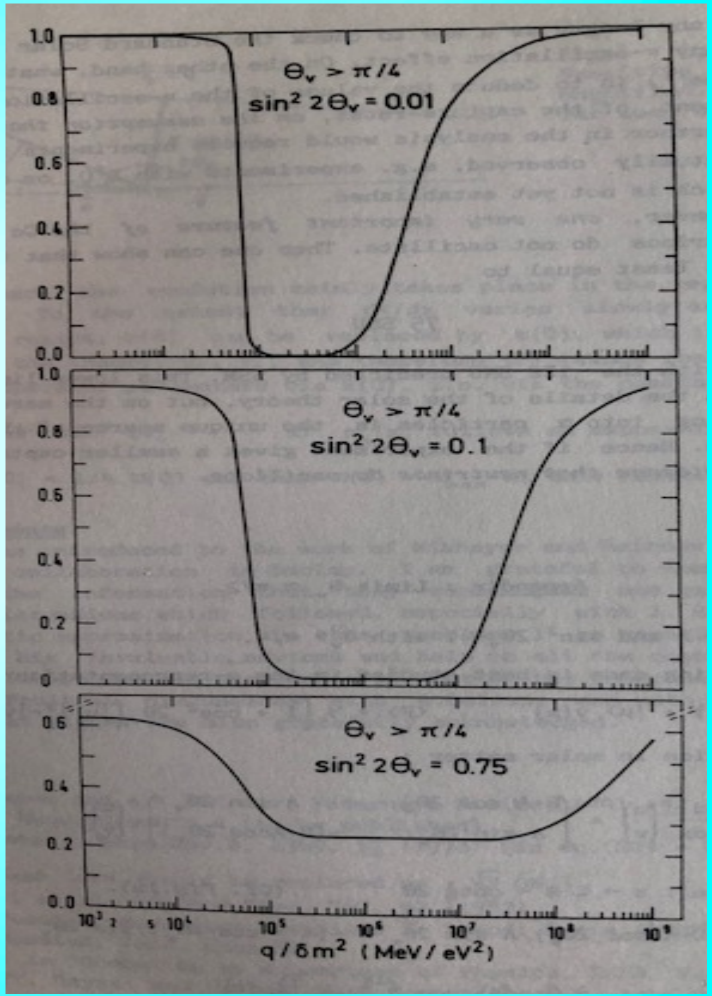
Adiabatic solution

xx

$$\nu_f = \cos\theta_m^0\, e^{i\,\phi_1}\nu_1 + \sin\theta_m^0\, e^{i\,\phi_2}\nu_2$$

↑

Mixing in the production point



Adiabaticity violation: L-Z

W.C. Haxton, Phys.Rev.Lett. 57 (1986) 1271-1274
S. J. Parke, Phys.Rev.Lett. 57 (1986) 1275-1278

Level crossing picture

Transitions between the levels are described by the Landau – Zenner probability valid for linear dependence of density

$$P_c = |\beta|^2 = P_{LZ} = e^{-\pi\gamma/2}$$

γ is the adiabaticity parameter.

Similar to level crossing problem in atomic physics

Haxton

Along diagonal line of the MSW triangle in the Δm^2 - $\sin^2 2\theta$ plane with $\theta_m \sim \pi/2$ for $\theta \sim 0$

$$P_{ee} = 1 - P_c$$

Parke

General expression for $\theta_m \neq \pi/2$ and non-zero θ

The same expression as in Messiah paper

Geometrical approach

J. Bouchez M. Cribier, W Hampel, J. Rich, M Spiro, D. Vignaud, Z.Phys. C 32, (1986) 499

Graphical representation which uses analogy with electron spin precession in the magnetic field

In the space of components of neutrino polarization vector (P, R, T) = (E, Y, X) → probabilities (in contrast to amplitudes we used)

$$\frac{d\theta_m / dx}{2W} = \text{const}$$

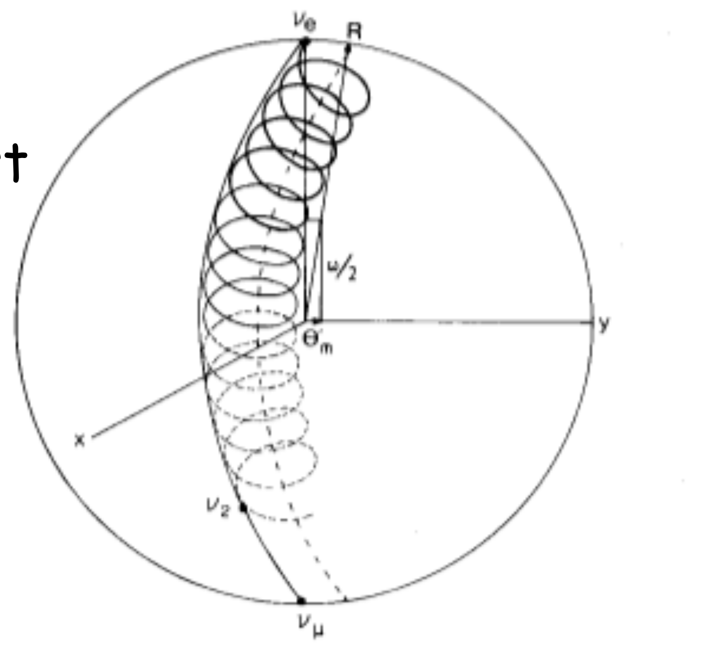


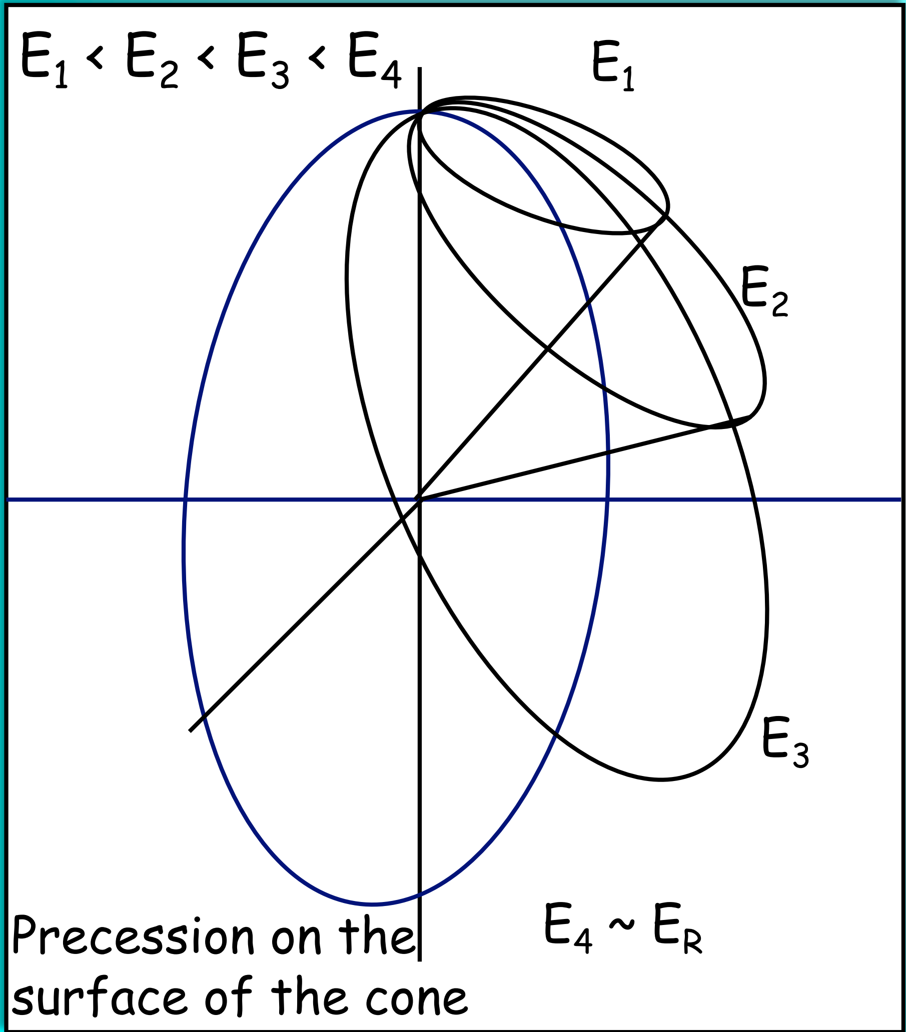
Fig. 3. Sphere representing any superposition of ν_e and ν_μ

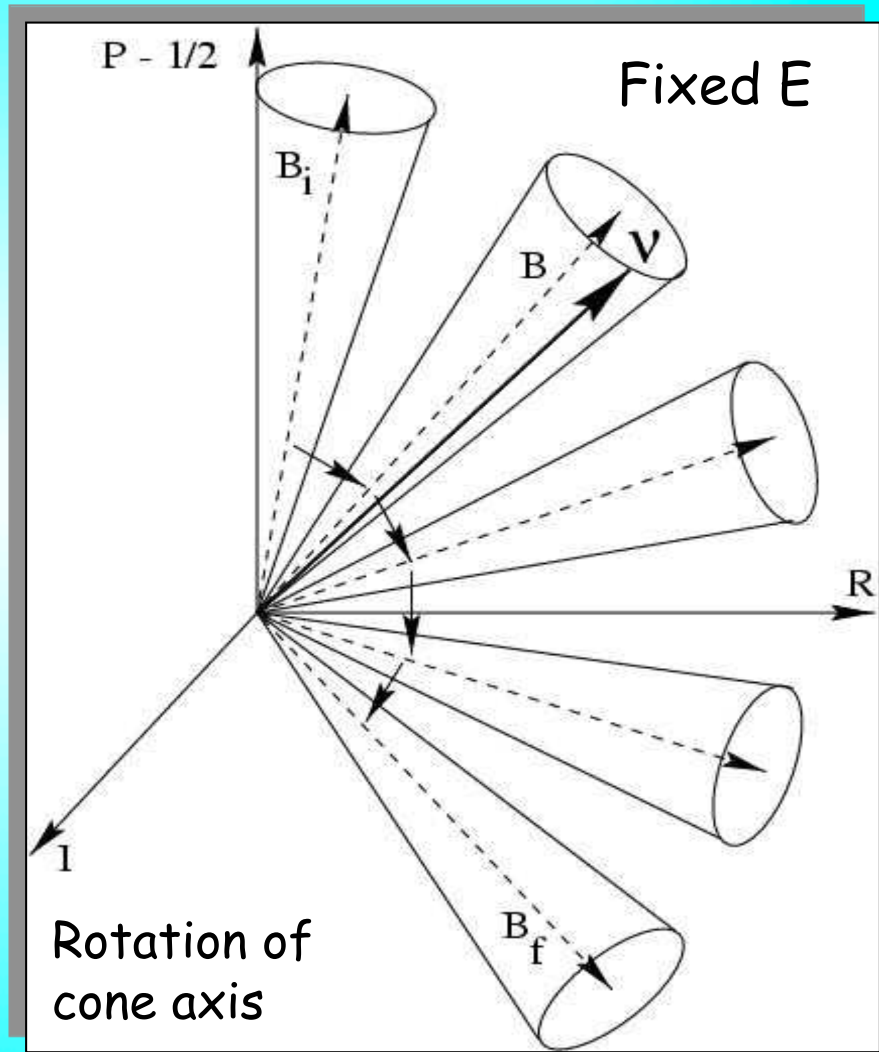
Remark:

Resonant oscillations -
As we shall see this is not exactly what happens in the Sun (varying density)

Graphic representation

xx





Resonance enhancement

Adiabatic conversion

Further developments

Dynamics

Adiabaticity violation
Fomalizm beyond LZ,
Exact solutions
For different density distributions

Different media:
thermal, polarized,
magnetized , moving,
fluctuating

Mixing induced by matter, interaction of neutrino magnetic moment with magnetic fields

Neutrino systems

$\nu_\mu \to \nu_s$
$\nu_\mu \to \nu_s$
$\nu_e \to \nu_s$
$\nu_e \to \bar{\nu}_\mu$

Applications

Solar neutrinos

Supernova neutrinos

Cosmic neutrinos in sources

Neutrinos in Early Universe

Summary

Wolfenstein

- Coherent forward scattering should be taken into account
- Induces oscillations of massless neutrinos and modify usual oscillations
- Strong non-trivial modification at $l_\nu \sim l_0$ can enhance transition probability
- Evolution equation

  Adiabaticity for massless neutrinos adiabatic formula

Mikheyev Smirnov

- Resonance, properties
- Adiabatic condition, adiabatic transitions
- Graphic representation

Further development

- MSW as level crossing
- Adiabaticity violation formalism

Flop /jump probability

Epilog

1998 in final Homestake publication – no reference to MSW. Neutrino spin-flip in magnetic field the main explanation

2002 – 2004 LMA MSW has been established as the solution of the solar neutrino problem

2008 Cabibbo Data confirmed original Pontecorvo solution of the solar neutrino (?) and reject spurious MSW solution

2015 In scientific background description Nobel committee put formula for oscillations in medium with CONSTANT density in connection to solar neutrinos

2017 BOREXINO further confirmed LMA MSW solution

2018 K Lande: Homestake did not observed time variations…

Back-up

In fact,

MSW: two effects

Both are related to modification of mixing in mater
But different dynamics
Different degrees of freedom involved

Resonance enhancement of oscillations

uniform medium with constant parameters

Phase difference increase between the eigenstates

$\phi(t)$

$\theta_m(E)$

Mixing does not change but depends on energy

Adiabatic conversion

Non-uniform medium or/and medium with varying in time parameters

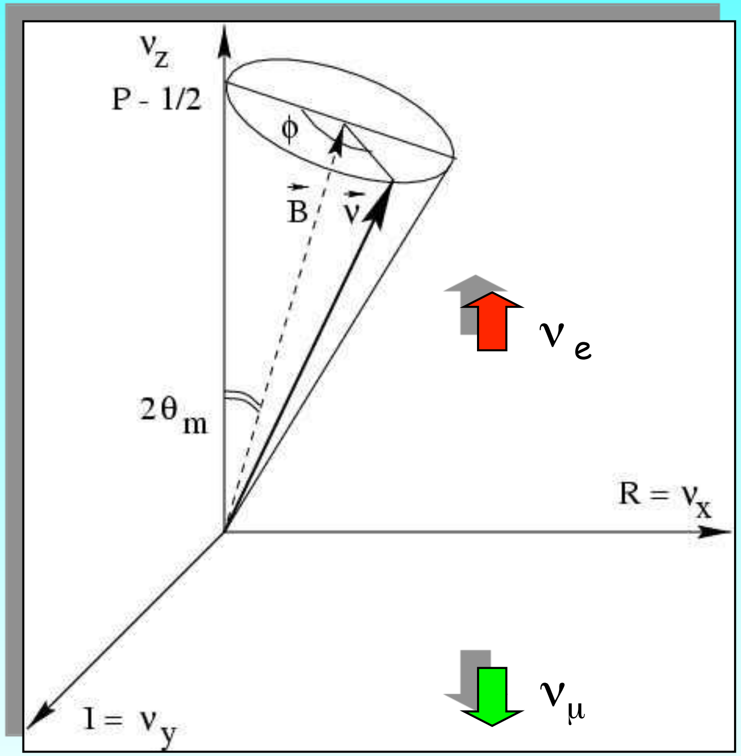
Change of mixing in medium → change of flavor of the eigenstates

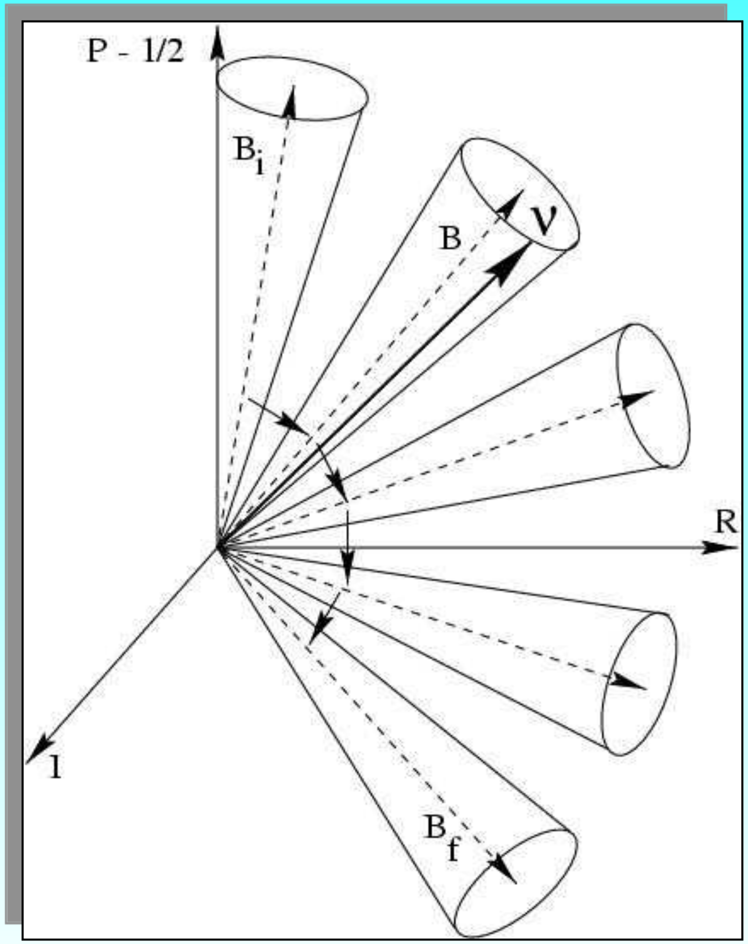
$\theta_m(t)$

Phase is irrelevant

Graphic representation

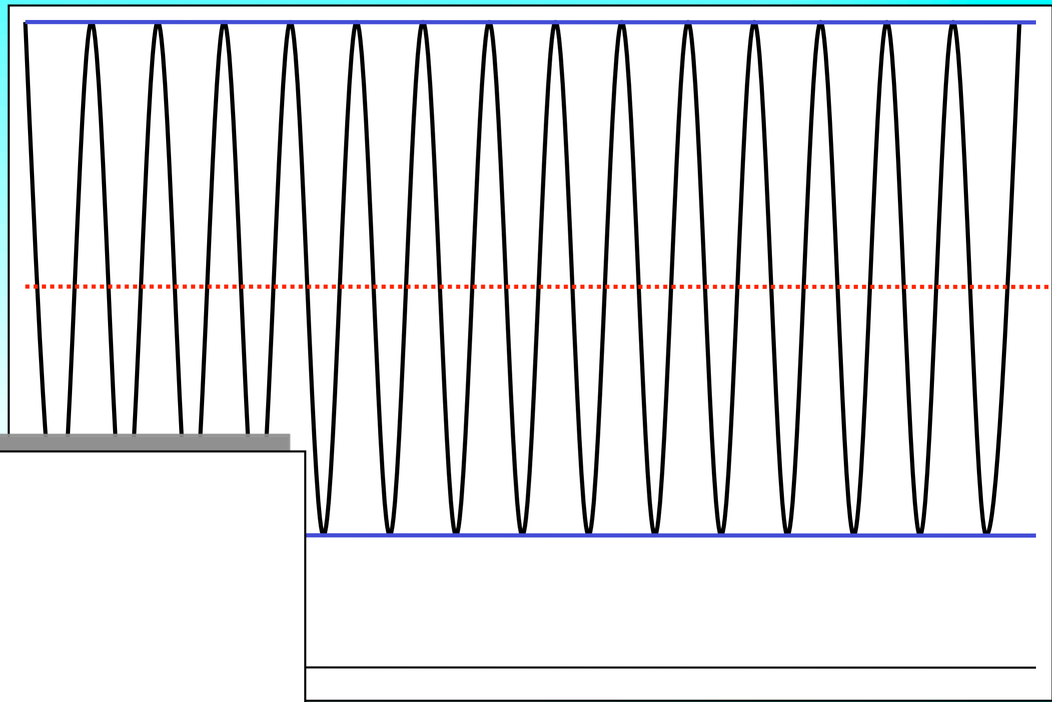
Adiabaticity



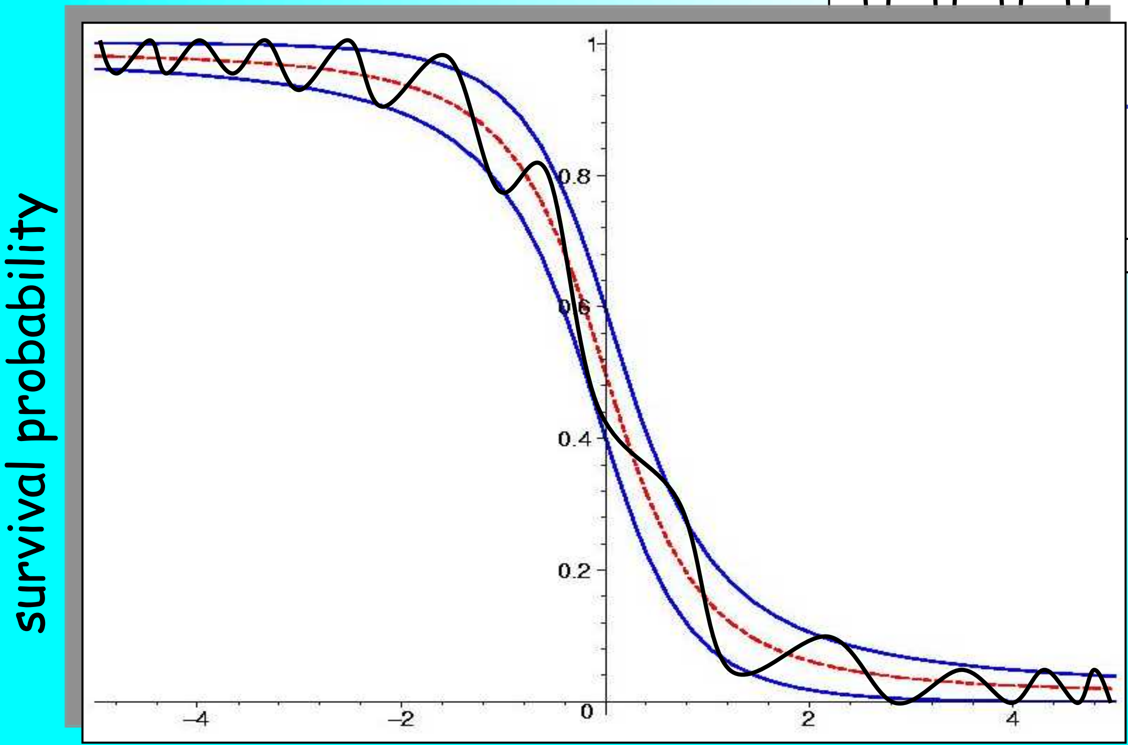


Spatial picture

Oscillations



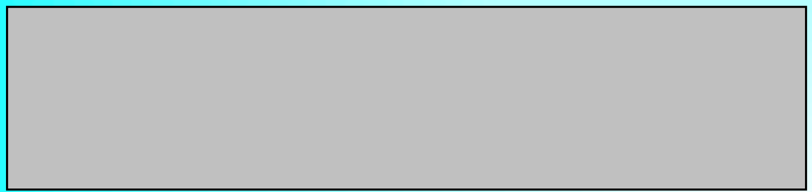
Adiabatic conversion



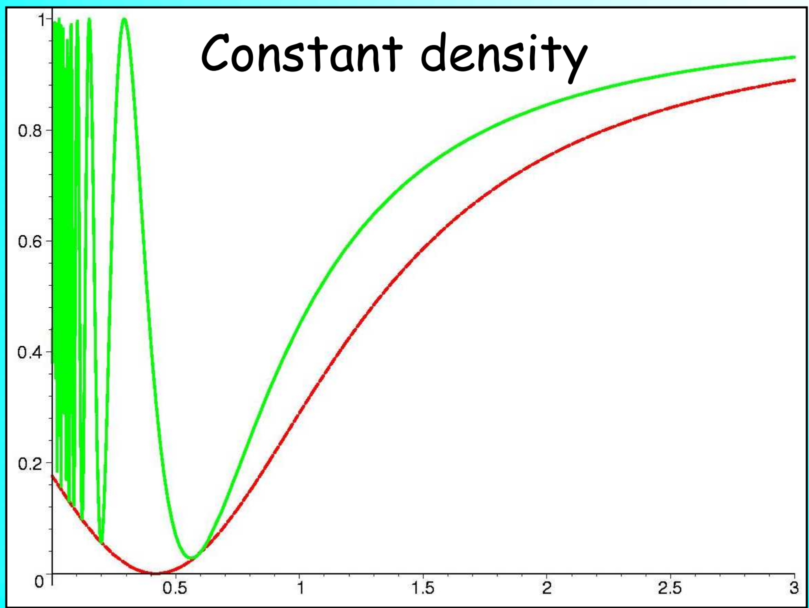
A Yu Smirnov

Energy dependence

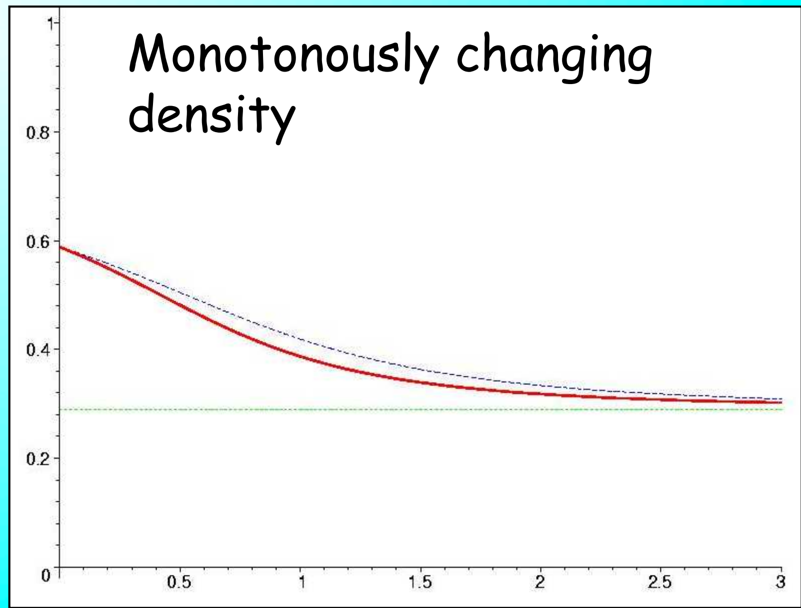
Resonance enhancement ν Adiabatic conversion



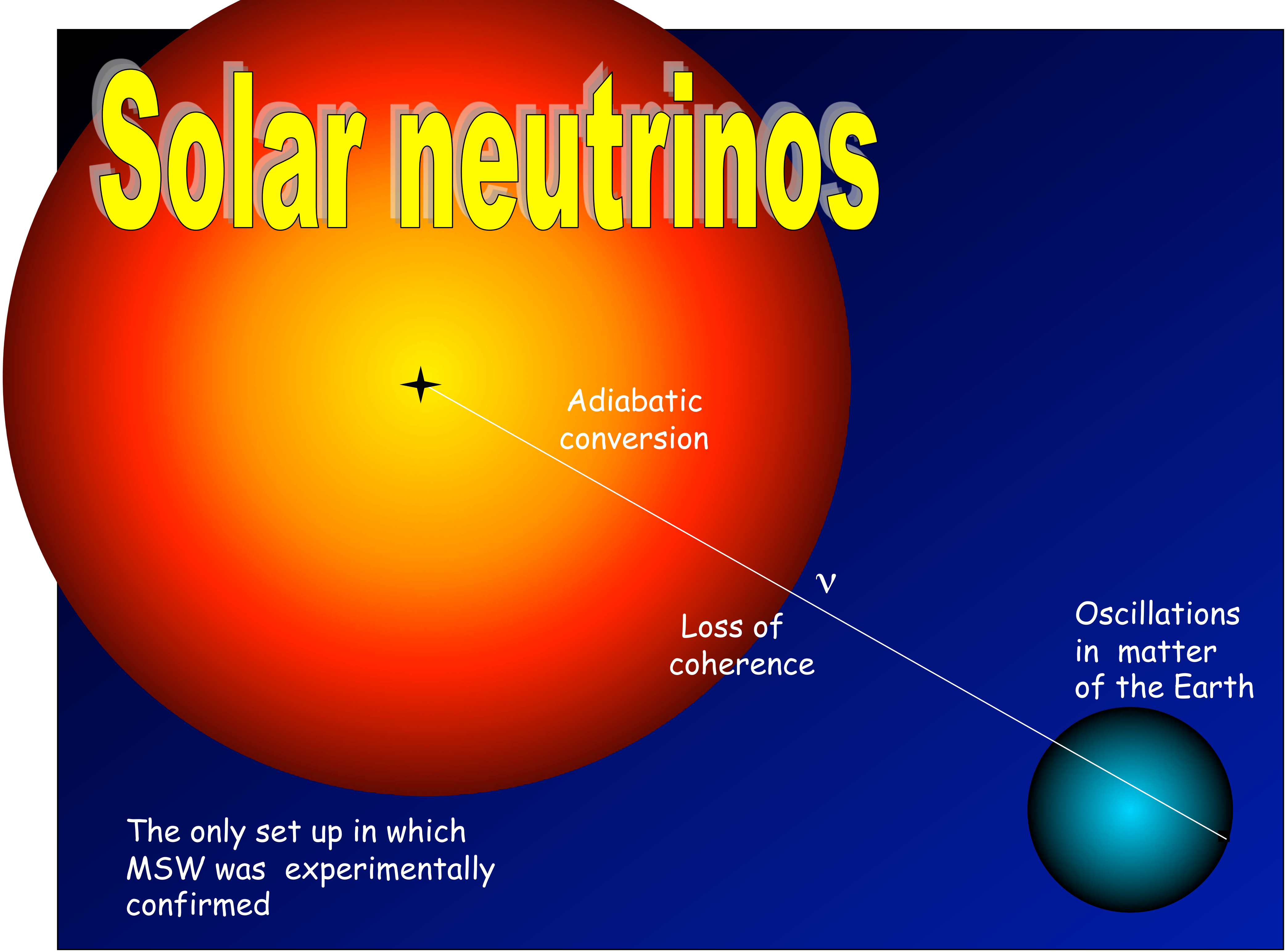
$\frac{F(E)}{F_0(E)}$



E/E_R



E/E_R


Solar neutrinos
Adiabatic conversion
ν
Loss of coherence
Oscillations in matter of the Earth
The only set up in which MSW was experimentally confirmed

Mixing in matter

Eigenstates and eigenvalues

Diagonalization of the Hamiltonian:

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

$$V = \sqrt{2}\, G_F\, n_e$$

Mixing is maximal if

$$V = \frac{\Delta m^2}{2E} \cos 2\theta$$

Resonance condition

$$H_e = H_\mu$$

$$\sin^2 2\theta_m = 1$$

Difference of the eigenvalues

$$H_{2m} - H_{1m} = \frac{\Delta m^2}{2E} \sqrt{(\cos 2\theta - 2EV/\Delta m^2)^2 + \sin^2 2\theta}$$

Original papers:

L. Wolfenstein, Phys. Rev. D17 (1978) 2369

L. Wolfenstein, in ``Neutrino-78'', Purdue Univ. C3, 1978.

adiabaticity

L. Wolfenstein, Phys. Rev. D20 (1979) 2634

V. D. Barger, K. Whisnant, S. Pakvasa, R.J.N. Phillips, Phys.Rev. D22 (1980) 2718

enhancement of oscillations

S.P. Mikheev, A.Yu. Smirnov, Sov. J. Nucl.Phys. 42 (1985) 913-917, Yad.Fiz. 42 (1985) 1441-1448

Resonance, Adiabaticity Solar nu

S.P. Mikheev, A.Yu. Smirnov, Nuovo Cim. C9 (1986) 17-26

S.P. Mikheev, A.Yu. Smirnov, Sov. Phys. JETP 64 (1986) 4-7, Zh.Eksp.Teor.Fiz. 91 (1986) 7-13, arXiv:0706.0454 [hep-ph]

adiabatic formulas

S.P. Mikheev, A.Yu. Smirnov, 6th Moriond workshop, Tignes, Jan. 1986 p. 355

Earth matter effects, day night, atmospheric

P. Langacker J.P. Leville and J Sheiman, On the detection of the cosmological neutrinos by coherent scattering, Phys. Rev D27, 1228 (1983)

H.A. Bethe, Phys.Rev.Lett. 56 (1986) 1305

A. Messiah, 6th Moriond workshop, Tignes Jan. 1986 p.373

Oscillations?

S. J. Parke, Phys.Rev.Lett. 57 (1986) 1275-1278

W.C. Haxton, Phys.Rev.Lett. 57 (1986) 1271-1274

S. P. Rosen, J. M. Gelb, Phys.Rev. D34 (1986) 969

P. Langacker, S.T. Petcov, G. Steigman, S. Toshev, Nucl.Phys. B282 (1987) 589

The MSW effect and matter effects in neutrino oscillations. A.Yu. Smirnov, Phys. Scripta T121 (2005) 57-64, hep-ph/0412391

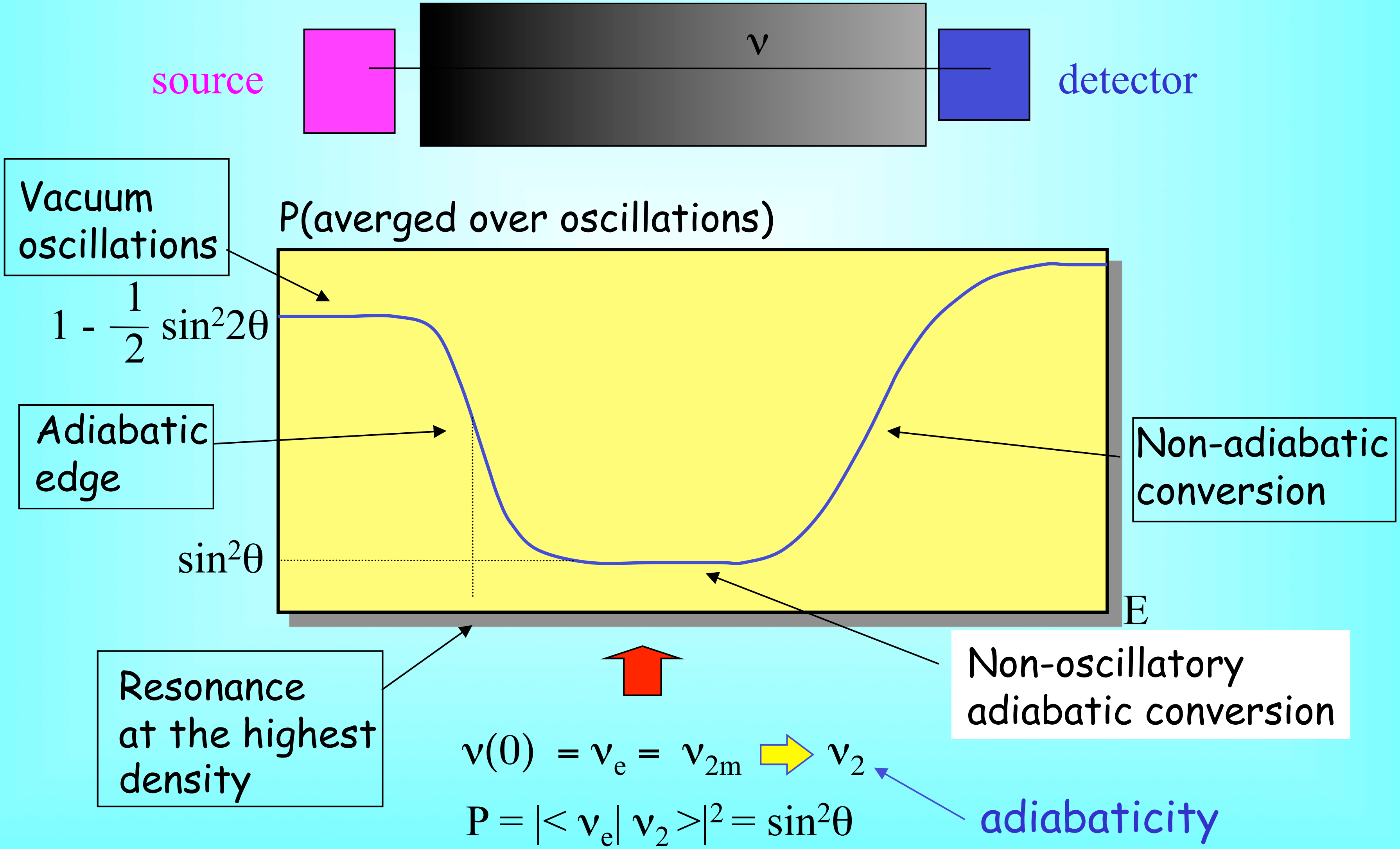
A. Y. Smirnov, hep-ph/0305106

P.C. de Holanda, A.Yu. Smirnov, Astropart.Phys. 21 (2004) 287, hep-ph/0309299

Quantum field theoretic approach to neutrino oscillations in matter. E. Kh. Akhmedov, A. Wilhelm, arXiv:1205.6231 [hep-ph]

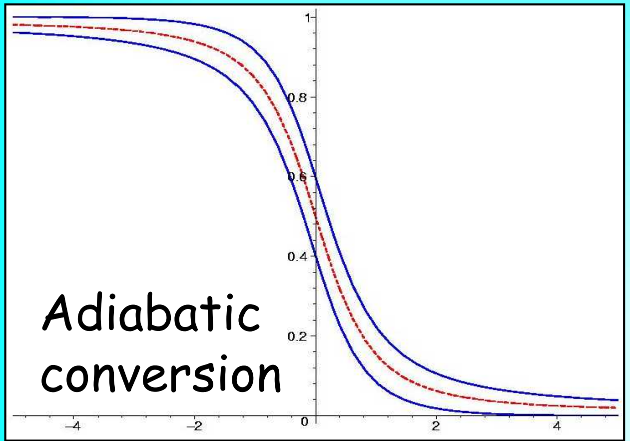
Survival probability

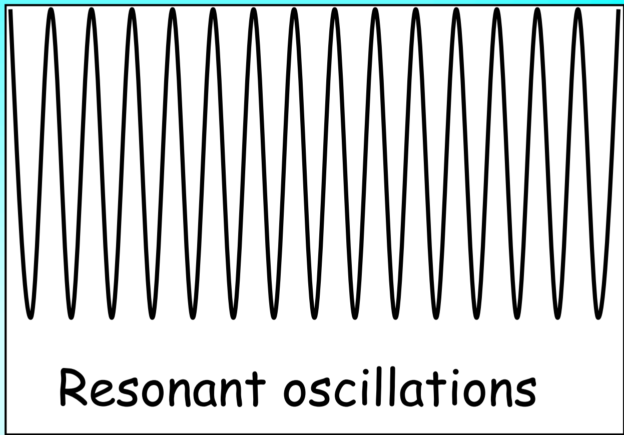
"Suppression bath"



Generalizations and

Realizations





the Sun
Supernovae
Early Universe
CR sources

Atmospheric
Accelerator
Supernova
neutrinos
propagating in
the Earth

Many others if physics beyond 3nu paradigm (e.g. sterile neutrinos exists)

Gianni Conforto

(CERN & Florence)

"Neutrino oscillations"

organizer of parallel session

1938 - 2003

Gargamelle, Crystal Ball
NOMAD, L3

Cecilia Jarlskog

(Stockholm)

"Status of electroweak theory"

Between 1980 and 1985

S. Pakvasa, 1981	Sun, Supernova
H.J. Haubold, 1982	Supernova
P.V Murthy, 1983	Earth