The Nature of the Neutrino (Dirac/Majorana) and Double Beta Decay with or without Neutrinos

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The history of the research associated with the fundamental problem of the nature of massive neutrinos, which can be Dirac particles or Majorana fermions, and the related processes of neutrinoless double beta decay, $(A,Z) \to (A,Z+2) + e^- + e^-$, and two neutrino double beta decay, $(A,Z) \to (A,Z+2) + e^- + e^- + \bar{\nu}_e + \bar{\nu}_e$, is reviewed.

1 Prologue

I would like to begin with some general observations.

Determining the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos is one of the most challenging and pressing fundamental problems in present day elementary particle physics. The question of the nature of massive neutrinos is directly related to the question about the underlying basic symmetries of particle interactions. The massive neutrinos can be Dirac particles if the particle interactions conserve some charge carried by neutrinos, for example, the total lepton charge $L = L_e + L_\mu + L_\tau$. In this case the neutrinos with definite mass differ from their respective anti-particles by the value of the conserved charge they carry. Massive Majorana neutrinos are only possible if particle interactions (including the neutrino mass terms) do not conserve any lepton charge. As is well known, the seesaw and many other models of neutrino mass generation predict massive neutrinos to be Majorana fermions.

An inherent problem of the assumption of conservation of the total lepton charge L is that it is associated with a global $U(1)_{\rm LC}$ symmetry. To quote E. Witten [1]: "In modern understanding of particle physics global symmetries are approximate." Similar ideas were expressed by S. Weinberg in [2]. Thus, the global $U(1)_{LC}$ symmetry leading to $L = const.$ and to massive Dirac neutrinos is expected to be broken, e.g., by quantum gravity effects. This implies L non-conservation, which in turn leads to massive Majorana neutrinos.

The Majorana nature of massive neutrinos manifests itself in the existence of processes in which the total lepton charge changes by two units, $|\Delta L| = 2$: $K^+ \to \pi^- + \mu^+ + \mu^+$, $\mu^- + (A,Z) \to \mu^+ + (A, Z-2)$, etc. Extensive studies have shown that the only feasible experiments having the potential of establishing that the massive neutrinos are Majorana particles are at present the experiments searching for neutrinoless double beta ($(\beta\beta)_{0\nu}-$) decay [a]:

$$(A, Z) \to (A, Z+2) + e^- + e^- \,. \tag{1}$$

Establishing that the total lepton charge L is not conserved in particle interactions by observing the $(\beta\beta)_{0\nu}-$decay would be a fundamental discovery (similar to establishing baryon number non-conservation (e.g., by observing proton decay)).

Establishing that the massive neutrinos ν_j are Majorana particles would be of fundamental importance, as important as the discovery of neutrino oscillations, and would have far reaching implications.

Determining the status of lepton charge conservation and the nature - Dirac or Majorana - of massive neutrinos are part of a program of research in neutrino physics, which aims at shedding light on some of the fundamental aspects of neutrino mixing and includes also (see, e.g., [3]):
– determination of the status of CP symmetry in the lepton sector;
– determination of the type of neutrino mass spectrum, or the "neutrino mass ordering";
– determination of absolute neutrino mass scale or the value of the lightest neutrino mass;
– high precision measurement of neutrino mixing parameters.

The program extends beyond the year 2030. Our ultimate goal is to understand at a fundamental level the mechanism giving rise to neutrino masses and mixing and to non-conservation of the lepton charges L_l, $l = e, \mu, \tau$. The indicated comprehensive experimental program of research in neutrino physics and the related theoretical efforts are stimulated by the fact that the existence of nonzero neutrino masses and the smallness of neutrino masses suggest the existence of new fundamental mass scale in particle physics, i.e., the existence of New Physics beyond that predicted by the Standard Theory. It is hoped that progress in the theory of neutrino mixing will also lead, in particular, to progress in the theory of flavour and to a better understanding of the mechanism of generation of the baryon asymmetry of the Universe.

2 The Early Period 1930 – 1960

The history of neutrino physics and double beta decay begin with the famous letter of Pauli of December 4, 1930, in which the hypothesis of existence of neutrinos was put forward [5]. The next important development was the seminal article by Fermi on the $\beta-$decay theory [6]. Both were reviewed at this Conference by C. Jarlskog [7].

The possibility of two neutrino double beta ($(\beta\beta)_{2\nu}-$) decay,

$$(A, Z) \to (A, Z+2) + e^- + e^- + \bar{\nu} + \bar{\nu} \,, \tag{2}$$

was discussed first in 1935 by M. Goeppert-Mayer in [8]. Goeppert-Mayer realised that some even-even nuclei, for which the ordinary $\beta-$decay is forbidden, can be still metastable since they can undergo a decay into a lighter nucleus with emission of two anti-neutrinos and two electrons, Eq. (2). She notices that the $(\beta\beta)_{2\nu}-$decay can proceed in $2^{\rm nd}$ order of Fermi $\beta-$decay Hamiltonian $\mathcal{H}^{\beta}_{\mathcal{I}}$ and thus that the process consists of "the simultaneous occurrence of two ($\beta-$decay) transitions, each of which does not fulfill the law of conservation of energy separately". Using the Fermi Hamiltonian $\mathcal{H}^{\beta}_{\mathcal{I}}$ she further performs a calculation of the rate of the decay (e.g., for a nucleus with $Z = 31$) and concludes that the $(\beta\beta)_{2\nu}-$decay half-life $T^{2\nu}_{1/2} > 10^{17}$ yr (the half-life calculated in [8] is estimated to be $T^{2\nu}_{1/2} \sim 10^{20}$ yr).

In 1936 Gamow and Teller pointed out [9] that the Fermi $\beta-$decay Hamiltonian $\mathcal{H}^{\beta}_{\mathcal{I}}$ should contain, in addition to the vector current $\bar{p}(x)\gamma_{\alpha}n(x)$ postulated by Fermi (describing transitions in which the spins and parities of the initial and final state nuclei coincide) also an

[a] For an example of a possible alternative experiment having the potential of establishing the Majorana nature of massive neutrinos, which, however, is far from being feasible at present, see, e.g., [4].

axial ($\bar{p}(x)\gamma_\alpha\gamma_5\, n(x)$) and/or tensor ($\bar{p}(x)\sigma_{\alpha\beta}\, n(x)$) currents that could describe the observed $\beta-$decays in which the spins of the initial and final state nuclei differ by one unit. The following general four-fermion parity conserving β-decay Hamiltonian including the Fermi and the Gamow-Teller terms was proposed in 1936 (see, e.g., [10]):

$$\mathcal{H}^{\beta}_{\mathcal{I}} = \sum_{i=S,P,V,A,T} G_i\, \bar{p}(x)\, O^i\, n(x)\, \bar{e}(x)\, O_i\, \nu(x) + h.c.,\;\; O^i = I, \gamma_5, \gamma^\alpha, \gamma^\alpha\gamma_5, \sigma^{\alpha\beta}\,. \qquad (3)$$

Thus, in addition to the Fermi vector term (V), the Hamiltonian in Eq. (3) includes scalar (S), pseudo-scalar (P), axial-vector (A) and tensor (T) terms [b]. It is characterised by five constants G_i having dimension of $(Mass)^{-2}$.

In 1937 Majorana published his highly original "Teoria Simmetrica dell'Elettrone e del Positrone" [12] in which he suggested, in particular, that neutrinos can be, what we call today, "Majorana" fermions. We are still discussing this possibility today. The article by Majorana was the subject of the talks by C. Jarlskog and F. Guerra at this Conference [7,13].

Soon after the publication of the article by Majorana [12], Racah discussed some of the physical implications of the Majorana neutrino hypothesis [14]. He pointed out that if neutrinos are Majorana particles, i.e., if they are identical with their antiparticles, an anti-neutrino produced in $\beta-$ decay together with an electron, when interacting via the inverse $\beta-$decay reaction with a nucleus, can produce an electron again:

$$(A,Z) \rightarrow (A,Z+1) + e^- + \bar{\nu}\,, \qquad (4)$$

$$\bar{\nu} + (A,Z+1) \rightarrow (A,Z+2) + e^-\,, \quad \bar{\nu} \equiv \nu\,. \qquad (5)$$

For a Dirac neutrino the chain in Eq. (5) is impossible: in the inverse $\beta-$decay process a Dirac anti-neutrino emitted in $\beta-$decay, Eq. (4), can produce only a positron in the final state. Racah notices that this can be used to determine whether neutrinos are Dirac or Majorana particles. Thus, Racah did not discuss neutrinoless double beta decay, but rather pointed out how it might be possible to distinguish between Majorana and Dirac neutrinos in the processes of inverse $\beta-$decay using free neutrino fluxes [15].

It was pointed out first by Furry in a well known paper from 1939 [16] that if neutrinos are Majorana particles the process of $(\beta\beta)_{0\nu}-$decay, Eq. (1), can take place. In this process two neutrons from the initial nucleus transform, by exchanging a virtual Majorana neutrino, into two protons of the final state nucleus and two free electrons. For Dirac neutrinos the process is impossible because the requisite neutrino propagator is identically equal to zero [c]. Using the Fermi type four-fermion interaction Hamiltonian and assuming that the two fermion currents involved are Lorentz i) scalars (S), ii) pseudo-scalars (P), iii) vectors (V) and iv) axial vectors (A), Furry performed calculations of the $(\beta\beta)_{0\nu}-$decay half-life, $\mathrm{T}^{0\nu}_{1/2}$, for zero-mass neutrino, $m_\nu = 0$. Comparing his results with the results of Goeppert-Mayer [8], Furry concluded that due to difference of the phase space factors in the two processes, Eqs. (2) and (1), the $(\beta\beta)_{0\nu}-$decay half-life can be shorter by up to five orders of magnitude than the $(\beta\beta)_{2\nu}-$decay half-life: $T^{0\nu}_{1/2} \sim 10^{-5} \times T^{2\nu}_{1/2}$.

2.1 The Early Experiments

The main motivation for the first experiments searching for $(\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ decays, we will call generically "$2\beta-$decays", was the same as is the motivation for the $(\beta\beta)_{0\nu}-$decay experiments today [d] – to answer the fundamental question about the nature (Dirac or Majorana) of

[b] The existence of five possible types of four-fermion interaction terms (without derivatives) was suggested earlier on general grounds, i.e., not in connection with β-decay theory, in [11].

[c] This is a consequence, of course, of the total lepton charge conservation.

[d] A very detailed account of the history of experimental searches for $(\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ decays is given in the review article [17].

the neutrino. Theory [8,16] was predicting $(\beta\beta)_{0\nu}-$decay half-life $T^{0\nu}_{1/2} \sim 10^{15}$ yr in the case of Majorana neutrino and $(\beta\beta)_{2\nu}-$decay half life $T^{2\nu}_{1/2} \sim 10^{20}$ yr for Dirac neutrino.

The first experiment searching for the two processes was performed in 1948 by Fireman [18] with 25 gr of Sn enriched to 54% with ^{124}Sn. Fireman used Geiger counters for detection of the final state electrons and obtained the lower limit $T^{2\beta}_{1/2} > 3 \times 10^{15}$ yr [e].

In the period 1949 – 1953 positive results of searches for $2\beta-$decays were claimed in experiments with ^{124}Sn [19], ^{100}Mo [20], ^{96}Zr [21] and ^{48}Ca [22]. They were disproved by later experiments.

In the same period the first searches for the related processes of $(A,Z) \to (A,Z-2)+e^+ +e^+$ and $e^- + (A,Z) \to (A,Z-2) + e^+$ were performed [20,23] with negative results.

In these first experiments enriched isotopes (^{48}Ca, ^{94}Zr, ^{96}Zr, ^{124}Sn) were routinely utilised, and most advanced (for the time) experimental methods and detectors were used (nuclear emulsion, Geiger, proportional, and scintillation counters, Wilson chamber). The detectors were installed relatively deep underground (to suppress cosmic ray induced background), and active and passive shieldings were employed. The sensitivity reached was $\sim 10^{17} - 10^{18}$ yr. The first experiment in the USSR searching for $2\beta-$decays was performed in 1956 [24].

In 1949, Inghram and Reynolds performed the first geochemical experiment searching for $^{130}Te \to^{130} Xe + e^- + e^-(+\bar{\nu} + \bar{\nu})$ [25]. The authors used ancient (~several billion years old) minerals from which Xenon was extracted and subjected to isotope analysis. The presence of excess amount of ^{130}Xe after accounting for various standard contributions from nuclear reactions triggered by cosmic rays, etc. would imply that the decay $(\beta\beta)_{0\nu}$ and/or $(\beta\beta)_{2\nu}$ of ^{130}Te took place. The authors obtained the limit $T^{2\beta}_{1/2}(^{130}Te) > 8\times 10^{19}$ yr, which was considerably stronger than the limits obtained in counter experiments.

In 1950 Inghram and Reynolds using the same method detected $2\beta^-$ decay of ^{130}Te with half-life [26]

$$T^{2\beta}_{1/2}(^{130}Te) = 1.4 \times 10^{21}\ \text{yr} \quad (\text{Inghram} + \text{Reynolds}, 1950)\,. \tag{6}$$

The result of CUORE-0 experiment on $T^{2\nu}_{1/2}(^{130}Te)$ obtained in 2016 reads [27]:

$$T^{2\nu}_{1/2}(^{130}Te) = [8.2 \pm 0.2(\text{stat.}) \pm 0.6(\text{syst.})] \times 10^{20}\ \text{yr}\,. \tag{7}$$

Thus, apparently the $(\beta\beta)_{2\nu}-$decay of ^{130}Te was shown to take place first in 1950 in the geochemical experiment of Inghram and Reynolds.

2.2 1956 - 1960, Related Developments

In the period 1956 - 1960 important developments took place, which had strong impact on the $(\beta\beta)_{0\nu}-$decay theory, that in turn had significant implications for the searches for $(\beta\beta)_{0\nu}-$decay.

In 1956 the neutrino was observed in the inverse $\beta-$decay experiment of Cowan, Reines et al. [28].

Also in 1956 Lee and Yang put forward the (revolutionary) hypothesis of non-conservation of parity in $\beta-$decay [29]. It was confirmed the same year in the experiment of Wu et al. [30].

In 1957 the two-component neutrino theory was proposed by Landau, Lee and Yang, and Salam [31,32,33]. Assuming that neutrino mass is zero, $m_\nu = 0$, Landau, Lee and Yang, and Salam notice that for a massless neutrino the left-handed (LH) and right-handed (RH) components of the neutrino field, $\nu_L(x)$ and $\nu_R(x)$, are independent and postulated that only one of them enters the expression of the parity non-conserving $\beta-$decay Hamiltonian, $\mathcal{H}^{\beta}_{\mathcal{I}}$.

In 1958 Goldhaber, Grodzins and Sunyar [34] have shown that the neutrino emitted in the process $e^- +^{152} Eu \to^{152} Sm^* + \nu \to^{152} Sm + \nu + \gamma$, has a helicity compatible with (-1) and thus it was concluded that the neutrino field in $\mathcal{H}^{\beta}_{\mathcal{I}}$ is left-handed, $\nu_L(x)$.

[e] Here and further $T^{2\beta}_{1/2}$ should be understood as $T^{2\beta}_{1/2} = (\Gamma^{2\nu} + \Gamma^{0\nu})^{-1}\, ln2$, where $\Gamma^{2\nu}$ and $\Gamma^{0\nu}$ are the $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ decay rates, respectively.

In 1958 Feynman and Gell-Mann [35], and Sudarshan and Marshak [36], proposed the *universal $V-A$ current×current theory of weak interaction.* These authors postulated that all fermion fields in the charged current (CC) weak interaction Hamiltonian $\mathcal{H}_{\mathcal{I}}^{\mathcal{CC}}$, and not only the neutrino fields [f], are left-handed, $f(x) \rightarrow f_L(x)$, $f = p, n, \nu, \mu, e$. They incorporated in $\mathcal{H}_{\mathcal{I}}^{\mathcal{CC}}$ also the idea of Pontecorvo [37] of $\mu - e$ universality of weak interaction.

Thus, $\mathcal{H}_{\mathcal{I}}^{\mathcal{CC}}$ was assumed to have the following elegant form:

$$\mathcal{H}_{\mathcal{WI}}^{\mathcal{CC}} = -\,\mathcal{L}_{\mathcal{WI}}^{\mathcal{CC}} = \frac{G_F}{\sqrt{2}}\, j^{\alpha}(x)\; (j_{\alpha}(x))^{\dagger}\,, \tag{8}$$

$$j_{\alpha}(x) = 2\left[\bar{p}_L(x)\gamma_{\alpha}n_L(x) + \bar{\nu}_L(x)\gamma_{\alpha}e_L(x) + \bar{\nu}_L(x)\gamma_{\alpha}\mu_L(x)\right]\,. \tag{9}$$

In 1957 Pontecorvo considered the possibility of existence of neutrino oscillations [38,39]. He introduced in [39] for the first time fermion mixing in $\mathcal{H}_{\mathcal{WI}}^{\mathcal{CC}}$ by assuming that the neutrino field $\nu(x)$ in $\mathcal{H}_{\mathcal{WI}}^{\mathcal{CC}}$ is a linear combination of the fields of two Majorana neutrinos χ_1 and χ_2 having definite but different masses $m_{1,2} > 0$, $m_1 \neq m_2$, and definite but opposite CP parities, $\eta_{1,2CP}$, $\eta_{1CP} = -\,\eta_{2CP}$:

$$\nu(x) = \frac{\chi_1(x) + \chi_2(x)}{\sqrt{2}}\,. \tag{10}$$

As it is clear from Eq. (10), Pontecorvo assumed maximal neutrino mixing.

In 1962 Maki, Nakagawa and Sakata proposed a modification of the Nagoya composite model of the proton, neutron and lambda baryons with baryon-lepton symmetry [40]. They assumed that the neutrinos coupled to the electron and the muon in $\mathcal{H}_{\mathcal{WI}}^{\mathcal{CC}}$, Eq. (9), are different – a possibility discussed since the second half of 1940s – and used the notation ν_e and ν_μ for them. This was done before the results of the two-neutrino Broohaven experiment were reported. It was further postulated in [40] that the ν_e and ν_μ fields in $\mathcal{H}_{\mathcal{WI}}^{\mathcal{CC}}$ are linear combinations of the fields of two massive Dirac neutrinos Ψ_1 and Ψ_2 having different masses, $m_{1,2}^D$, $m_1^D \neq m_2^D$:

$$\nu_{eL}(x) = \Psi_{1L}\cos\theta_C -\; \Psi_{2L}\sin\theta_C\,, \tag{11}$$

$$\nu_{\mu L}(x) = \Psi_{1L}\sin\theta_C + \Psi_{2L}\cos\theta_C\,. \tag{12}$$

By construction (due to the implemented baryon-lepton symmetry) the mixing angle in (12) is what is universally called today "the Cabibbo angle" [g].

In view of the pioneering articles [39,40], the neutrino mixing matrix is usually called today the Pontecorvo, Maki, Nakagawa and Sakata (PMNS) matrix. In the current reference 3-neutrino mixing scheme (see, e.g., [3]) the weak lepton charged current has the form:

$$(j_{\alpha}^{\rm lep}(x))^{\dagger} = \sum_{l=e,\mu,\tau} 2\,\bar{l}_L(x)\gamma_{\alpha}\nu_{lL}(x)\,, \quad \nu_{lL}(x) = \sum_{j=1,2,3} U_{lj}\nu_{jL}(x)\,, \tag{13}$$

where $U \equiv U_{\rm PMNS}$ is the 3×3 unitary PMNS matrix and $\nu_{jL}(x)$ is the LH component of the field of the neutrino ν_j having mass m_j.

3 The Period 1960 - 1980

The V-A structure of the CC weak interaction Hamiltonian (Lagrangian), which was largely confirmed experimentally, implied that it is possible (and most likely) to have $T^{0\nu}_{1/2} >> T^{2\nu}_{1/2}$ instead of $T^{0\nu}_{1/2} \sim 10^{-5} T^{2\nu}_{1/2}$.

[f] In addition to $\beta-$decay, $\mathcal{H}_{\mathcal{I}}^{\mathcal{CC}}$ described also the related processes of $e^{\pm}$ capture, $e^- + p \rightarrow n + \nu$, $e^+ + n \rightarrow p + \bar{\nu}$, inverse $\beta-$decay, $\bar{\nu} + p \rightarrow n + e^+$, neutrino quasi-elastic scattering $\nu + n \rightarrow p + e^-$, and the analogous processes involving $\mu^{\pm}$ as well as $\mu-$decay, $\mu^- \rightarrow e^- + \bar{\nu} + \nu$, and $\bar{\nu} + e^- \rightarrow \mu^- + \bar{\nu}$. In [35,36] distinction between ν_e and ν_μ was not made.

[g] In [40] the authors quote the 1960 article by M. Gell-Mann and M. Levy, Nuovo Cim. 16, 605 (1960), in which a small mixing parameter analogous to the Cabibbo angle was introduced in a footnote. The article by N. Cabibbo was published in 1963 in Phys. Rev. Lett. 10, 531 (1963).

Greuling and Whitten were the first to perform a calculation of $(\beta\beta)_{0\nu}-$decay rate, $\Gamma^{0\nu}$, in the $V-A$ theory assuming that ν is a Majorana particle[41]. They had the modern understanding of the connection between the lepton charge non-conservation and the existence of $(\beta\beta)_{0\nu}-$decay. These authors showed, in particular, that if ν is a massless Majorana fermion, $m(\nu)=0$, and the electron current in $\mathcal{H}_{\mathcal{I}}^{\mathcal{CC}}$ has the established $V-A$ structure, denoting this current as $(V-A)_e$, then $\mathcal{H}_{\mathcal{I}}^{\mathcal{CC}}$ conserves a lepton charge (L_e) and $(\beta\beta)_{0\nu}-$decay is forbidden. The $(\beta\beta)_{0\nu}-$decay would be allowed for Majorana ν and $(V-A)_e$ if $m(\nu)\neq 0$, or if $m(\nu)=0$, but the electron current had a small $V+A$ admixture denoted schematically as $(V-A)_e+\eta(V+A)_e$, where η is a free constant parameter. Greuling and Whitten showed that for $\eta=0$, the $(\beta\beta)_{0\nu}-$decay rate $\Gamma^{0\nu}\propto|m(\nu)|^2$, while if $m(\nu)=0$ but $\eta\neq 0$, $\Gamma^{0\nu}\propto|\eta|^2$.

In 1966 Mateosian and Goldhaber performed a counter experiment with 11.4 g of ^{48}Ca (enriched to 96.6%) [42]. In this experiment the scheme "detector ≡ source" was realised for the first time. The following limit was obtained: $T_{1/2}^{2\beta}(^{48}Ca)>2\times10^{20}$ yr and thus the limit of 10^{20} yr was reached for the first time in a counter experiment.

In 1967 Fiorini *et al.* [43] made use for the first time a Ge(Li) detector in a $(\beta\beta)_{0\nu}-$decay experiment. The following limit on the $(\beta\beta)_{0\nu}-$decay half-life of ^{76}Ge was reported: $T_{1/2}^{0\nu}(^{76}Ge)>3\times10^{20}$ yr. This limit was improved significantly in 1973 [44]: $T_{1/2}^{0\nu}(^{76}Ge)>5\times10^{21}$ yr. These experiments represented a proof of the feasibility of using Ge detectors for high sensitivity searches of $(\beta\beta)_{0\nu}-$decay. As a consequence, Ge detectors were used later with increasing sensitivity in a large number of experiments (see [45] for a complete list and a rather detailed description of these experiments).

The first experiments employing the "tracking technique" (visualization of tracks and measurement of the energy of two electrons) in the searches for $(\beta\beta)_{0\nu}-$decay were performed by Wu et al. with ^{48}Ca and ^{82}Se using a streamer chamber (placed in a magnetic field) and plastic scintillators [46]. The following lower limits were obtained: $T_{1/2}^{0\nu}(^{48}Ca)>2.0\times10^{21}$ yr and $T_{1/2}^{0\nu}(^{82}Se)>3.1\times10^{21}$ yr.

In the period 1966-1975 several geochemical experiments with ^{130}Te, ^{82}Se and ^{128}Te were performed [47,48,49,50]. A major contribution was made by Kirsten et al. Kirsten and Muller [47] obtained the first compelling evidence for the double beta decay of ^{82}Se and determined the value of $T_{1/2}^{2\beta}(^{82}Se)$. These authors found $T_{1/2}^{2\beta}(^{82}Se)=1.37\times10^{20}$ yr with an error on the value of $T_{1/2}^{2\beta}(^{82}Se)$ estimated to be about 20%. Earlier Kirsten et al. [48] determined also the value of $T_{1/2}^{2\beta}(^{130}Te)$ (under conditions which practically did not allow any ambiguity in the interpretation of the result obtained). The following value was reported in [48]: $T_{1/2}^{2\beta}(^{130}Te)=10^{21.34\pm0.12}$ yr. A determination of $T_{1/2}^{2\beta}(^{130}Te)$ was reported also in [49] where the authors found: $T_{1/2}^{2\beta}(^{130}Te)=(8.20\pm0.64)\times10^{20}$ yr. These new determinations of $T_{1/2}^{2\beta}(^{130}Te)$ confirmed the 1950 result obtained in [26], Eq. (6). In 1975, Manuel et al. [50] measured $T_{1/2}^{2\beta}(^{130}Te)$ and the ratio $T_{1/2}^{2\beta}(^{130}Te)/T_{1/2}^{2\beta}(^{128}Te)$ and used them for the first determination of $T_{1/2}^{2\beta}(^{128}Te)$. These authors found: $T_{1/2}^{2\beta}(^{130}Te)/T_{1/2}^{2\beta}(^{128}Te)=(1.59\pm0.05)\times10^{-3}$ and $T_{1/2}^{2\beta}(^{128}Te)=(1.54\pm0.17)\times10^{24}$ yr.

The quoted geochemically determined values of $T_{1/2}^{2\beta}(^{82}Se)$ and $T_{1/2}^{2\beta}(^{130}Te)$ are impressively compatible with measurements of the $(\beta\beta)_{2\nu}-$decay half-lives of ^{82}Se and ^{130}Te performed in counter experiments much later (see Eq. (6) and Table 1).

4 1960 - 1980: Related Developments

In 1962, following the proposals of Pontecorvo[51] and Schwartz[52], Danby et al. have demonstrated in an experiment performed at the Broohaven National Laboratory [53] that the neutrinos coupled to the electron and the muon in the weak charged lepton current are different particles, i.e., that

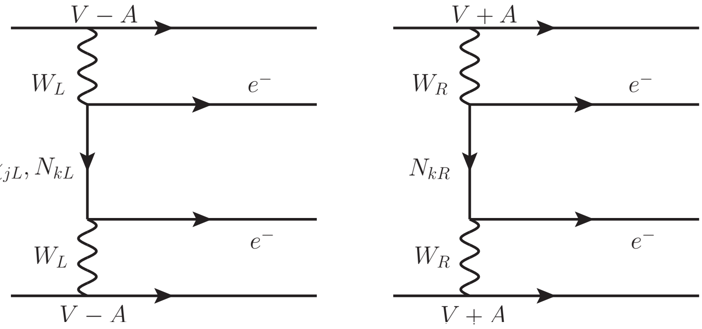

Figure 1 – Schematic diagrams with exchange of virtual light and heavy Majorana neutrinos χ_{jL} and N_{kL} (N_{kR}) giving contributions to the $(\beta\beta)_{0\nu}-$decay amplitude in the case of $V-A$ ($V+A$) CC weak interaction.

$\nu_e \neq \nu_\mu$.

In 1969 Gribov and Pontecorvo [54] considered a Majorana mass term for the flavour neutrinos (neutrino fields) ν_e and ν_μ ($\nu_{eL}(x)$ and $\nu_{\mu L}(x)$):

$$\mathcal{L}^{\nu}_{M}(x) = \frac{1}{2}\,\nu^{\mathrm{T}}_{l'L}(x)\; C^{-1}\; M_{l'l}\;\nu_{lL}(x) + h.c., \quad l, l' = e, \mu\,, \tag{14}$$

where C is the charged conjugation matrix, $C^{-1}\gamma_\alpha C = -\gamma^{\mathrm{T}}_\alpha$. They showed, in particular, that i) as a consequence of $\nu_{lL}(x)$ being fermion fields, the mass matrix M in Eq. (14) satisfies $M^{\mathrm{T}} = M$, and that ii) the diagonalisation of $\mathcal{L}^{\nu}_{M}(x)$ leads to two neutrinos with definite and, in general, different masses which are Majorana particles.

In 1976 Bilenky and Pontecorvo [55] introduced for the first time Dirac+Majorana neutrino mass term which is the most general mass term that can be built from active LH and sterile (singlet) RH neutrino fields $\nu_{\ell L}(x)$ and $\nu_{\ell R}(x)$. The Dirac+Majorana neutrino mass term, as is well known, is at the basis of the seesaw mechanism of neutrino mass generation. It was shown in [55] that in the case of n LH and n RH neutrino fields the diagonalisation of the Dirac+Majorana mass term leads to $2n$ massive Majorana neutrinos.

The neutrino Majorana and Dirac+Majorana mass terms are integral parts of the mechanisms of neutrino mass generations leading to massive Majorana neutrinos in gauge theories of electroweak interaction and in GUTs.

In 1977 Halprin, Primakoff and Rosen [56] have performed a calculation of the $(\beta\beta)_{0\nu}-$ decay rate assuming the presence of an RH admixture in the LH weak charged electron current:

$$j^{(e)}_{\alpha}(x) = \bar{e}(x)\gamma_\alpha\left[(1-\gamma_5) + \eta(1+\gamma_5)\right]\nu_e(x)\,, \tag{15}$$

where η is a constant. The authors of [56] further assumed that the neutrino ν_e is a "γ_5 non-invariant Majorana particle" (i.e., that the neutrino field satisfies $\nu_e = C(\bar{\nu}_e)^T$). Using the existing experimental constraints on the $(\beta\beta)_{0\nu}$-decay half-life they derived the following limits: i) $\eta < 5\times 10^{-4}$ for $m(\nu_e) = 0$ ("RH mechanism"), and ii) $\Gamma^{0\nu}_{th} < 300\,\Gamma^{0\nu}_{exp.lim}$ for η=0 and $m(\nu_e) = 60$ eV (the upper limit on $m(\nu_e)$ at the time), $\Gamma^{0\nu}_{th}$ and $\Gamma^{0\nu}_{exp.lim}$ being the theoretically predicted and the maximal experimentally allowed decay rates. In [56] the first $(\beta\beta)_{0\nu}-$decay bounds were obtained on the mass M_N of a heavy Majorana neutrino N_e coupled to the electron in the weak charged lepton current (see Fig. 1 where $\chi_{jL} \equiv \nu_{eL}$ and $N_{kR(L)} \equiv N_{eR(L)}$):

$$j^{(e)}_{\alpha}(x) = \bar{e}(x)\,\gamma_\alpha(1-\gamma_5)\,\nu_e(x) + U_{eN}\,\bar{e}(x)\,\gamma_\alpha(1\pm\gamma_5)\,N_e(x)\,. \tag{16}$$

Using the existing data on $(\beta\beta)_{0\nu}$-decay the following bounds were reported for $U_{eN} = 1$ [56]: $M_N < 1$ keV, or $M_N > 3$ GeV. The current combined $|U_{eN}|^2 - M_N$ limits are shown in Fig. 2.

Starting from 1976 gauge theories (extensions of the Standard Model, GUTs) naturally incorporating L-non-conservation and massive Majorana neutrinos were proposed (see [65], for

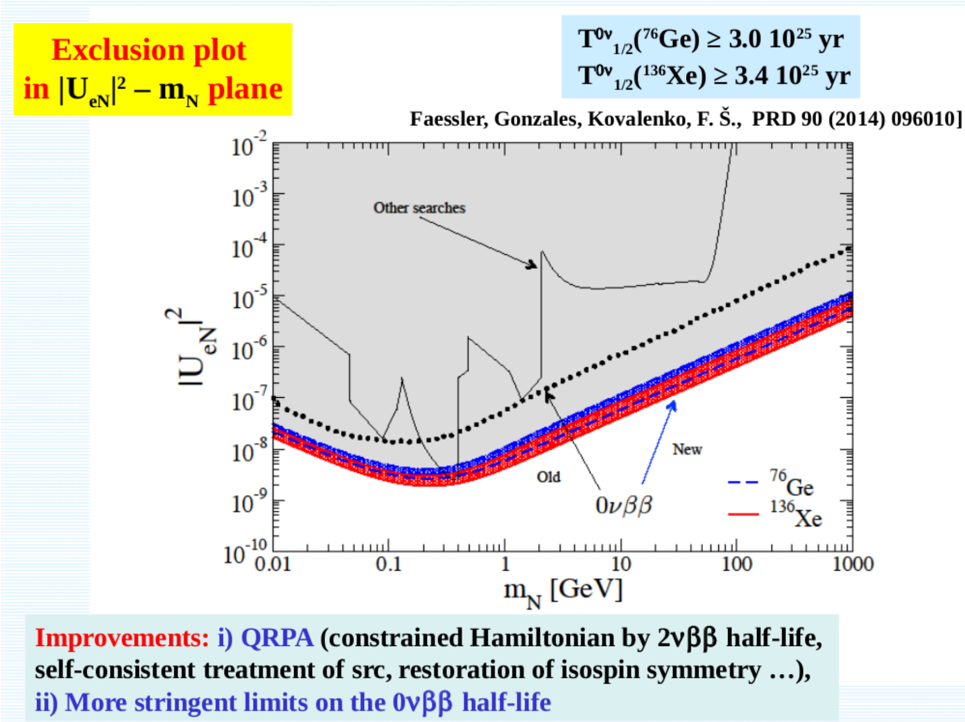

Figure 2 – The exclusion plot in the $|U_{eN}|^2 - M_n$ plane ($m_N \equiv M_N$) from the limits on the half-lives of ^{76}Ge and ^{136}Xe indicated in the figure. (Figure from [57] updated by F. Šimkovic in 2017.)

example). In particular, the enormous disparity between the magnitudes of the neutrino masses and the masses of the charged leptons and quarks found a natural explanation by the seesaw mechanism [66] and the Weinberg dimension 5 effective operator [67], both of which implied $L-$non-conservation and massive Majorana neutrinos.

5 The Period 1980 - 1990: Theoretical Activity

In this period there was a renewed interest in the process of $(\beta\beta)_{0\nu}-$decay (see further). It was stimulated, in particular, by the development of gauge theories of electroweak interactions and GUTs in which, as we have indicated, massive Majorana neutrinos appear naturally. In addition, the leptogenesis scenario of the origin of baryon asymmetry of the Universe, relating the neutrino mass generation via the seesaw mechanism (leading to Majorana neutrinos) with the generation of the matter-antimatter asymmetry of the Universe, was proposed [68]. A massive neutrino (with a mass of $\sim$ 40 eV) was considered in that period as a plausible (hot) dark matter candidate. And Lubimov et al. claimed in 1980 the observation of a neutrino mass of $\sim$ 20 eV in an ITEP tritium β-decay experiment [h].

We will discuss next some relevant theoretical developments which took place in the period 1980 - 1990.

5.1 The Majorana Phases

In 1980 in a JINR (Dubna) preprint published in May, Bilenky et al. [58] pointed out that if the massive neutrinos are Majorana particles, the neutrino mixing matrix contains additional physical CP violation (CPV) "Majorana" phases in comparison to the number of "Dirac" phases present in the case of massive Dirac neutrinos. The authors of [58] showed, in particular, that

[h] This claim was proven to be incorrect much later by independent experiments (see the contribution of M. Goodman in these Proceedings [69]).

in the case of n lepton families, the $n \times n$ unitary neutrino mixing matrix $U_{\rm PMNS}$ contains, in general, $(n-1)$ Majorana CPV phases in addition to the $(n-1)(n-2)/2$ Dirac CPV phases, or altogether $n(n-1)/2$ CPV phases. It was noticed in [58] (see also [59]) that in this case the PMNS matrix can be cast in the form:

$$U_{\rm PMNS} = V P\,,\ V:\ n \times n \text{ unitary, includes } (n-1)(n-2)/2 \text{ Dirac CPV}\,, \tag{17}$$

$$P = {\rm diag}(1, e^{i\frac{\alpha_{21}}{2}}, e^{i\frac{\alpha_{31}}{2}}, ..., e^{i\frac{\alpha_{n1}}{2}})\ :\ \alpha_{k1},\ k = 2, ..., n\ -\ (n-1) \text{ Majorana CPV}\,. \tag{18}$$

In the reference case of 3 families, these results imply that $U_{\rm PMNS}$ contains one Dirac phase (in V) and two Majorana CPV phases (in P). Even in the mixing involving only 2 massive Majorana neutrinos there is one physical CPV Majorana phase. In contrast, the CC weak interaction is automatically CP-invariant in the case of mixing of two massive Dirac neutrinos or of two quarks.

Similar conclusions concerning the existence of Majorana CPV phases were reached in [60] and [61], which appeared in preprint forms respectively in June and July of 1980.

In [58] it was proven also that the flavour neutrino and antineutrino vacuum oscillation probabilities, $P(\nu_l \to \nu_{l'})$ and $P(\bar{\nu}_l \to \bar{\nu}_{l'})$, are not sensitive to the Majorana phases. It was shown in [62] that the same result holds also when the flavour neutrino and antineutrino oscillations take place in matter. These results implied that it is impossible to get information about the nature of massive neutrinos in experiments studying $\nu_l \to \nu_{l'}$ and/or $\bar{\nu}_l \to \bar{\nu}_{l'}$ oscillations.

The authors of [61] realised that in the case when the $(\beta\beta)_{0\nu}-$decay is induced by exchange of light Majorana neutrinos χ_j (with masses $m_j \lesssim 1$ MeV, see Fig. 1), the $(\beta\beta)_{0\nu}-$decay rate $\Gamma^{0\nu} \propto |\langle m \rangle|^2$, where $\langle m \rangle$ is the $(\beta\beta)_{0\nu}-$decay effective Majorana mass parameter,

$$|{<}m{>}| = |\sum_{j=1,2,3} m_j\, U_{ej}^2| = \left| m_1 |U_{e1}|^2 + m_2 |U_{e2}|^2\ e^{i\alpha_{21}} + m_3 |U_{e3}|^2\ e^{i(\alpha_{31} - 2\delta)} \right|\,, \tag{19}$$

and we have given the expression of $|{<}m{>}|$ for 3-neutrino mixing and the standard parametrisation of the PMNS matrix [3]. In Eq. (19) δ is the Dirac CPV phase. It was also noticed in [61] that because of the presence of the Majorana phases there is a possibility of cancellation between the different terms in $|{<}m{>}|$, and thus of a strong suppression of $\Gamma^{0\nu}$.

In 1981 Wolfenstein derived the CP invariance constraints on massive Majorana neutrinos and on the Majorana CPV phases [63] (see also [59,64]). He showed that in the case of CP invariance, the Majorana neutrinos with definite mass χ_j have definite CP parities equal to $\pm i$:

$$U_{CP}\, \chi_j(x)\, U_{CP}^{\dagger} = \eta_j^{CP}\, \gamma_0\, \chi_j(x_p)\,, \quad \eta_j^{CP} = \pm i\,, \tag{20}$$

where $\chi_j(x)$ is the field of χ_j and U_{CP} is the unitary operator of CP transformation. The Majorana phases α_{j1}, $j = 2, 3, ...$, take the CP conserving values $k_{j1}\pi$, $k_{j1} = 0, 1, 2, ...$ Correspondingly, the phase factors in the expression for $|{<}m{>}|$ take values $e^{i\alpha_{j1}} = \pm 1$, which represent the relative CP parities of the Majorana neutrinos χ_j and χ_1.

It follows from Eq. (19) and the preceding discussion, in particular, that the maximal and minimal values of $|{<}m{>}|$ are obtained in the case of CP invariance and are determined by the CP conserving values of the Majorana phases [i] which thus play very important role in the phenomenology of $(\beta\beta)_{0\nu}-$decay.

5.2 The Process with Majoron Emission

In 1981 it was pointed out that the spontaneous breaking of the global $U(1)_{\rm LC}$ symmetry, associated with the L conservation, leads (in accordance with the Goldstone theorem [70]) to the

[i] The CP conserving values of $\delta = 0, \pi$ do not have an effect on $|{<}m{>}|$, Eq. (19).

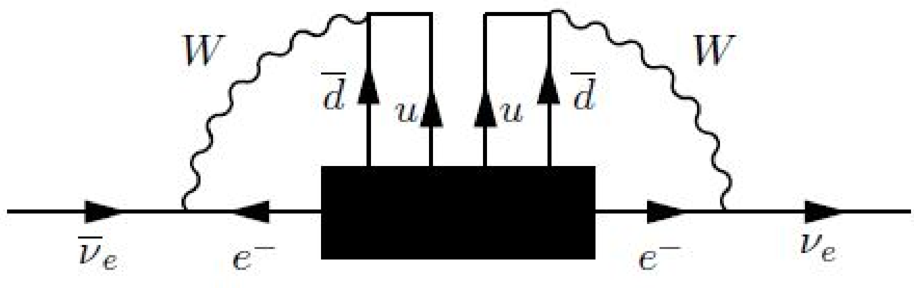

Figure 3 – The diagram generating Majorana mass term of ν_e assuming the existence of $(\beta\beta)_{0\nu}$−decay.

presence in the theory of a massless scalar particle, ϕ^0, called "Majoron" [71]. In this case the process of neutrinoless double beta decay with emission of a Majoron becomes possible:

$$(A, Z) \to (A, Z+2) + e^- + e^- + \phi^0 \,. \tag{21}$$

The searches for this process continue today.

5.3 The "Black Box" Theorem

In 1982 Schechter and Valle published the "Black Box" theorem [72] which stated that the observation of $(\beta\beta)_{0\nu}$−decay implies (under very general conditions) the existence of a Majorana mass term of ν_e and thus massive Majorana neutrinos independently of the mechanism which triggers the decay (Fig. 3). This theorem became serious theoretical motivation for continuing the experimental searches for $(\beta\beta)_{0\nu}$−decay with increasing sensitivity.

In 2011 a study of the magnitude of the Majorana mass term of ν_e generated according to the "Black Box" theorem was performed in [73]. The Black Box operators induce Majorana neutrino mass term at four-loop level (see Fig. 3). This warrants the statement that an observation of neutrinoless double beta decay guarantees the Majorana nature of neutrinos. The authors of [73] evaluated the Schechter-Valle (Black Box) theorem quantitatively by considering the most general Lorentz invariant Lagrangian consisting of point-like operators for neutrinoless double beta decay and calculating the corresponding radiatively generated neutrino masses. They found that these neutrino masses "are many orders of magnitude smaller than the observed neutrino masses and splittings". As an example they give the following prediction of the value of the Majorana neutrino mass generated by the Black Box diagram (see [73] for details of the relevant calculation): $\delta m_\nu = 5 \times 10^{-28}$ eV. The authors of [73] thus concluded: "Although the principal statement of the Schechter-Valle theorem remains valid, we conclude that the Black Box diagram itself generates radiatively only mass terms which are many orders of magnitude too small to explain neutrino masses. Therefore, other operators must give the leading contributions to neutrino masses, which could be of Dirac or Majorana nature."

5.4 Nuclear Matrix Element Calculations

In the period 1984 – 1986 the first more detailed calculations of the nuclear matrix elements (NME) of $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ decays were performed.

In [74] it was pointed out, in particular, that the NME of $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ decays are very different in structure and are not related. The NME were obtained within the shell model by considering the weak coupling approximation scheme [j].

[j] The NME obtained in [74] are significantly smaller (up to factor 2-3) when compared with nuclear shell model matrix elements calculated recently with larger model space, more advanced nucleon-nucleon interaction and by avoiding the previous many-body simplifications.

The 1985 article by Doi et al. [75] represents a basic theoretical work on the calculation of the NME for the $((\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ decays. The energy and angular distributions of the two e^- were calculated for the "standard" light Majorana neutrino exchange and the RH current mechanisms. They authors pointed out to the possibility to distinguish experimentally between the two mechanisms using the difference between the respective distributions.

In 1986 in was shown in [76] that the inclusion of the particle-particle interaction in a nucleus, characterised by a constant g_{pp}, within the quasi-particle random phase approximation (QRPA) method (which was usually disregarded in the QRPA calculations of the ordinary beta decay matrix elements - only particle-hole interaction was considered) permits to calculate the NME and the rate of the $(\beta\beta)_{2\nu}-$decay with relatively small uncertainties and to reproduce the measured $(\beta\beta)_{2\nu}-$decay half-lives[k]. This result led to an extensive use of the QRPA models for the calculation of NMEs for $(\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ decays.

6 Experimental Activity: 1980-2000

The experimental activity related to $2\beta-$decays increased significantly in the period 1980–2000. The use of detectors with passive and active shielding located deep underground, built with low background materials led to a substantial reduction of background. Large (several to few$\times$10 kg) relatively cheap high purity germanium detectors became available and several experiments were performed with ^{76}Ge. These developments led to an increase of sensitivity by several orders of magnitude: several collaborations reported limits on $\mathrm{T}^{0\nu}_{1/2}$ of $\sim 10^{23}$-10^{25} yr.

The limit $T^{0\nu}_{1/2}(^{76}Ge) > 1.2 \times 10^{24}$ yr (90% C.L.) was reported by D. Caldwell [77] using 7.2-kg high-purity natural Ge (7.8% ^{76}Ge) semiconductor detectors (inside of NaI anticoincedence shield, which was inside a high purity Pb shield, at a depth of 600 m w.e.)

In 1987 the ITEP/YePI collaboration used for the first time semiconductor Ge(Li) detectors grown from Ge enriched in ^{76}Ge at 85% to search for $(\beta\beta)_{0\nu}$, $(\beta\beta)_{2\nu}$ and Majoron decays [78]. The following results were obtained: $T^{0\nu}_{1/2}(^{76}Ge) > 2\times 10^{24}$ yr (68% C.L.), $T^{2\nu}_{1/2}(^{76}Ge) = (9\pm 1)\times 10^{20}$ yr, $T^{0\nu,M}_{1/2}(^{76}Ge) > 1.2 \times 10^{20}$ yr, $T^{0\nu,M}_{1/2}(^{76}Ge)$ being the half-life of the decay with Majoron emission, Eq. (21). This progress paved the way for the Heidelberg–Moscow [80] and IGEX [79] experiments with enriched ^{76}Ge, which reached record sensitivity to $\mathrm{T}^{0\nu}_{1/2}(^{76}\mathrm{Ge})$ of $\sim 10^{25}$ yr.

The limit $T^{0\nu}_{1/2}(^{136}Xe) > 3.4\times 10^{23}$ y was obtained [81] in a time projection chamber experiment with 3.3-kg Xe enriched in ^{136}Xe to 62%, in which the sum of the energies and the individual energies of the two electrons were measured, events with the simultaneous emission of $2e^-$ from one point were selected and the tracks of electrons were reconstructed.

In 1984 Fiorini and Niinikoski discussed the possibility (following the idea of G.V. Mizelmaher, B.S. Neganov, V.N. Trofimov from JINR Dubna (Communication JINR P8-82-549, Dubna, 1982 (in Russian)) to use low-temperature (bolometer) detectors to search for $(\beta\beta)_{2\nu}$ and $(\beta\beta)_{0\nu}$ decays [82]. The proposed method was successfully realised by the Milano group in CUORICINO and CUORE experiments.

A breakthrough experimental result was reported in 1987 by Moe et al. [83]: for the first time the $(\beta\beta)_{2\nu}-$decay was observed in a laboratory experiment. 36 events of $(\beta\beta)_{2\nu}-$decay of ^{82}Se were detected in a direct counter experiment with a time projection chamber. The following half-life was measured: $T^{2\nu}_{1/2}(^{82}Se) = 1.1^{+0.8}_{-0.3} \times 10^{20}$ yr.

In the 1990s the $(\beta\beta)_{2\nu}-$decay was observed and its half-life measured of i) ^{76}Ge [78], ii) ^{100}Mo, ^{150}Nd and ^{48}Ca [84], iii) ^{100}Mo and ^{116}Cd [85], iv) ^{100}Mo, ^{116}Cd, ^{82}Se and ^{96}Zr by NEMO–2 experiment [86] (in which the energy spectra and angular distributions of electrons were also measured).

In the discussed period a number of geochemical experiments were performed as well. In 1993 the first geochemical experiment with ^{96}Zr was realised and the following half-life for the

[k] The authors found also that the results obtained exhibit very strong dependence on the constant g_{pp}.

^{96}Zr– ^{96}Mo transition was obtained [87]: $T^{2\beta}_{1/2}(^{96}Zr -^{96}Mo) = (3.9 \pm 0.9) \times 10^{19}$ yr. The values of $T^{2\beta}_{1/2}(^{82}Se)$, $T^{2\beta}_{1/2}(^{130}Te)$ and $T^{2\beta}_{1/2}(^{128}Te)$ where again discussed in the period of interest. Kirsten et al. [88] reported $T^{2\beta}_{1/2}(^{82}Se) = (1.30 \pm 0.50) \times 10^{20}$ yr, $T^{2\beta}_{1/2}(^{130}Te) = (1.63 \pm 0.14) \times 10^{21}$ yr, and stated that in what concerns $T^{2\beta}_{1/2}(^{130}Te)$, "the majority of data from all sources is $> 1.5 \times 10^{21}$ yr." At the same time Manuel reported [89]: $T^{2\beta}_{1/2}(^{130}Te) = (7 \pm 2) \times 10^{20}$ yr, $T^{2\beta}_{1/2}(^{82}Se) = 1 \times 10^{20}$ yr and $T^{2\beta}_{1/2}(^{128}Te)/T^{2\beta}_{1/2}(^{130}Te) = 2 \times 10^3$. The inverse of the last ratio was determined with a high precision by Bernatowicz et al. [90]: $T^{2\beta}_{1/2}(^{130}Te)/T^{2\beta}_{1/2}(^{128}Te) = (3.52 \pm 0.11) \times 10^{-4}$. Takaoka et al. [91] used this result together with $T^{2\beta}_{1/2}(^{130}Te) = (7.9 \pm 1.0) \times 10^{20}$ yr obtained in their experiment to determine the half-life of ^{128}Te: $T^{2\beta}_{1/2}(^{128}Te) = (2.2 \pm 0.3) \times 10^{24}$ yr. Bernatowicz et al. [90] reported also: $T^{2\beta}_{1/2}(^{130}Te) = (2.7 \pm 0.1) \times 10^{21}$ yr and $T^{2\beta}_{1/2}(^{128}Te) = (7.7 \pm 0.4) \times 10^{24}$ yr. The origins of the discrepancies between the results on $T^{2\beta}_{1/2}(^{130}Te)$ and $T^{2\beta}_{1/2}(^{128}Te)$ obtained by the different authors were not understood at the time; later the smaller values were proven to be correct.

Let us add that in the 1980s compelling experimental evidences have been obtained, which showed that the neutrino ν_τ coupled to the τ lepton in the weak charged lepton current is of a new type (see [92]), i.e., that $\nu_\tau \neq \nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu$. In 2000 the DONUT experiment had directly observed the ν_τ interaction with matter in which the τ lepton was produced [93].

7 Developments in the Period 2000 – 2010

In 1998 the Super Kamiokande experiment provided compelling evidence for oscillations of the atmospheric ν_μ and $\bar{\nu}_\mu$ caused by non-zero neutrino masses and neutrino mixing [94], which was further elaborated in [95]. In 2001 – 2002 flavour conversion of the solar ν_e was proven to take place [96]. These remarkable discoveries, for which Takaaki Kajita (from the Super-Kamiokande Collaboration) and Arthur McDonald (from the SNO Collaboration) were awarded the Nobel Prize for Physics in 2015, had far reaching implications in particle physics in general, and for the searches for $(\beta\beta)_{0\nu}$−decay in particular. Further, disappearance of reactor $\bar{\nu}_e$ due to oscillations has been observed in the KamLAND experiment [97], while strong (to compelling) evidence for ν_μ disappearance due to oscillations were obtained also in the long-baseline accelerator neutrino experiments K2K [98] and MINOS [99].

In the reference 3-neutrino mixing framework and in the standard parametrisation of the PMNS matrix (see, e.g., [3]) the oscillations of the atmospheric and accelerator ν_μ and $\bar{\nu}_\mu$ and the flavour conversion of the solar ν_e to a good approximation are driven respectively by the parameters Δm^2_{31}, θ_{23} and Δm^2_{21}, θ_{12}, where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$, m_i, $i = 1, 2, 3$, being the mass of the neutrino ν_i, $\sin^2\theta_{23} \equiv |U_{\mu 3}|^2/(1 - |U_{e3}|^2)$ and $\sin^2\theta_{12} \equiv |U_{e2}|^2/(1 - |U_{e3}|^2)$. Here U_{lj}, $l = e, \mu, \tau$, $j = 1, 2, 3$, are elements of the PMNS matrix. In 1999 the CHOOZ experiment obtained a strong limit on the third mixing angle in the 3-neutrino mixing matrix, θ_{13} [100]: $\sin^2 2\theta_{13} < 0.10$ (90% CL) at $|\Delta m^2_{31}| = 2.5 \times 10^{-3}$ eV^2, where $\sin^2\theta_{13} = |U_{e3}|^2$ (Fig. 4).

The data accumulated in the neutrino oscillation experiments by 2003 allowed to determine the parameters $|\Delta m^2_{31}|$, Δm^2_{21} and $\sin^2\theta_{12}$ with a relatively good precision, while $\sin^2\theta_{13}$ was tightly constrained: $|\Delta m^2_{31}| \cong 2.6 \times 10^{-3}$ eV^2, $\Delta m^2_{21}| \cong 7.2 \times 10^{-5}$ eV^2, $\sin^2\theta_{12} \cong 0.3$. The sign of Δm^2_{31} could not (and still cannot) be determined from the data and the two possible signs correspond, as it is well known, to two possible types of neutrino mass spectrum (see, e.g., [3]): with normal ordering (NO), $m_1 < m_2 < m_3$, and with inverted ordering (IO), $m_3 < m_1 < m_2$. Depending on the value of the lightest neutrino mass, the spectrum can be i) normal hierarchical (NH), $m_1 \ll m_2 < m_3$, ii) inverted hierarchical (H), $m_3 \ll m_1 < m_2$, or iii) quasi-degenerate (QD), $m_1 \cong m_2 \cong m_3$ with $m^2_{1,2,3} >> |\Delta m^2_{31}|$. The latter requires $m_{1,2,3} \gtrsim 0.10$ eV.

These developments led to the important realisation that the $(\beta\beta)_{0\nu}$−decay experiments have

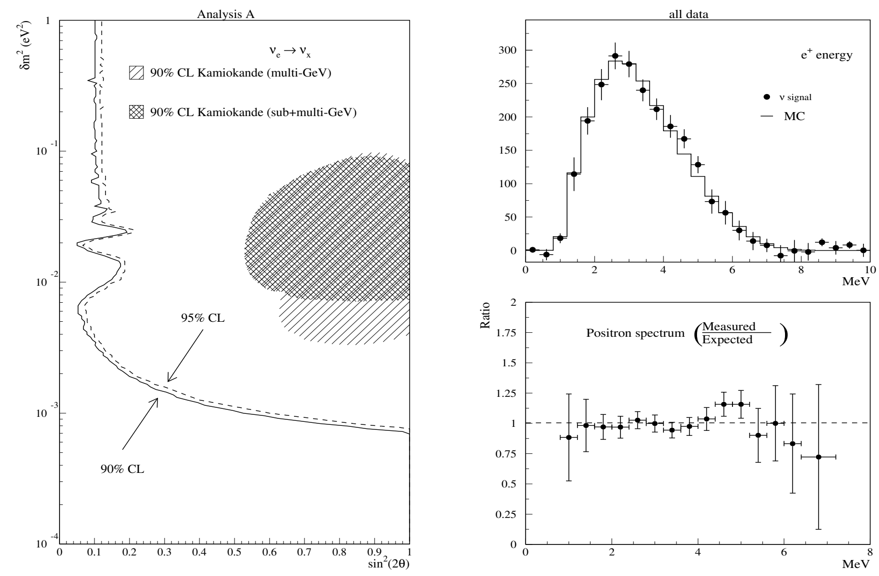

Figure 4 – The CHOOZ exclusion plot in the $|\Delta m^2_{31}| - \sin^2 2\theta_{13}$ plane (left panel) and the measured e^+ spectrum normalised to the expected spectrum (right panel). See [100] for details.

a remarkable physics potential [101,102,103,104]. They can probe the Majorana nature of massive neutrinos, and if neutrinos are proven to be Majorana particles, via the measurement of $|{<}m{>}|$ they can provide information on:
– the neutrino mass spectrum (NH, IH, QD),
– absolute neutrino mass scale,
– and with input on the value of $\min(m_j)$ – on the Majorana phases in the PMNS matrix.
This best is illustrated by Fig. 5 in which $|{<}m{>}|$ is shown as a function of $\min(m_j)$ [l].

In 2006 the quantitative studies performed in [110], which were based on advances in leptogenesis theory [111], have shown that the CP violation necessary for the generation of the observed baryon asymmetry of the Universe in the leptogenesis scenario can be provided exclusively by the Majorana phases of the neutrino mixing matrix U.

Two experiments with ^{76}Ge, IGEX [79] and Heidelberg-Moscow [80], published significantly improved limits on $T^{0\nu}_{1/2}(^{76}Ge)$ in 2002 and 2001, respectively. The IGEX experiment announced first results in 1996. In its final configuration it took data in the Canfranc tunnel, Spain, and used 6.5 kg Ge enriched in ^{76}Ge at 86%. The following lower limit was obtained [79]: $T^{0\nu}_{1/2}(^{76}Ge) > 1.57 \times 10^{25}$ yr (90% C.L.). A somewhat better limit was reported in 2001 by the Heidelberg–Moscow collaboration whose detector consisted of 11 kg of Ge enriched in ^{76}Ge at 86% [80] and was located in Gran Sasso Laboratory, Italy: $T^{0\nu}_{1/2}(^{76}Ge) > 1.9 \times 10^{25}$ yr (90% C.L.). This result was based on 47.4 kg y of data.

Analising a somewhat larger data set (54.98 kg y), H.V. Klapdor- Kleingrothaus with three members of the Heidelberg–Moscow collaboration claimed the observation of $(\beta\beta)_{0\nu}$−decay first (in 2001) with a half-life [112] $T^{0\nu}_{1/2}(^{76}Ge) = 1.5 \times 10^{25}$ yr, later (in 2004 using 71.7 kg y of data) with a half-life [113] $\mathrm{T}^{0\nu}_{1/2}(^{76}\mathrm{Ge}){=}1.19{\times}10^{25}$ yr, and finally in 2006 (reanalising the 2004 data with one co-author) with a half-life [114] $T^{0\nu}_{1/2}(^{76}Ge) = 2.23^{+0.44}_{-0.31} \times 10^{25}$ yr. The Moscow part of the Heidelberg–Moscow collaboration disagreed with this claim [115].

The analyses leading to the claims of observation of $(\beta\beta)_{0\nu}$−decay by Klapdor-Kleingrothaus et al. were criticised in a number of articles (see [69] for further details). The claimed positive

[l] Figures exhibiting $|{<}m{>}|$ versus $\min(m_j)$ appeared first in [102] in a form in which (in contrast to Fig. 5) only the contours delimiting the areas corresponding to NO and IO spectra are shown.

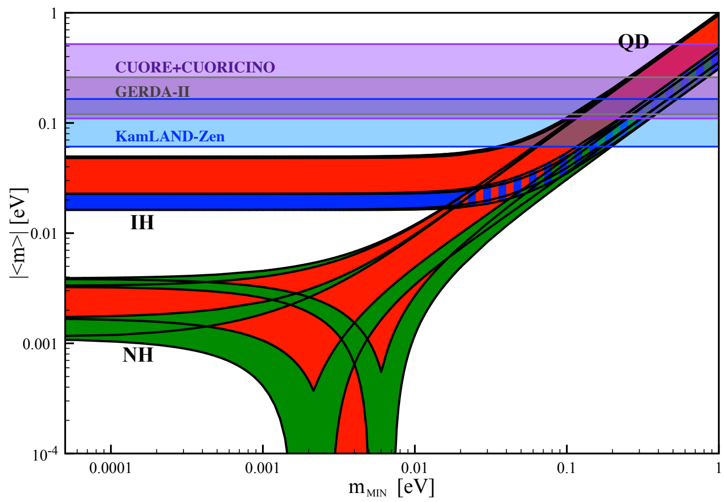

Figure 5 – The effective Majorana mass $|{<}\,m\,{>}|$ (including a 2σ uncertainty), as a function of min(m_j). The figure is obtained using the best fit values and the 1σ ranges of allowed values of Δm^2_{21}, $\sin^2\theta_{12}$, $\sin^2\theta_{13}$ and $|\Delta m^2_{31(32)}|$ from [105] (propagated to $|{<}\,m\,{>}|$ and then taking a 2σ uncertainty). The phases α_{21} and $(\alpha_{31}-2\delta)$ are varied in the interval [0,2π]. The predictions for the NH, IH and QD spectra as well as the GERDA-II [107], KamLAND-Zen [106] and the combined CUORE+CUORICINO [108] limits (Eqs. (23), (25) and (24)), are indicated. The black lines determine the ranges of values of $|{<}\,m\,{>}|$ for the different pairs of CP conserving values of α_{21} and $(\alpha_{31}-2\delta)$: $(0,0)$, $(0,\pi)$, $(\pi,0)$ and (π,π). The red regions correspond to at least one of the phases α_{21} and $(\alpha_{31}-2\delta)$ having a CP violating value, while the blue and green areas correspond to α_{21} and $(\alpha_{31}-2\delta)$ possessing CP conserving values (Update by S. Pascoli in [3] of a figure from [109].)

results on $(\beta\beta)_{0\nu}-$decay published by Klapdor-Kleingrothaus et al. in [112,113,114] were definitely proven to be incorrect by the results of GERDA II experiment with ^{76}Ge [107]:

$$\mathrm{T}^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 8.0\times 10^{25}\ \mathrm{yr}\ (90\%\ \mathrm{C.L})\,, \tag{22}$$

$$|{<}\,m\,{>}| < (0.16-0.26)\ \mathrm{eV}\,,\quad \mathrm{GERDA\ II}\,, \tag{23}$$

where we have quoted also the limit on the effective Majorana mass $|{<}\,m\,{>}|$ reported by the GERDA II collaboration. The interval reflects the estimated uncertainty in the relevant NME used to extract the limit on $|{<}\,m\,{>}|$.

Two experiments CUORICINO (using cryogenic detector with bolometers) and NEMO-3 (employing tracking device) took data in the discussed period and were completed respectively in 2008 and 2011. The CUORICINO collaboration used 40.7 kg of ^{130}Te and obtained the following final result [116]: $T^{0\nu}_{1/2}(^{130}Te) > 2.8\times 10^{24}$ yr.

The NEMO-3 experiment searched for $(\beta\beta)_{0\nu}$-decay and observed the $(\beta\beta)_{2\nu}-$decay of ^{48}Ca, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{130}Te and ^{15}Nd [117]. The best limit was on $(\beta\beta)_{0\nu}$-decay half-life was obtained with ^{100}Mo (6.914 kg source): $T^{0\nu}_{1/2}(^{100}Mo) > 1.1\times 10^{24}$ yr (90% C.L.). In this experiment more than 700 000 (!) $(\beta\beta)_{2\nu}-$decays of ^{100}Mo were detected with almost zero background and the individual energy spectra and angular distribution of electrons were measured [m]. The $(\beta\beta)_{2\nu}$-decay half-lives of ^{48}Ca, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{130}Te and ^{15}Nd were also determined with impressive precision [n].

Values of $(\beta\beta)_{2\nu}$ half-life $T^{2\nu}_{1/2}$ of different nuclei measured by 2010 are shown in Table 1 (taken from [118]).

[m] The images of $(\beta\beta)_{2\nu}-$decay from NEMO-3 detector have certain delicate elegance, see Fig. 6.

[n] These results are published in a large number of publications the last of which appeared in 2018. In the present article only four are quoted (see [117]).

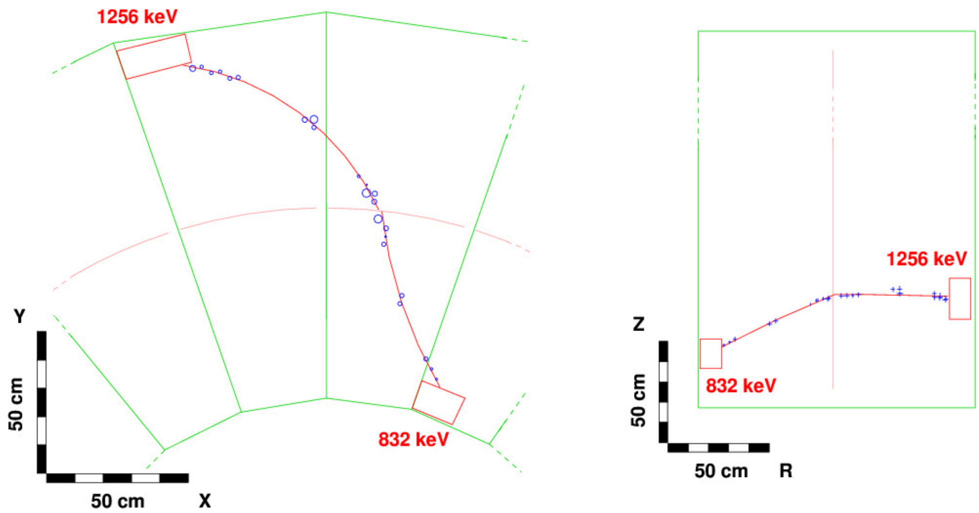

Figure 6 – A $(\beta\beta)_{2\nu}-$decay event as seen by the NEMO-3 detecor.

8 Latest Results, Current Problems and Prospects

We have quoted the latest result from the Gerda II experiment in Eq. (23), which represents the best limit on $(\beta\beta)_{0\nu}-$decay half-life of ^{76}Ge. We quote next two more best limits on $(\beta\beta)_{0\nu}-$decay half-lives of ^{130}Te and ^{13e}Xe, obtained respectively in the combined analysis of CUORICINO and CUORE-0 data [108] and in the KamLAND-Zen experiment [106]:

$$\begin{aligned} & \mathrm{T}^{0\nu}_{1/2}(^{130}\mathrm{Te}) > 1.5\times 10^{25}\ \mathrm{yr}\ (90\%\ \mathrm{C.L})\,, \\ & |{<}\,m\,{>}| < (0.11 - 0.52)\ \mathrm{eV}\,, \quad \mathrm{CUORICINO} + \mathrm{CUORE} - 0\,; \end{aligned} \tag{24}$$

$$\begin{aligned} & \mathrm{T}^{0\nu}_{1/2}(^{136}\mathrm{Xe}) > 1.07\times 10^{26}\ \mathrm{yr}\ (90\%\ \mathrm{C.L})\,, \\ & |{<}\,m\,{>}| < (0.061 - 0.165)\ \mathrm{eV}\,, \quad \mathrm{KamLAND} - \mathrm{Zen}\,, \end{aligned} \tag{25}$$

The intervals reflect the estimated uncertainties in the relevant NMEs used to extract the limits on $|{<}\,m\,{>}|$ from the experimentally obtained lower bounds on the ^{130}Te and ^{136}Xe $(\beta\beta)_{0\nu}-$decay half-lives [o].

Important role in the searches for $(\beta\beta)_{0\nu}-$decay in the discussed period was played by the EXO experiment with liquid ^{136}Xe time projection chamber [120]. In its final phase (EXO-200), the data was obtained with $\sim$100 kg of ^{136}Xe. The following best limit was reported by EXO collaboration [121]: $T^{0\nu}_{1/2}(^{136}Xe) > 1.8\times 10^{25}$ yr (90% C.L).

The "conservative" upper limit $|{<}\,m\,{>}| = 0.165$ eV (Eq. (25)), which is in the range of the QD neutrino mass spectrum, implies the following upper limit on the absolute Majorana neutrino mass scale [122]: $m_0 \cong m_{1,2,3} \lesssim 0.60$ eV.

A large number of experiments of a new generation aims at a sensitivity to $|{<}\,m\,{>}| \sim (0.01 - 0.05)$ eV [p], which will allow to probe the whole range of the predictions for $|{<}\,m\,{>}|$ in the case of IO neutrino mass spectrum [103] shown in Fig. 5 (for reviews of the currently running and future planned $(\beta\beta)_{0\nu}$-decay experiments and their prospective sensitivities see, e.g., [119,123,124]).

[o] For a review of the limits on $|{<}\,m\,{>}|$ obtained in other $(\beta\beta)_{0\nu}$-decay experiments and a detailed discussion of the NME calculations for $(\beta\beta)_{0\nu}$-decay and their uncertainties see, e.g., [119].

[p] An incomplete list includes CUORE (^{130}Te), GERDA II (^{76}Ge), MAJORANA (^{76}Ge), LEGEND (^{76}Ge), KamLAND-ZEN (^{136}Xe), nEXO (^{136}Xe), SNO+ (^{130}Te), AMoRE (^{100}Mo), CANDLES (^{48}Ca), SuperNEMO (^{82}Se, ^{150}Nd), NEXT (^{136}Xe), DCBA (^{82}Se, ^{150}Nd), PANDAX-III (^{136}Xe), ZICOS (^{96}Zr), MOON (^{100}Mo).

Table 1: Values of $T_{1/2}^{2\nu}$ measured by 2010. (Table taken from [118] where one can find details of how the values quoted in the table were obtained.)

Isotope	$T_{1/2}^{2\nu}$ y
^{48}Ca	$4.4^{+0.6}_{-0.5} \times 10^{19}$
^{76}Ge	$(1.5 \pm 0.1) \times 10^{21}$
^{82}Se	$(0.92 \pm 0.07) \times 10^{20}$
^{96}Zr	$(2.3 \pm 0.2) \times 10^{19}$
^{100}Mo	$(7.1 \pm 0.4) \times 10^{18}$
^{100}Mo–^{100}Ru(0_1^+)	$5.9^{+0.8}_{-0.6} \times 10^{20}$
^{116}Cd	$(2.8 \pm 0.2) \times 10^{19}$
^{128}Te	$(1.9 \pm 0.4) \times 10^{24}$
^{130}Te	$6.8^{+1.2}_{-1.1} \times 10^{20}$
^{150}Nd	$(8.2 \pm 0.9) \times 10^{18}$
^{150}Nd–^{150}Sm(0_1^+)	$1.33^{+0.45}_{-0.26} \times 10^{20}$
^{238}U	$(2.0 \pm 0.6) \times 10^{21}$
^{130}Ba, ECEC(2ν)	$(2.2 \pm 0.5) \times 10^{21}$

Obtaining quantitative information on the neutrino mixing parameters from a measurement of $(\beta\beta)_{0\nu}$-decay half-life would be impossible without sufficiently precise knowledge of the corresponding NME of the process. At present the variation of the values of different $(\beta\beta)_{0\nu}$-decay NMEs calculated using various currently employed methods is typically by factors $\sim (2-3)$ (for a discussion of the current status of the calculations of the NMEs for the $(\beta\beta)_{0\nu}$-decay see, e.g., [119,124,125]).

Additional source of uncertainty is the effective value of the axial-vector coupling constant g_A in $(\beta\beta)_{0\nu}-$decay. This constant is related to the weak charged axial current (Gamow-Teller transitions), which is not conserved and therefore can be and is renormalised, i.e., quenched, by the nuclear medium. This implies that g_A is reduced from its free value of $g_A = 1.269$. The reduction of g_A can have important implications for the $(\beta\beta)_{0\nu}-$decay searches since to a good approximation $T_{1/2}^{0\nu} \propto (g_A^{eff})^{-4}$. The problem of the g_A quenching arose in connection with the efforts to describe theoretically the experimental data on $(\beta\beta)_{2\nu}-$decay [126]. The physical origin of the quenching is not fully understood, and the magnitude of the quenching of g_A in $(\beta\beta)_{0\nu}-$decay is subject to debates (for further details see, e.g., [119]).

The $(\beta\beta)_{0\nu}-$decay can be generated, in principle, by a $\Delta L = 2$ mechanism other than the light Majorana neutrino exchange considered here, or by a combination of mechanisms one of which is the light Majorana neutrino exchange (for a discussion of different mechanisms which can trigger $(\beta\beta)_{0\nu}$-decay, see, e.g., [127,128] and the articles quoted therein). Actually, the predictions for $|<\!m\!>|$ in the cases of the NH, IH and QD neutrino mass spectra (shown in Fig. 5) can be drastically modified by the existence of lepton charge non-conserving ($|\Delta L| = 2$) new physics beyond that predicted by the SM: eV or GeV to TeV scale RH Majorana neutrinos, etc. (see, e.g., [129]). There is a potential synergy between the searches for $(\beta\beta)_{0\nu}-$decay and the searches for neutrino-related $|\Delta L| = 2$ beyond the SM physics at LHC: $(\beta\beta)_{0\nu}-$decay experiments with a sensitivity to half-lives of $T_{1/2}^{0\nu} = 10^{25}$ yr probe approximately values of $|<\!m\!>| \sim 0.1$ eV and "new physics" at the scale $\Lambda_{LNV} \sim 1$ TeV (see, e.g., [130] and references quoted therein).

9 Epilogue

The authors of the first experiment searching for $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ decays in the USSR [q], Dobrokhotov et al., wrote in 1956 [24]: "The search for double beta-decay is an amazing example of a fantastic succession of periods of hope and of disillusionment. Two times in the course of a single decade this phenomenon has been "discovered", and both times the discovery has been found to be erroneous. The history of the question is still not complete; the phenomenon has not been observed experimentally, and the succession of journal articles in recent years only gradually sets larger and larger lower limits on the lifetime of a nucleus capable of double beta-decay. In the present research we have again not succeeded in observing the event, but the limit of the half-life of the process has been raised to about 0.7×10^{19} years, and further steps in this direction (if indeed they are worth-while) will depend on achieving considerable increases in the amount of material subjected to study."

The situation regarding the experimental searches for $(\beta\beta)_{0\nu}-$decay is essentially the same today. The quest for one of the deepest "secrets" of Nature – the nature (Dirac or Majorana) of massive neutrinos and for the status of lepton charge conservation – continues.

Acknowledgements

The author would like to thank M. Cribier, J. Dumarchez, D. Vignaud and their colleagues for the organisation of the timely and very interesting conference on "History of the Neutrino". Useful discussions with F. Šimkovic of the NME calculations for $(\beta\beta)_{0\nu}$ and $(\beta\beta)_{2\nu}$ decays are acknowledged with gratefulness. This work was supported in part by the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreements No 674896 (ITN Elusives) and No 690575 (RISE InvisiblesPlus), by the INFN program on Theoretical Astroparticle Physics and by the World Premier International Research Center Initiative (WPI Initiative, MEXT), Japan.

References

1. E. Witten, "Symmetry and Emergence," Nature Phys. **14** (2018) 116 [arXiv:1710.01791 [hep-th]].
2. S. Weinberg, CERN Courier, 13 October 2017.
3. K. Nakamura and S.T. Petcov in M. Tanabashi *et al.* [Particle Data Group], "Review of Particle Physics," Phys. Rev. D **98** (2018) 030001.
4. D. N. Dinh, S. T. Petcov, N. Sasao, M. Tanaka and M. Yoshimura, "Observables in Neutrino Mass Spectroscopy Using Atoms," Phys. Lett. B **719**, 154 (2013).
5. W. Pauli, "Open letter to the group of radioactive people at the Gauverein meeting in Tubingen", December 4, 1930. The text of the letter (translated in English) is cited in: L.M. Brown, *Physics Today*, September 1978, p.23.
6. E. Fermi, "Versuch einer theorie der β-strahlen", Z. Physik **88**, 161 (1934).
7. C. Jarlkog, these Proceedings.
8. M. Goeppert-Mayer, "Double-beta disintegration", *Phys. Rev.* D **48**, 512 (1935).
9. G. Gamow and E. Teller, "Selection Rules for the β-Disintegration", *Phys. Rev.* D **49**, 895 (1936).
10. H.A. Bethe and R.F. Bacher, "Nuclear Physics A. Stationary States of Nuclei", Rev. Mod. Phys. **8**, 82 (1936).
11. J. von Neumann, "Einige Bemerkungen zur Diracschen Theorie des relativistischen Drehelektrons", Zeitschrift für Physik **48**, 868 (1928).

[q] The experiment was performed with ^{48}Ca.

12. E. Majorana, "Teoria simmetrica dell'elettrone e del positrone", *Nuovo Cimento* **14**, 171 (1937)
13. F. Guerra, these Proceedings.
14. G. Racah, "Sulla simmetria tra particelle e antiparticelle", *Nuovo Cimento* **14**, 322 (1937).
15. B. Pontecorvo, Priroda **1**, 43 (1983) as quoted in [17].
16. W.H. Furry, "On transition probabilities in double-beta disintegration", *Phys. Rev.* D **56**, 1184 (1939).
17. A.S. Barabash, "Double Beta Decay: Historical Review of 75 Years of Research," Phys. Atom. Nucl. **74**, 603 (2011) (arXiv:1104.2714).
18. E.L. Fireman, "Double beta decay", *Phys. Rev.* **74**, 1238 (1948).
19. E.L. Fireman, "A measurement of the half-life of double-beta decay of ^{124}Sn", *Phys. Rev.* **75**, 323 (1949).
20. J.H. Fremlin and M.C. Walters, Proc. Phys. Soc. London A **65**, 911 (1952).
21. J.A. McCarthy, "Search for Double Beta-Decay in ^{124}Sn and ^{96}Zr", *Phys. Rev.* **90**, 853 (1953).
22. J.A. McCarthy, "Search for Double Beta Decay in ^{48}Ca", *Phys. Rev.* **97**, 1234 (1955).
23. A. Berthelot *et al.*, Compt. Rend. **236**, 1769 (1953).
24. E.N. Dobrokhotov, V.R. Lazarenko and S.Yu. Luk'yanov, Dokl. Akad. Nauk SSSR **110**, 966 (1956) (Sov. Phys. Dokl. **1**, 600 (1956)).
25. M.G. Inghram and J.H. Reynolds, "On the Double Beta-Process", *Phys. Rev.* **76**, 1265 (1949).
26. M.G. Inghram and J.H. Reynolds, "Double-beta decay of ^{130}Te", *Phys. Rev.* **78**, 822 (1950).
27. C. Alduino *et al.*, "Measurement of the Two-Neutrino Double Beta Decay Half-life of ^{130}Te with the CUORE-0 Experiment", EPJ C **77**, 13 (2017) [arXiv:1609.01666].
28. C.L. Cowan Jr., F. Reines *et al.*, "Detection of the Free Neutrino: a Confirmation", Science **124**, 103 (1956).
29. T. D. Lee and C. N. Yang, "Question of Parity Conservation in Weak Interactions", *Phys. Rev.* D **104**, 254 (1956).
30. C. S. Wu *et al.*, "Experimental test of parity conservation in beta decay", *Phys. Rev.* D **104**, 1413 (1957).
31. L.D. Landau, "On the conservation laws for weak interactions", Nucl. Phys. **3**, 127 (1957).
32. T.D. Lee and C.N. Yang, "Parity non-conservation and a two-component theory of the neutrino ", *Phys. Rev.* D **105**, 1671 (1957).
33. A. Salam, "On parity conservation and neutrino mass", Nuovo Cim. **5**, 299 (1957).
34. M. Goldhaber, L. Grodzins and A.W. Sunyar, "Helicity of neutrinos", *Phys. Rev.* D **109**, 1015 (1958).
35. R.P. Feynman and M. Gell-Mann, "Theory of the Fermi Interaction", *Phys. Rev.* D **109**, 193 (1958).
36. E.C.G. Sudarshan and R.E. Marshak, "Chirality Invariance and the Universal Fermi Interaction", *Phys. Rev.* D **109**, 1860 (1958).
37. B. Pontecorvo, "Nuclear capture of mesons and the meson decay", *Phys. Rev.* D **72**, 246 (1947).
38. B. Pontecorvo, "Mesonium and anti-mesonium," Sov. Phys. JETP **6**,429 (1957) [Zh. Eksp. Teor. Fiz. **33**, 549 (1957)].
39. B. Pontecorvo, "Inverse beta processes and nonconservation of lepton charge," Sov. Phys. JETP **7**, 172 (1958) [Zh. Eksp. Teor. Fiz. **34**, 247 (1957)].
40. Z. Maki, M. Nakagawa and S. Sakata, "Remarks on the unified model of elementary particles," Prog. Theor. Phys. **28**, 870 (1962).
41. E. Greuling, R.C. Whitten, "Lepton conservation and double beta-decay", Ann. Phys (N.Y.) **11**, 510 (1960).

42. E. der Mateosian, M. Goldhaber, “Limits for Lepton-Conserving and Lepton-Nonconserving Double Beta Decay in ^{48}Ca”, Phys. Rev. **146**, 810 (1966).
43. E. Fiorini *et al.*, “A search for lepton non-conservation in double beta decay with a germanium detector”, Phys. Lett. B **25**, 602 (1967).
44. E. Fiorini *et al.*, “Neutrinoless double-beta decay of ^{76}Ge”, Nuovo Cimento A **13**, 747 (1973).
45. A.A. Smolnikov, these Proceedings.
46. R.K. Bardin *et al.*, “A search for the double beta decay of ^{48}Ca and lepton conservation”, Nucl. Phys. A **158**, 337 (1970); B.T. Cleveland *et al.*, “Lepton Conservation in the Double β Decay of ^{82}Se”, Phys. Rev. Lett. **35**, 757 (1975); see also R.K. Bardin *et al.*, “Double beta decay in ^{48}Ca and the conservation of leptons”, Phys. Lett. B **26**, 112 (1967).
47. T. Kirsten and H.W. Muller, Earth and Planet Science letters **6**, 271 (1969).
48. T. Kirsten et al., “Experimental evidence for the double-beta decay of ^{130}Te”, Phys. Rev. Lett. **20**, 1300 (1968).
49. N. Takaoka and K. Ogata, “The half-life of ^{130}Te double-beta decay”, Z. Naturforsch. A **21**, 84 (1966).
50. E.W. Hennecke, O.K. Manuel and D.D. Sabu, Phys. Rev. C **11**, 1378 (1975).
51. B. Pontecorvo, “Electron and Muon Neutrinos,” Sov. Phys. JETP **10**, 1236 (1960) [Zh. Eksp. Teor. Fiz. **37**, 1751 (1959)].
52. M. Schwartz, “Feasibility of using high-energy neutrinos to study the weak interactions,” Phys. Rev. Lett. **4**, 306 (1960).
53. G. Danby *et al.*, “Observation of High-Energy Neutrino Reactions and the Existence of Two Kinds of Neutrinos,” Phys. Rev. Lett. **9**, 36 (1962).
54. V. Gribov, B. Pontecorvo, “Neutrino astronomy and lepton charge”, Phys.Lett. B **28**, 493 (1969).
55. S.M. Bilenky and B. Pontecorvo, Lett. Nuovo Cim. **17**, 569 (1976).
56. A. Halprin, H. Primakoff and S.P. Rosen, “Double-beta decay and a massive Majorana neutrino”, Phys. Rev. D **13**, 2567 (1977).
57. A. Faessler *et al.*, “Arbitrary mass Majorana neutrinos in neutrinoless double beta decay”, Phys. Rev. D **90**, 096010 (2014).
58. S.M. Bilenky, J. Hosek and S.T. Petcov, “On the oscillations of neutrinos with Dirac and Majorana masses”, Phys. Lett. B **94**, 495 (1980).
59. S.M. Bilenky, N.P. Nedelcheva and S.T. Petcov, “Some implications of CP invariance for mixing of Majorana neutrinos”, Nucl. Phys. B **247**, 61 (1984).
60. J. Schechter and J.W.F. Valle, “Neutrino Masses in SU(2) $\times$ U(1) Theories,” Phys. Rev. D **22**, 2227 (1980) 2227.
61. M. Doi *et al.*, “CP Violation in Majorana Neutrinos,” Phys. Lett. **102B** (1981) 323.
62. P. Langacker *et al.*, “On the Mikheev-Smirnov-Wolfenstein (MSW) Mechanism of Amplification of Neutrino Oscillations in Matter,” Nucl. Phys. B **282**, 589 (1987).
63. L. Wolfenstein, “CP properties of Majorana neutrinos and double beta decay”, Phys. Lett. B **107**, 77 (1981).
64. B. Kayser, “CPT, CP, and C phases, and their effects, in Majorana-particle processes”, Phys. Rev. D **30**, 1023 (1984).
65. W. Konetschny and W. Kummer, “Nonconservation of Total Lepton Number with Scalar Bosons,” Phys. Lett. **70B** (1977) 433;
66. P. Minkowski, “Mu $\rightarrow$ e Gamma At A Rate Of One Out Of 1-Billion Muon Decays?,” *Phys. Lett. B* **67**, 421 (1977); T. Yanagida in *Proc. of the Workshop on Unified Theory and Baryon Number of the Universe*, KEK, Japan, 1979; M. Gell-Mann, P. Ramond and R. Slansky in Sanibel Talk, CALT-68-709, Feb 1979, and in *Supergravity* (North Holland, Amsterdam 1979); S.L.Glashow, Cargese Lectures (1979). R. N. Mohapatra and G. Senjanovic, “Neutrino Mass And Spontaneous Parity Nonconservation,” Phys. Rev. Lett.

44, 912 (1980).
67. S. Weinberg, "Baryon- and Lepton-Nonconserving Processes", Phys. Rev. Lett. **43**, 1566 (1979).
68. M. Fukugita and T. Yanagida, "Barygenesis without grand unification", Phys. Lett. B**174**, 45 (1986).
69. M.C. Goodman, these Proceedings.
70. J. Goldstone, "Field Theories with Superconductor Solutions," Nuovo Cim. **19**, 154 (1961)
71. H.M. Georgi, S.L. Glashow, S. Nussinov, "Unconventional model of neutrino masses", Nucl. Phys. B**193**, 297 (1981); Y. Chikashige, R.N. Mohapatra, R.D. Peccei, "Are there real Goldstone bosons associated with broken lepton number?", Phys. Lett. B**98**, 265 (1981); G.B. Gelmini, M. Roncadelli, "Left-handed neutrino mass scale and spontaneously broken lepton number", Phys. Lett. B**99**, 411 (1981).
72. J. Schechter and J.F.W. Valle, "Neutrinoless double-β decay in SU(2) $\times$ U(1) theories", Phys. Rev. D**25**, 2951 (1982).
73. M. Duerr, M. Lindner and A. Merle, "On the Quantitative Impact of the Schechter-Valle Theorem," JHEP **1106**, 091 (2011).
74. W. Haxton and G.J. Stephenson,Jr., "Double beta decay", Prog. Part. Nucl. Phys. **12**, 409 (1984).
75. M. Doi, T. Kotani and E. Takasugi, "Double Beta Decay and Majorana Neutrino", Prog. Theor. Phys. Suppl. **83**, 1 (1985).
76. P. Vogel and M.R. Zirnbauer, "Suppression of the two-neutrino double-beta decay by nuclear-structure effects", Phys. Rev. Lett. **57**, 3148 (1986).
77. D.O. Caldwell, J. Phys. G **17**, S137 (1991).
78. A.A. Vasenko *et al.*, in *Proc. of the 2nd Int. Symposium on Underground Physics'87*, Baksan Valley, USSR, Aug. 17–19, 1987 (Nauka, Moscow, 1988), p. 288; Mod. Phys. Lett. A **5**, 1299 (1990).
79. C.E. Aalseth *et al.* [IGEX Collab.], "The IGEX ^{76}Ge Neutrinoless Double-Beta Decay Experiment: Prospects for Next Generation Experiments", Phys. Rev. C**65**, 092007 (2002).
80. H.V. Klapdor-Kleingrothaus *et al.* [Heidelberg-Moscow Collab.], "Latest results from the Heidelberg-Moscow double beta decay experiment", Eur. Phys. J. A**12**, 147 (2001).
81. R. Luescher *et al.*, "Search for $\beta\beta$ decay in ^{136}Xe: new results from the Gotthard experiment", Phys. Lett. B**434**, 407 (1998).
82. E. Fiorini and T.O. Niinikoski, "Low-temperature calorimetry for rare decays", Nucl. Instrum. Methods Phys. Res. **224**, 83 (1984).
83. S.R. Elliott, A.A. Hahn and M.K. Moe, "Direct evidence for two-neutrino double-beta decay in ^{82}Se", Phys. Rev. Lett. **59**, 2020 (1987).
84. S.R. Elliott *et al.*, J. Phys. G**17**, S145 (1991); A. De Silva *et al.*, Phys. Rev. C**56**, 2451 (1997); A. Balysh *et al.*, "Double Beta Decay of ^{48}Ca", Phys. Rev. Lett. **77**, 5186 (1996).
85. H. Ejiri *et al.*, "Double beta decays of ^{100}Mo", Phys. Lett. B **258**, 17 (1991); J. Phys. Soc. Jpn **64**, 339 (1995).
86. D. Dassié *et al.*, "Two-neutrino double-β decay measurement of ^{100}Mo Phys. Rev. D**51**, 2090 (1995); R. Arnold *et al.*, JETP Lett. **61**, 170 (1995), Z. Phys. C**72**, 239 (1996), Nucl. Phys. A**636**, 209 (1998) and Nucl. Phys. A**658**, 299 (1999).
87. A. Kawashima, K. Takahashi and A. Masuda, Phys. Rev. C **47**, R2452 (1993).
88. T. Kirsten *et al.*, in *Proc. of the Int. Symposium on Nuclear Beta Decay and Neutrino (Osaka'86)* (World Sci., Singapore, 1986), p. 81.
89. O.K. Manuel, in *Proceedings o f the International Symposium on Nuclear Beta Decay and Neutrino (Osaka'86)* (World Sci., Singapore, 1986), p. 71.
90. T. Bernatowicz *et al.*, "Precise determination of relative and absolute $\beta\beta$-decay rates of ^{128}Te and ^{130}Te", Phys. Rev. C **47**, 806 (1993).

91. N. Takaoka, Y. Motomura and K. Nagano, “Half-life of ^{130}Te double-β decay measured with geologically qualified samples”, Phys. Rev. C **53**, 1557 (1996).
92. A. Blondel, these Proceedings (arXiv:1812.11362).
93. K. Kodama *et al.* [DONUT Collab.], “Observation of tau neutrino interactions,” Phys. Lett. B **504**, 218 (2001).
94. Y. Fukuda *et al.* [Super-Kamiokande Collaboration], “Evidence for oscillation of atmospheric neutrinos,” Phys. Rev. Lett. **81**, 1562 (1998).
95. Y. Ashie *et al.* [Super-Kamiokande Collab.], “Evidence for an Oscillatory Signature in Atmospheric Neutrino Oscillations”, Phys. Rev. Lett. **93**, 101801 (2004).
96. Q.R. Ahmad *et al.* [SNO Collab.], “Measurement of the Rate of ν_e + d $\rightarrow$ p+p+e$^-$ Interactions Produced by ^{8}B Solar Neutrinos at the Sudbury Neutrino Observatory”, Phys. Rev. Lett. **87**, 071301 (2001) and **89**, 011301 (2002).
97. K. Eguchi *et al.* [KamLAND Collab.], “First Results from KamLAND: Evidence for Reactor Antineutrino Disappearance”, Phys. Rev. Lett. **90**, 021802 (2003), and T. Araki *et al.* **94**, 081801 (2005).
98. M.H. Ahn *et al.* [K2K Collab.], “Measurement of neutrino oscillation by the K2K experiment”, Phys. Rev. D **74**, 072003 (2006).
99. D.G. Michael *et al.* [MINOS Collab.], “Observation of Muon Neutrino Disappearance with the MINOS Detectors in the NuMI Neutrino Beam”, Phys. Rev. Lett. **97**, 191801 (2006), and P. Adamson *et al.*, Phys. Rev. Lett. **101**, 131802 (2008).
100. M. Apollonio *et al.* [CHOOZ Collab.], “Limits on neutrino oscillations from the CHOOZ experiment”, Phys. Lett. B **466**, 415 (1999).
101. S.M. Bilenky *et al.*, Phys. Rev. D **54**, 4432 (1996); K. Czakon *et al.*, hep-ph/0003161; H.V. Klapdor-Kleingrothaus, H. Päs and A.Yu. Smirnov, Phys. Rev. D **63**, 073005 (2001); S.M. Bilenky, S. Pascoli, and S.T. Petcov, Phys. Rev. D **64**, 053010 and 113003 (2001).
102. F. Vissani, “Signal of neutrinoless double beta decay, neutrino spectrum and oscillation scenarios”, JHEP **9906**, 022 (1999).
103. S. Pascoli and S.T. Petcov, “The SNO solar neutrino data, neutrinoless double beta-decay and neutrino mass spectrum”, Phys. Lett. B **544**, 239 (2002); addendum in Phys. Lett. B **580**, 280 (2004).
104. S. Pascoli, S.T. Petcov and L. Wolfenstein, Phys. Lett. B **524**, 319 (2002). S. Pascoli, S.T. Petcov and T. Schwetz, Nucl. Phys. B **734**, 24 (2006); A. Faessler *et al.*, “Quasiparticle random phase approximation uncertainties and their correlations in the analysis of $0\nu\beta\beta$ decay”, Phys. Rev. D **79**, 053001 (2009).
105. F. Capozzi *et al.*, “Global constraints on absolute neutrino masses and their ordering”, Phys. Rev. D **95**, 096014 (2017).
106. A. Gando *et al.* [KamLAND-Zen Collab.], “Search for Majorana Neutrinos near the Inverted Mass Hierarchy Region with KamLAND-Zen,” Phys. Rev. Lett. **117**, 082503 (2016) and **117** (2016) 109903.
107. M. Agostini *et al.* [GERDA Collab.], “Improved Limit on Neutrinoless Double-β Decay of ^{76}Ge from GERDA Phase II,” Phys. Rev. Lett. **120**, 132503 (2018).
108. C. Alduino *et al.* [CUORE Collab.], “First Results from CUORE: A Search for Lepton Number Violation via $0\nu\beta\beta$ Decay of ^{130}Te,” Phys. Rev. Lett. **120**, 132501 (2018).
109. S. Pascoli and S.T. Petcov, “Majorana neutrinos, neutrino mass spectrum, and the m $\sim 10^{-3}$ eV frontier in neutrinoless double beta decay”, Phys. Rev. D **77**, 113003 (2008).
110. S. Pascoli, S.T. Petcov and A. Riotto, “Connecting low energy leptonic CP violation to leptogenesis”, Phys. Rev. D **75**, 083511 (2007) and Nucl. Phys. B **774**, 1 (2007).
111. A. Abada *et al.*, “Flavour issues in leptogenesis”, JCAP **0604**, 004 (2006); E. Nardi *et al.*, JHEP **0601**, 164 (2006).
112. H.V. Klapdor-Kleingrothaus *et al.* [Heidelberg-Moscow Collab.], “Evidence for neutrino-

less double beta decay", Mod. Phys. Lett. A **16**, 2409 (2001).
113. H.V. Klapdor-Kleingrothaus *et al.*, "Search for neutrinoless double beta decay with enriched ^{76}Ge in Gran Sasso 1990-2003", Phys. Lett. B **586**, 198 (2004).
114. H.V. Klapdor-Kleingrothaus and I.V. Krivosheina, "The evidence for the observation of $0\nu\beta\beta$ decay: The identification of $0\nu\beta\beta$ events from the full spectra", Mod. Phys. Lett. A **21**, 1547 (2006).
115. A.M. Bakalyarov *et al.*, "Results of the experiment on investigation of Germanium-76 double beta decay. Experimental data of Heidelberg-Moscow collaboration November 1995 - August 2001", Phys. Part. Nucl. Lett. **2**, 77 (2005) (hep-ex/0309016).
116. C. Arnaboldi *et al.* [CUORICINO Collab.], "New Limit on the Neutrinoless $\beta\beta$ Decay of ^{130}Te Phys. Rev. Lett. **95**, 142501 (2005); E. Andreotti *et al.*, Astropart. Phys. **34**, 822 (2011).
117. R. Arnold *et al.* [NEMO-3 Collab.], "Limits on different majoron decay modes of ^{100}Mo and ^{82}Se for neutrinoless double beta decays in the NEMO-3 experiment", Nucl. Phys. A **765**, 483 (2006) 483; Phys. Rev. D **92**, 072011 (2015), and Eur. Phys. J. C **78**, 821 (2018); J. Argyriades *et al.*, Nucl. Phys. A **847**, 168 (2010).
118. A.S. Barabash, "Precise half-life values for two-neutrino double-β decay", Phys. Rev. C **81**, 035501 (2010).
119. J. D. Vergados, H. Ejiri and F. Šimkovic, "Neutrinoless double beta decay and neutrino mass," Int. J. Mod. Phys. E **25**, 1630007 (2016).
120. G. Gratta, "Neutrinoless Double Beta Decay with EXO-200 and nEXO", Talk at XXVIII International Conference on Neutrino Physics and Astrophysics, 4-9 June 2018, Heidelberg, Germany, doi: 10.5281/zenodo.1286892.
121. J. B. Albert *et al.* [EXO Collab.], "Search for Neutrinoless Double-Beta Decay with the Upgraded EXO-200 Detector," Phys. Rev. Lett. **120**, 072701 (2018).
122. J.T. Penedo and S.T. Petcov, "The 10^{-3} eV frontier in neutrinoless double beta decay," Phys. Lett. B **786**, 410 (2018).
123. N. Abgrall *et al.* [LEGEND Collab.], "The Large Enriched Germanium Experiment for Neutrinoless Double Beta Decay (LEGEND)," AIP Conf. Proc. **1894**, 020027 (2017) (arXiv:1709.01980).
124. M.J. Dolinski, A.W.P. Poon and W. Rodejohann, "Neutrinoless Double-Beta Decay: Status and Prospects," arXiv:1902.04097.
125. J. Engel and J. Menèndez, "Status and Future of Nuclear Matrix Elements for Neutrinoless Double-Beta Decay: A Review," Rep. Prog. Phys. **80**, 046301 (2017).
126. F. Iachello, J. Kotila and J. Barea, "Quenching of g_A and its impact in double beta decay," PoS NEUTEL **2015**, 047 (2015)
127. A. Halprin *et al.*, "Double-beta Decay and a Massive Majorana Neutrino", Phys. Rev. D **13**, 2567 (1976); H. Päs *et al.*, Phys. Lett. B **453**, 194 (1999) and **498**, 35 (2001); F.F. Deppisch, M. Hirsch and H. Päs, Neutrinoless Double Beta Decay and Physics Beyond the Standard Model J. Phys. **G39**, 124007 (2012); L.C. Helo *et al.*, "Double beta decay and neutrino mass models," JHEP **1505**, 092 (2015).
128. A. Halprin, S.T. Petcov and S.P. Rosen, "Effects of Light and Heavy Majorana Neutrinos in Neutrinoless Double Beta Decay", Phys. Lett. B **453**, 335 (1983); V.M. Gehman and S.R. Elliott, " Multiple-Isotope Comparison for Determining $(\beta\beta)_{0\nu}$ Mechanisms", J. Phys. G **34**, 667 (2007), Erratum **35**, 029701 (2008); A. Faessler *et al.*, "Uncovering Multiple CP-Nonconserving Mechanisms of $(\beta\beta)_{0\nu}$ Decay," Phys. Rev. D **83**, 113003 (2011).
129. S.M. Bilenky, S. Pascoli and S.T. Petcov, "Majorana neutrinos, neutrino mass spectrum, CP violation and neutrinoless double beta decay. 2. Mixing of four neutrinos," Phys. Rev. D **64**, 113003 (2001); M. Blennow *et al.*, "Neutrinoless double beta decay in seesaw models," JHEP **1007**, 096 (2010); J. Barry, W. Rodejohann and H. Zhang, "Light Sterile Neutrinos: Models and Phenomenology," JHEP **1107**, 091 (2011).

130. J.C. Helo *et al.*, “Short-range mechanisms of neutrinoless double beta decay at the LHC,” Phys. Rev. D **88**, 073011 (2013); L. Gonzales *et al.*, “Scalar-mediated double beta decay and LHC,” JHEP **1612**, 130 (2016).