Naturalness in the Multicritical Universe

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T. Griffin, K.T. Grosvenor, P.H. and Z. Yan,
Multicritical Symmetry Breaking and Naturalness of Slow Nambu-Goldstone Bosons,
arXiv:1308.5967, Phys. Rev. **D88** (2013) 101701;

T. Griffin, K.T. Grosvenor, P.H. and Z. Yan,
Scalar Field Theories with Polynomial Shift Symmetries,
arXiv:1412.1046, Commun. Math. Phys. **340** (2015) 1720;

T. Griffin, K.T. Grosvenor, P.H. and Z. Yan,
Cascading Multicriticality in Nonrelativistic Spontaneous Symmetry Breaking,
arXiv:1507.06992, Phys. Rev. Lett. **115** (2015) 241601;

K.T. Grosvenor, P.H., C. Mogni and Z. Yan,
Nonrelativistic Short-Distance Completions of a Naturally Light Higgs,
arXiv:1608.06937;

A brief recent review: P.H., *Surprises with Nonrelativistic Naturalness*,
arXiv:1608.06287, Int. J. Mod. Phys. **D25** (2016)1645007;

& in progress with Christopher Mogni, Stephen Randall, Alexander Frenkel

Puzzles of Naturalness

Some of the most fascinating open problems in modern physics are all problems of naturalness:

- The cosmological constant problem
- The Higgs mass hierarchy problem
- in slow-roll inflation: The η problem
- The linear resistivity of strange metals, in the regime above T_c in high-T_c superconductors [Bednorz&Müller '86; Polchinski '92]

The first three puzzles involve gravity; and can all be phrased as questions about the size of various terms in the potential of a scalar field.

What is Naturalness?

Technical Naturalness: 't Hooft (1979)

"The concept of causality requires that macroscopic phenomena follow from microscopic equations."

"The following dogma should be followed: At any energy scale μ, a physical parameter or a set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system."

"Pursuing naturalness beyond 1000 GeV will require theories that are immensely complex compared with some of the grand unified schemes."

Example 1: Massive $\lambda\phi^4$ in $3+1$ dimensions.

$$S = \frac{1}{2}\int d^4x \left(\partial_\mu\phi\partial^\mu\phi - m^2\phi^2 - \frac{1}{12}\lambda\phi^4\right)$$

Recall technical naturalness:

$$\lambda \sim \varepsilon, \quad m^2 \sim \mu^2\varepsilon, \quad \mu \sim m/\sqrt{\lambda}.$$

Symmetry: The constant shift $\phi \to \phi + a$.

Example 2: Einstein gravity (with cosmological constant).

$$S = \frac{1}{16\pi G_N}\int d^4x \,(R - 2\Lambda)$$

No known good symmetry protects small Λ:

$$G_N \sim M_P^{-2},\ \Lambda \sim M_P^2.$$

Gravity without Relativity

(a.k.a. gravity with anisotropic scaling, or Hořava-Lifshitz gravity)

Gravity on spacetimes with a preferred time foliation (cf. FRW!)

Opens up the possibility of new RG fixed points, with improved UV behavior due to anisotropic scaling.

Field theories with anisotropic scaling:

$$x^i \to \lambda x^i, \quad t \to \lambda^z t.$$

z: dynamical critical exponent – characteristic of RG fixed point. Many interesting examples in condensed matter, dynamical critical phenomena, quantum critical systems, ..., with $z = 1,\ 2,\ \ldots,\ n,\ \ldots$, or fractions ($z = 3/2$ for KPZ surface growth in $D = 1$), ..., continous families ...

... and now gravity as well, with propagating gravitons, formulated as a quantum field theory of the metric.

What is Multicritical Universe?

A scenario for “minimal” reconciliation of particle physics and gravity, based on traditional principles of QFT & RG, in a framework that can potentially be UV complete.

Ingredients:

1. Particle side: the standard model, or your favorite BSM extension. Lorentz invariant. Already UV complete.

2. Gravity side: multicritical gravity, with anisotropic scaling in UV (= “HL gravity”). Flows to isotropic scaling in IR.

Robust consequences:

Anisotropic scaling communicated to the particle sector only via universal coupling to gravity.
Generic, high-energy Lorentz violations, also induced (and suppressed) by this irrelevant coupling.

Go “Nonrelativistic”: Aristotelian QFT

Motivation:

- inherited from nonrelativistic quantum gravity,
- new short-distance completions of relativistic QFTs,
- curiosity about new tools for technical naturalness in Standard Model & beyond, in cosmology,
- spin-off applications to condensed matter,
- interesting from math-ph perspective,
- curiosity about how far can string & M theory extend ...

The Aristotelian Spacetime

By the Aristotelian spacetime, we will mean $\mathbf{R}^{D+1}$ with the Cartesian coordinates (t, x^i), $i = 1, \dots D$, and with the flat metric $g_{ij} = \delta_{ij}$, $N = 1$, $N_i = 0$, and with the preferred foliation by the flat spatial slices of constant t.

By the Aristotelian symmetry, we will mean the isometries of the Aristotelian spacetime:

$$x^i \to \Lambda^i_j x^j + b^i, \qquad t \to t + b.$$

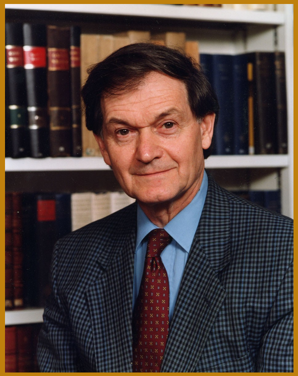
- These are *derived*, as all foliation-preserving diffeomorphisms that preserrve the metric.
- The isometries respect an emergent rest frame.
- Such spacetimes are solutions of HL gravities with zero Λ.
- Often called the "Lifshitz spacetime" in modern literature . . .

Lifshitz or Aristotelian Spacetime?

Evgenii Mikhailovich Lifshitz (February 21, 1915 – October 29, 1985)

Lifshitz or Aristotelian Spacetime?

History: in the mid-1960's, Andrzej Trautman, Roger Penrose



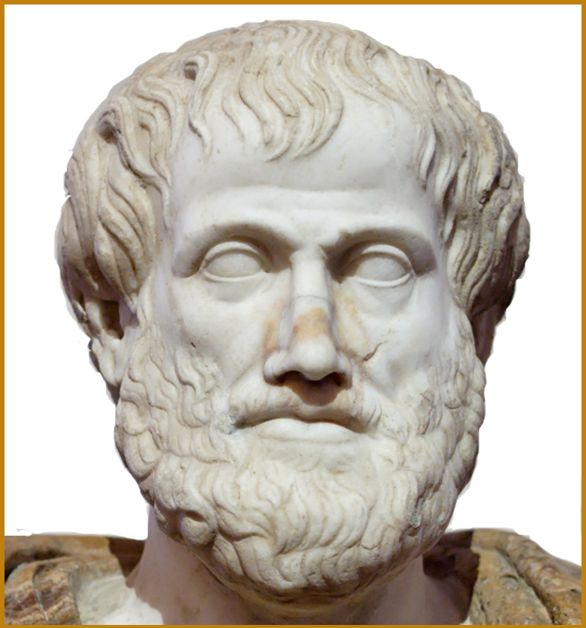
talked about the "Aristotelian spacetime": in Penrose's 1968 Structure of Space-Time (Battelle Rencontres), he begins with

- Before Einstein (curved relative space-time) and Minkowski (rigid relative space-time), there was
- Galilean spacetime (relative space and absolute time), and before that,
- Aristotelian spacetime (absolute space, absolute time)!

Lifshitz or Aristotelian Spacetime?

Aristotle?

Lifshitz or Aristotelian Spacetime?



Aristotle (384 – 322 BCE)

The Aristotelian Spacetime

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By the Aristotelian symmetry, we will mean the isometries of the Aristotelian spacetime:

$$x^i \to \Lambda^i_j x^j + b^i, \qquad t \to t + b.$$

The isometries respect an emergent rest frame.

If a QFT with Aristotelian symmetries is at an RG fixed point, it develops an extra symmetry, anisotropic conformal symmetry:

$$x^i \to \lambda x^i, \qquad t \to \lambda^z t.$$

Spontaneous Symmetry Breaking

Global internal symmetry breaking leads to Nambu-Goldstone modes. Phenomenon is remarkably universal, across many fields dealing with many-body systems.

But how many NG modes, and what is their low-energy dispersion relation?

- Relativistic case: All questions answered by Goldstone's theorem: One NG per broken generator, gapless=massless, $z = 1$ dispersion $\omega = k$.

- Nonrelativistic case: Classify NG modes by classifying their low-energy effective QFTs [Murayama&Watanabe, '12,'13].

Let's focus for definiteness on systems in Aristotelian spacetime. Write down possible EFT's for NG modes π^I.

Nonrelativistic Goldstone Theorem?

Assume Aristotelian symmetry. Then [Murayama&Watanabe]: the EFTs are

$$S = \int dt\, d^D\mathbf{x} \left(\Omega_I(\pi)\dot{\pi}^I + g_{IJ}(\pi)\dot{\pi}^I\dot{\pi}^J - h_{IJ}(\pi)\partial_i\pi^I\partial_i\pi^J + \ldots \right).$$

Hence, this yields two types of NG modes:

- Type A, $z = 1$ dispersion $\omega = ck$ (those unpaired by Ω, with no T-reversal breaking). As in the relativistic case, one Type A NG mode per one broken generator.

- Type B, dispersion $\omega \sim k^2$. Each associated with a *pair* of broken symmetry generators, as paired by Ω. Minimal T-reversal symmetry is broken.

Anything else would be fine tuning . . . or would it?

Is there a gap in the argument? Consider $z = 2$ theory:

$$S_{\rm eff} = \int dt\, d^D\mathbf{x}\, \left(g_{IJ}\dot{\pi}^I \dot{\pi}^J - \tilde{g}_{IJ}\Delta\pi^I \Delta\pi^J - c^2 \hat{g}_{IJ}\partial_i\pi^I \partial_i\pi^J \right).$$

If the relevant deformation is not generated, we can have new NG modes, with $z = 2$, associated with just one broken symmetry and not a pair, and with no time reversal breaking.

Example: Start with $z = 2$ $O(N)$ LSM in $3+1$ dimensions,

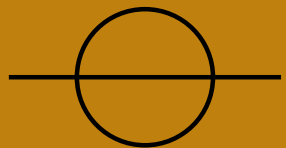
$$S_{\rm eff} = \int dt\, d^3\mathbf{x} \left(\dot{\phi}\cdot\dot{\phi} - \Delta\phi\cdot\Delta\phi - g(\phi^2)^2\partial_i\phi\partial_i\phi - \ldots - \lambda_5(\phi^2)^5 \right.$$

$$\left. - \ldots - c^2\partial_i\phi\cdot\partial_i\phi - \ldots - \lambda(\phi^2)^2 + m^4\phi^2 \right).$$

Naturalness of Slow NG Modes

How small is the quantum correction δc^2?

Consider the LSM, for simplicity in the unbroken phase. The first quantum correction to $\delta c^2 = 0$ comes at two loops, from



$$\sim \left(\frac{\lambda^2}{m^4}\right) |\mathbf{k}|^2 + \dots,$$

and it is finite. What does this mean?

Assume a hidden symmetry, broken by ε at scale μ:

$$\lambda \sim \mu^3 \varepsilon, \quad m^4 \sim \mu^4 \varepsilon, \quad c^2 \sim \mu^2 \varepsilon.$$

This implies $\mu \sim m/\lambda$ and $c^2 \sim \lambda^2/m^4$, just as we found by the explicit calculation!

Polynomial Shift Symmetry

So, there must be a new symmetry at play:

the "quadratic shift" symmetry

$$\pi^I(t, x^i) \to \pi^I(t, x^i) + a^I_{ij} x^i x^j + a^I_j x^j + a_0.$$

Note it depends only on spatial coordinates, not on time.
compare the Galileon cosmology: linear spacetime shifts)

This construction naturally iterates:

The higher "polynomial shift symmetry,"

$$\pi^I(t, x^i) \to \pi^I(t, x^i) + a^I_{j_1 j_2 \ldots j_{2z-2}} x^{j_1} x^{j_2} \ldots x^{j_{2z-2}} + \ldots$$

protects the $\omega \sim k^z$ low-energy dispersion for Type A modes
(and the $\omega \sim k^{2z}$ low-energy dispersion for Type B modes).

Refining the Classification of Nonrelativistic NG Modes

Refined classification of technically natural NG modes with Aristotelian spacetime symmetries:

• Type A tower of multicritical NG modes with $z = 1, 2, \ldots$, until one hits against the multicritical analog of the Coleman Hohenberg-Mermin-Wagner theorem at $z = D$;

• Type B tower of multicritical NG modes with $z = 2, 4, \ldots$ (and no analog of the MCW theorem).

These IR fixed points describe the free limit of multicritical NG modes, and imply low-energy theorems for scattering etc.

Generic interactions break the polynomial shift symmetry to the constant shift. But: Corrections are controllably small, if couplings are small.

Coleman-Hohenberg-Mermin-Wagner & Cascading Multicriticality

Recall relativistic CMW theorem: In $1+1$ dimensions, SSB which would require a NG mode ϕ can never happen, since ϕ does not exist: $\langle\phi(0)\phi(x)\rangle \sim \log(\mu_{\rm IR}x)$.

Multicritical analog of CMW theorem: Type A $D=z$ NG modes do not exist as quantum objects at the fixed point.

Take say $z=3$ in $3+1$ dimensions, below some scale μ. Naive CMW theorem: no symmetry breaking, no condensate?

Novelty: Cascading hierarchical multicriticality of NG modes.

At some *physical* crossover scale $\mu_{\rm IR} \ll \mu$, turn on a $z<3$ deformation. The theory self-regulates in IR, with $\omega \sim |\mathbf{k}|^3 + \ldots \mu_{\rm IR}^2|\mathbf{k}|$. And SSB is possible, after all!

Lab implications in condensed matter? Cosmology?

Field Theories with Polynomial Shift Symmetries

So, we found examples where a new symmetry

$$\phi(t, x^i) \to \phi(t, x^i) + a_{j_1 j_2 \ldots j_P} x^{j_1} x^{j_2} \ldots x^{j_P} + \ldots$$

protects the smallness of leading terms in the dispersion relation, and protects hierarchies.

In the examples shown, the symmetry is broken by interactions.

Now we can turn this around, and ask for the classification of scalar theories in which the polynomial shift symmetry is exact.

This is a very cute mathematical problem!

The simplest case of linear shift is related to the Galileon.

Polynomial Shift Invariants

It is natural to organize the invariants by their dimension at the free RG fixed point.

Task: Classify all terms in the Lagrangian containing n fields and $\Delta \equiv 2m$ derivatives, invariant under the degree-P shift symmetry up to a total derivative:

$$\delta_P L = \partial_i L_i.$$

This is essentially a cohomological problem.
It defines a vector space of invariants $H_{P,n,\Delta,D}$.

How to solve it?

use Graph Theory!

Graph Theory

Represent each $\partial_{i_1} \dots \phi \dots \partial_{i_{2m}} \phi$ term by a graph:

(1) Each field ϕ is represented by a vertex ($\bullet$);

(2) Each pair $\partial_i [\dots] \partial_i$ is represented by a link: ———

Consider only "loopless graphs" – classification up to integration by parts.

To formulate $\delta_P L = \partial_i L_i$, two more vertices needed:

(3) each $a_{j_1 \dots j_P} x^{j_1} \dots x^{j_P}$ is represented by "$\otimes$";

(4) The ∂_i on the RHS is represented by a "free end": $\star$.

Examples: Galileons and Beyond

Start with $P = 1$, linear shift symmetry.

Problem of (minimal) n-point 1-invariants is equivalent to Galileons.

The minimal value for Δ is $\Delta = n - 1$, the invariant is unique:

$$L_n \propto T^{i_1 \ldots i_{n-1} j_1 \ldots j_{n-1}} \, \partial_{i_1}\phi \, \partial_{j_1}\phi \, \partial_{i_2 j_2}\phi \ldots \partial_{i_{n-1} j_{n-1}}\phi,$$

where

$$T^{i_1 \ldots i_{n-1} j_1 \ldots j_{n-1}} = \varepsilon^{i_1 \ldots i_{n-1} k_n \ldots k_D} \, \varepsilon^{j_1 \ldots j_{n-1}}{}_{k_n \ldots k_D}.$$

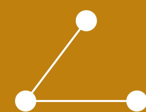
These are known (essentially in the Galileon literature).
Yet, the graph-theory representation reveals new patterns in these 1-invariants.

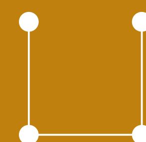
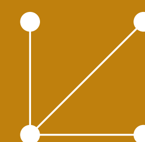
Examples: Galileons and Beyond

The n-point 1-invariants are:

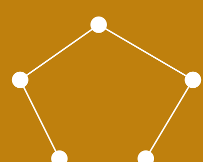
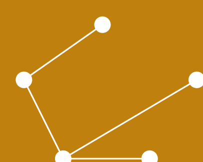
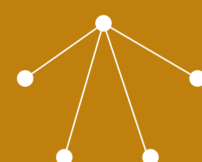
$L_1 = \phi = \bullet,$

$L_2 = \partial_i\phi\partial_i\phi =$  ,

$L_3 = 3\partial_i\phi\partial_j\phi\partial_i\partial_j\phi = 3$  ,

$L_4 = 12[i][ij][jk][k] + 4[i][j][k][ijk] = 12$  $+\, 4$  ,

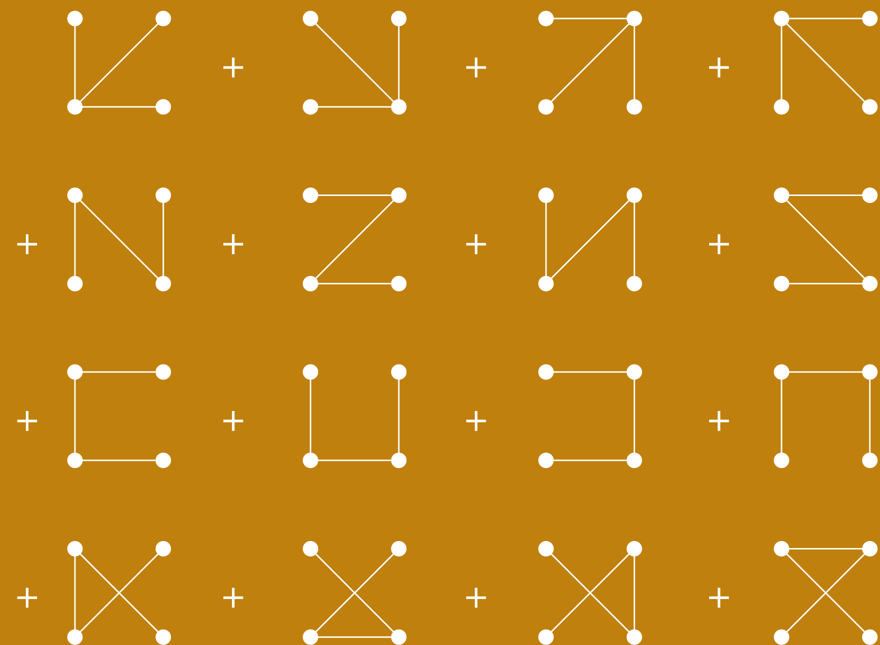
$L_5 = 60[i][ij][jk][kl][l] + 60[i][ij][jkl][k][l] + 5[i][j][k][l][ijkl]$

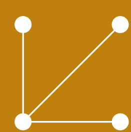
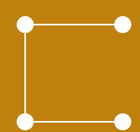
$= 60$  $+\, 60$  $+\, 5$  .

In graph theory, these unique 1-invariants correspond to the sum over all (spanning) trees, with equal weight!

Examples: Galileons and Beyond

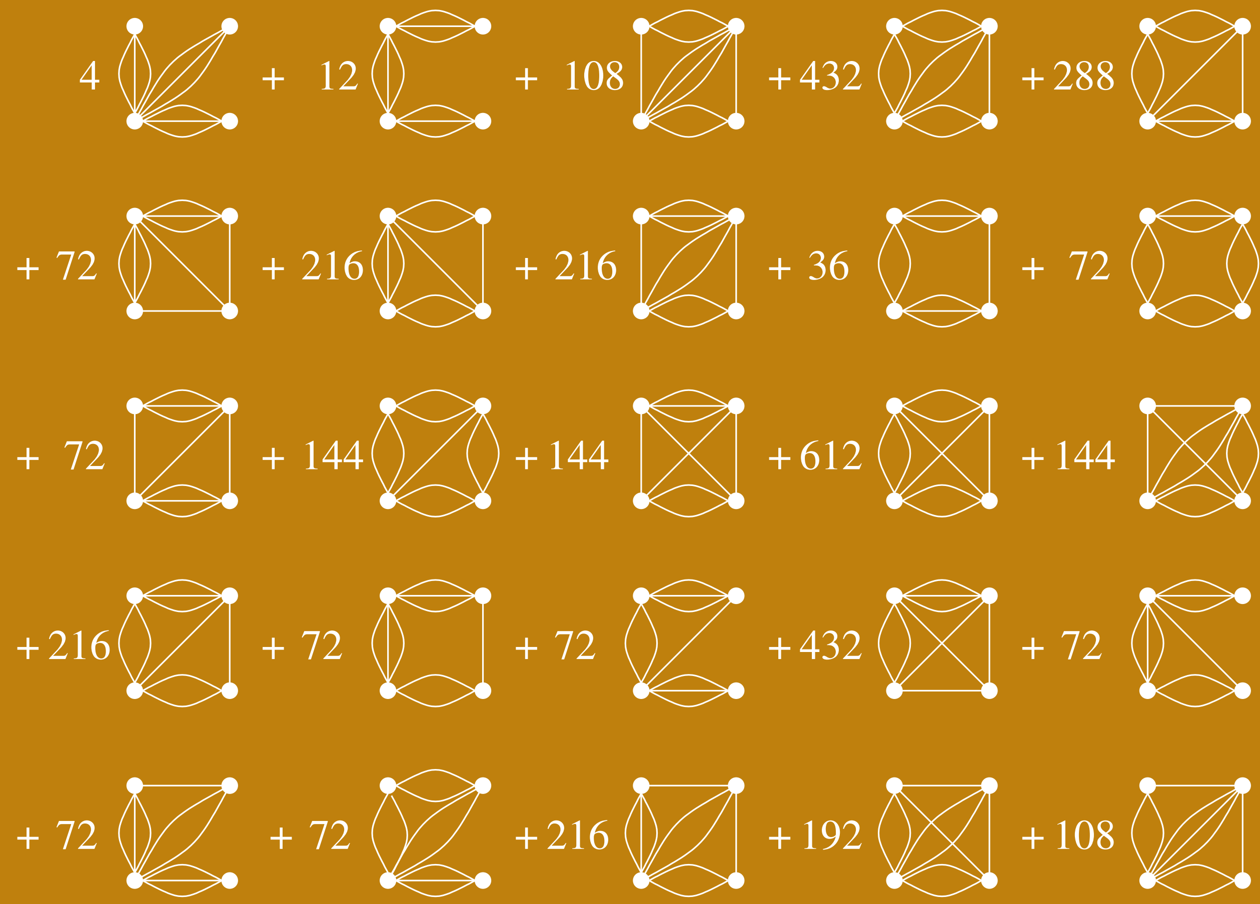
For example, the 4-point 1-invariant L_4 is:



$= 4$  $+ 12$ 

$P > 0$ Invariants: Superposition of Trees

Example: The most relevant quintic-shift 4-pt invariant is



Cascading Multicriticality: Examples

$2+1$ dimensions:

$$S = \frac{1}{2}\int dt\, d^2\mathbf{x} \left\{ \dot{\phi}^2 - (\partial_i\partial_j\phi)^2 - c^2(\partial_i\phi)^2 - m^2\phi^2 - g(\partial_i\phi\partial_i\phi)^2 \right\}$$

$$g \sim \varepsilon_1, \quad c^2 \sim \varepsilon_1\mu^2, \quad m^2 \sim \varepsilon_0\mu^4, \quad \varepsilon_0 \ll \varepsilon_1 \ll 1.$$

$3+1$ dimensions:

$$S = \frac{1}{2}\int dt\, d^3\mathbf{x} \quad \left\{ \dot{\phi}^2 - (\partial_i\partial_j\partial_k\phi)^2 - \zeta_2^2(\partial_i\partial_j\phi)^2 - c^2(\partial_i\phi)^2 - m^2\phi^2 \right.$$
$$\left. - \lambda\epsilon_{ijk}\epsilon_{\ell mp}\partial_i\phi\partial_j\partial_\ell\phi\partial_k\partial_m\phi\partial_p\phi \right\}$$

$$\zeta_2^2 \sim \varepsilon_2\mu^2, \quad \lambda \sim \varepsilon_1, \quad c^2 \sim \varepsilon_1\mu^4, \quad m^2 \sim \varepsilon_0\mu^6,$$
$$\varepsilon_0 \ll \varepsilon_1 \ll \varepsilon_2 \ll 1.$$

Now, Higgs Naturalness

Let's start with a simple scalar field theory first.

Recall one of our earlier examples of a cascading hierarchy:

$$S = \frac{1}{2}\int dt\, d^3\mathbf{x}\left\{\dot{\phi}^2 - (\partial_i\partial_j\partial_k\phi)^2 - \zeta_2^2(\partial_i\partial_j\phi)^2 - c^2(\partial_i\phi)^2 - m^2\phi^2 \right.$$
$$\left. - \lambda\epsilon_{ijk}\epsilon_{\ell mp}\partial_i\phi\partial_j\partial_\ell\phi\partial_k\partial_m\phi\partial_p\phi\right\}$$

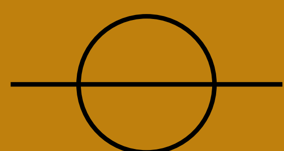
$$\zeta_2^2 \sim \varepsilon_2\mu^2, \quad \lambda \sim \varepsilon_1, \quad c^2 \sim \varepsilon_1\mu^4, \quad m^2 \sim \varepsilon_0\mu^6,$$
$$\varepsilon_0 \ll \varepsilon_1 \ll \varepsilon_2 \ll 1.$$

Quantum properties of the Model

- interaction 4-vertex: $-i\lambda|[\mathbf{kpq}]|^2$, where $[\mathbf{kpq}] \equiv \varepsilon_{ij\ell} k_i p_j q_\ell$.

- Nonrenormalization theorems: For λ and c^2 (and no wave-function renormalization). Proof: Simple, from the form of the vertex.

- Renormalization of 2-point fn:



$$\sim A \log \Lambda |\mathbf{k}|^6 + B\Lambda^2 |\mathbf{k}|^4;$$

hence, logaritmic running of ζ_3^2 (so, $\zeta_3^2 \neq 1$), and quadratically divergent ζ_2^2.

Quantum properties of the Model

- Asymptotic freedom? No; the effective coupling is $\bar{\lambda} = \lambda/\zeta_3^2$, runs to strong.

- Vacuum instability: Theory perturbatively stable, the λ self-interaction has energy unbounded from below. The $\phi = 0$ vacuum decays by tunnelling.

- Instability of ϕ particle: Damping. Finite life-time, $\propto \lambda^2$ (and $|\mathbf{k}|^3$ at high momenta). Narrow for small λ. Classicalization?

- Difference between Aristotelian and Wilsonian observers: Observers can differ by their definition of space vs. time.

The $O(N)$ Extension & Large N Limit

$O(N)$ generalization: Promote $\phi \to \phi^I$, $I = 1, \ldots, N$. There is a unique $O)(N)$ invariant generalization of the 4-pt λ vertex.

Large N limit: Define the 't Hooft coupling $\lambda' = \lambda N$. Take $N \to \infty$, holding λ' fixed.

- Self-interactions: As measured by λ', they are nonzero and finite.
- Vacuum instability: Suppressed by $1/N$.
- Particle decay: Suppressed by $1/N$. (The divergent parts of the 2-pt fn suppressed by $1/N$.)

The large N limit: what is its holographic dual??

Aristotelian, Wilsonian and Lorentzian Observers

Different observers: Different ways how to relate space to time.

Examples:

- Aristotelian observers. Choose t and y^i once and for all, regardless of the dynamics of fields in spacetime. Renormalization generates $\zeta_3^2(\mu)$, but $\lambda(\mu) = \lambda_{\rm bare}$. Running dispersion relation.

- Wilsonian observers. Anticipating $z \approx 3$ in UV, redefine $\tilde{t} = t$, $\tilde{y}^i$ by setting $\zeta_3^2 = 1$; equivalent to μ-dependent rescaling of space: Running spacetime. Now ϕ develops an anomalous dimension, $\lambda(\mu)$ depends on RG scale.

Aristotelian, Wilsonian and Lorentzian Observers

... and finally,

- Lorentzian observers. The low-energy observer anticipates Lorentz invariance, and sets $c = 1$.

This is equivalent to redefining the coordinates to $x^\mu =\equiv (x^0, x^i)$, with $x^0 = t$ and $x^i = y^i/c$.

Dimensions: Measuring in the units of energy,

$$[t] = -1, \quad [y^i] = -1/3, \quad [c] = 2/3, \quad [x^i] = -1.$$

Technical Naturalness: Imposed in the UV, "microscopic" theory. Cascading hierarchy of scales is possible, protected by the pattern of partial breakings of polynomial shift symmetries.

The Low-Energy Lorentzian Perspective

Compare the perspective of the UV Aristotelian observer:

$$S_{\mathrm{UV}} = \frac{1}{2}\int dt\, d^3\mathbf{y}\,\Big\{\dot\phi^2 - (\partial_i\partial_j\partial_k\phi)^2 - \zeta_2^2(\partial_i\partial_j\phi)^2 - c^2(\partial_i\phi)^2 + m^2\phi^2 - g\phi^4 - \lambda\epsilon_{ijk}\epsilon_{\ell mp}\partial_i\phi\partial_j\partial_\ell\phi\partial_k\partial_m\phi\partial_p\phi\Big\}$$

$$\zeta_2^2 \sim \varepsilon_2\mu^2, \quad \lambda\sim\varepsilon_1, \quad c^2\sim\varepsilon_1\mu^4, \quad m^2\sim\varepsilon_0\mu^6, \quad g\sim\varepsilon_0\mu^6,$$
$$\varepsilon_0 \ll \varepsilon_1 \ll \varepsilon_2 \ll 1.$$

and the IR Lorentzian observer:

$$S_{\mathrm{IR}} = \frac{1}{2}\int d^4x\,\Big\{\nabla_\mu\Phi\nabla^\mu\Phi + m^2\Phi^2 - \lambda_h\Phi^4 - \tilde\zeta_3^2(\nabla_i\nabla_j\nabla_k\Phi)^2 - \tilde\zeta_2^2(\nabla_i\nabla_j\Phi)^2 - \tilde\lambda\nabla_i\Phi\ldots\Phi\Big\}$$

where $\Phi = c^{3/2}\phi$ and $\nabla_\mu = \partial/\partial x^\mu$.

The Low-Energy Lorentzian Perspective

Couplings in the IR picture, in terms of the UV variables:

$$m^2 = m^2, \quad \lambda_h = \frac{g}{c^3}, \quad \tilde{\zeta}_3^2 = \frac{1}{c^6}, \quad \tilde{\zeta}_2^2 = \frac{\zeta_2^2}{c^4}, \quad \tilde{\lambda} = \frac{\lambda}{c^9}.$$

Recall the cascading hierarchy from the UV perspective:

$$\zeta_2^2 \sim \varepsilon_2 \mu^2, \quad \lambda \sim \varepsilon_1, \quad c^2 \sim \varepsilon_1 \mu^4, \quad m^2 \sim \varepsilon_0 \mu^6, \quad g \sim \varepsilon_0 \mu^6,$$

Define $\mu^3 \equiv M$, translate into the IR variables:

$$m^2 \sim \varepsilon_0 M^2, \qquad \lambda_h \sim \frac{\varepsilon_0}{\varepsilon_1^{3/2}},$$

$$\tilde{\zeta}_3^2 \sim \frac{\varepsilon_0^2}{\varepsilon_1^3} \frac{1}{m^4}, \qquad \tilde{\zeta}_2^2 \sim \frac{\varepsilon_0 \varepsilon_2}{\varepsilon_1^2} \frac{1}{m^2}, \quad \tilde{\lambda} \sim \frac{\varepsilon_0^3 \varepsilon_2}{\varepsilon_1^{9/2}} \frac{1}{m^6}.$$

Technically Natural Mass Hierarchy

$$m^2 \sim \varepsilon_0 M^2, \qquad \lambda_h \sim \frac{\varepsilon_0}{\varepsilon_1^{3/2}},$$

$$\tilde{\zeta}_3^2 \sim \frac{\varepsilon_0^2}{\varepsilon_1^3}\frac{1}{m^4}, \qquad \tilde{\zeta}_2^2 \sim \frac{\varepsilon_0 \varepsilon_2}{\varepsilon_1^2}\frac{1}{m^2}, \qquad \tilde{\lambda} \sim \frac{\varepsilon_0^3 \varepsilon_2}{\varepsilon_1^{9/2}}\frac{1}{m^6}.$$

We want $m = M_{\mathrm{EW}} \sim 1\mathrm{TeV}$, $M = M_P \sim 10^{18}\mathrm{GeV}$. We also want $\lambda_h \sim 0.1$ or 1. Take the "10-20-30" model:

$$\varepsilon_0 \sim 10^{-30}, \qquad \varepsilon_1 \sim 10^{-20}, \qquad \varepsilon_2 \sim 10^{-10}.$$

The nonrelativistic corrections are small,

$$\tilde{\zeta}_2^2 \sim \frac{1}{m^2}, \quad \tilde{\zeta}_3^2 \sim \frac{1}{m^4}, \quad \tilde{\lambda} \sim 10^{-10}\frac{1}{m^6}.$$

Fermions and Yukawa Couplings

Microscopic theory vs. low-energy relativistic picture:

$$\sum_f Y_f \int d^3\mathbf{y}\, dt\, \phi \Psi_f^\dagger \Psi_f = \sum_f y_f \int d^4x\, \Phi \bar{\psi}_f \psi_f,$$

where $[Y_f] = 1$, $\psi_f = c^{3/2}\Psi_f$ and $y_f = Y_f/c^{3/2}$.

Microscopic naturalness: $Y_f \sim \varepsilon_0 \mu^3$? Actually, there is more wiggling room:

$$\varepsilon_0 \mu^3 \leq Y_f \leq \sqrt{\varepsilon_0}\mu^3.$$

The low-energy observer sees this window of naturalness as

$$\varepsilon_0/\varepsilon_1^{3/4} \leq y_f \leq \varepsilon_0^{1/2}/\varepsilon_1^{3/4}.$$

In the 10-20-30 model, this gives the Yukawa range $10^{-15} \leq y_f \leq 1$, accommodating all the fermions of the SM!

Gauging

The grain of salt?

So far, we found the technically natural light scalar with non-derivative self-coupling, but only in the "gaugeless" limit of the SM.

Gauging, in the microscopic theory:

$\partial_i \to \partial_i + ie\mathcal{A}_i, \quad \partial_t \to \partial_t + ie\mathcal{A}_0$; go to $\mathcal{A}_0 = 0$ gauge.

Action: $\int d^3\mathbf{y}\, dt\, \dot{\mathcal{A}}_i \dot{\mathcal{A}}_i + \ldots$; implies $[\mathcal{A}_i] = 0$, $[e] = 1/3$.
Low-energy relativistic perspective:

$$A_i = c^{3/2}\mathcal{A}_i, \qquad g_{\mathrm{YM}} = e/c^{1/2}.$$

If $e^2 \sim \varepsilon_0 \mu^2$, then $g_{\mathrm{YM}}^2 \sim \varepsilon_0/\varepsilon_1^{1/2}$ is still way too small.

But: Bottom-up pheno approach followed by Berthier, Grosvenor, Yan; encouraging natural hierarchy by 2 OofM.

Conclusions I: Naturally Light Scalars

- Technical naturalness exhibits surprising features in the nonrelativistic settings of Aristotelian spacetime.
- We presented a new mechanism for a naturally light scalar with non-derivative self-couping:

$$m^2 \sim \varepsilon M^2, \quad \lambda_h \sim \varepsilon/\varepsilon_1^{3/2},$$

  in contrast with the relativistic naturalness:

$$m^2 \sim \varepsilon M^2, \qquad \lambda_h \sim \varepsilon.$$

- The crucial new small parameter ε_1 controls the size of the speed of light in the microscopic theory with $z > 1$.
- Higgs phenomenology looks quite promising, a large hierarchy with $m = M_{EW}$ and $M = M_P$ is possible at least in the "gaugeless" limit of SM. All fermion masses also natural! Need to learn more about the gauge sector.

Can we do this with the Cosmological Constant?

Consider the Multicritical Universe scenario.

$$S_{\rm UV} \;=\; \frac{1}{\kappa^2}\int dt\, d^3\mathbf{y}\sqrt{g}\,\{\dot{g}^2 - \zeta_3^2(\nabla R)^2 + \ldots - \zeta_2^2 R^2 + \ldots \\ - c^2 R - H^2\} + S_{\rm SM\ matter}$$

What is the natural scale of gravity? Planck scale? The Hubble scale!

Canonical dimensions around the $z = 3$ UV fixed point:

$$[\kappa^2] = 0,\ [\zeta_3^2] = 0,\ [\zeta_2^2] = 2/3,\ [c^2] = 4/3,\ [H^2] = 2.$$

Define again the IR coordinates x^i: $y^i = cx^i$, get:

$$S_{\rm IR} = \frac{c^3}{\kappa^2}\int dt\, d^3\mathbf{x}\sqrt{g}\,\{\dot{g}^2 - \xi_3^2(\nabla R)^2 - \xi_2^2 R^2 - R - H^2\}\,.$$

Can we do this with the Cosmological Constant?

Thus, we get the low-energy Newton constant as a derived object:

$$G_N = \frac{\kappa^2}{c^3}.$$

Make a natural dynamical assumption: In the absence of fancy symmetries, all ζ's are set by the same scale m:

$$\zeta_3^2 \sim 1,\ \zeta_2^2 \sim m^{2/3},\ c^2 \sim m^{4/3},\ \zeta_0^2 \sim m^2 = H^2$$

Then our expression for Newton's constant can be written as

$$\kappa^2 = G_N H^2.$$

Fit to Nature:

$$M_P \sim 10^{18}\mathrm{GeV},\ m \sim 10^{-42}\mathrm{GeV},\ c \sim m^{2/3},\ \kappa \sim 10^{-60},$$

and small $\Lambda \sim 10^{-120} M_P^2$ is technically natural!

Not so fast ... there are other constraints:

Higher-derivative corrections:

$$\xi_2^2 \sim \frac{\zeta_2^2}{c^4} \sim \frac{1}{m^2} \sim \frac{1}{H^2},$$

$$\xi_3^2 \sim \frac{1}{c^6} \sim \frac{1}{m^4} \sim \frac{1}{H^4},$$

which would lead to modifications to Newton's law at the scale of H!

Can we suppress such corrections?

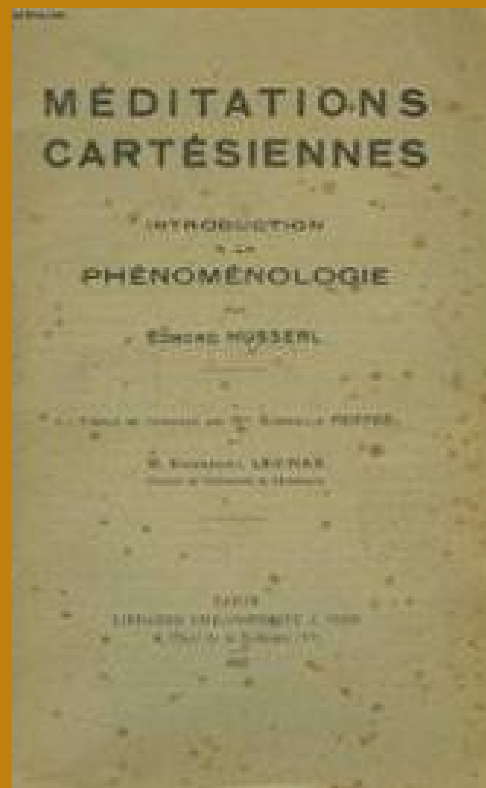
Using weak coupling $\kappa \ll 1$, yes for ζ_2 but not for ζ_3.

In order to satisfy observational constraints, we must lower m to the scale of current probes of Newton's law.

This still buys us about a half of the 120 orders of magnitude!

The remaining 60 OofM still fine tuning, an additional mechanism needed ...

Why Phenomenology?



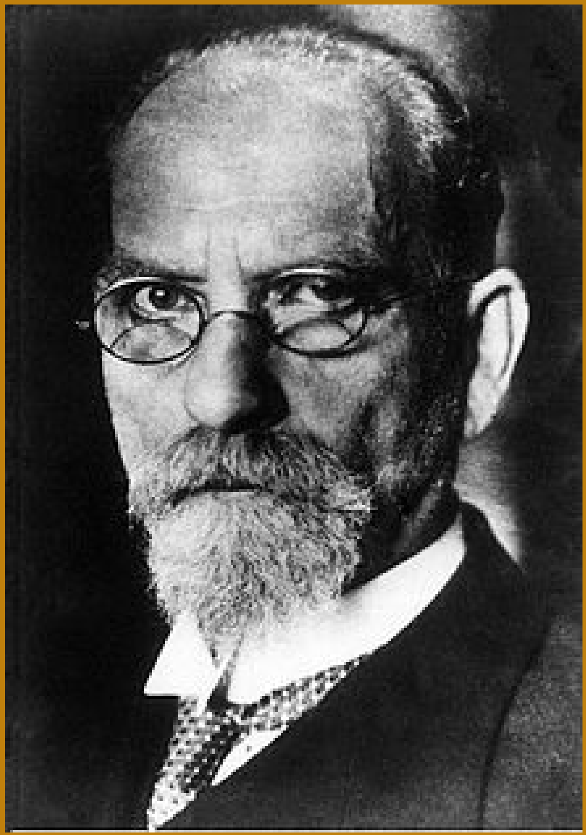
Cartesian Meditations: An Introduction to Phenomenology
(Paris lectures 1929, published 1931)

The Founding Father of Phenomenology

Edmund Husserl (April 8, 1859, in Prostějov, Moravia – 27 April 1938)

Prostějov, Moravia, Czech Republic

Introduction to Phenomenology



Edmund Husserl (April 8, 1859, in Prostějov, Moravia – 27 April 1938)

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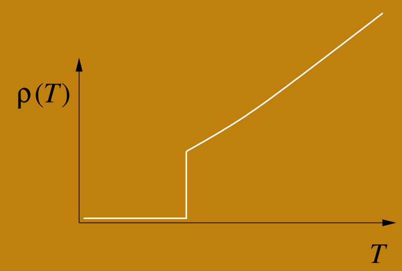
Back to Naturalness in Physics

In high-T_c superconductors, the phase above T_c exhibits "unnatural" properties:

Resistivity $\rho(T) \sim T$ over several orders of magnitude in temperature:



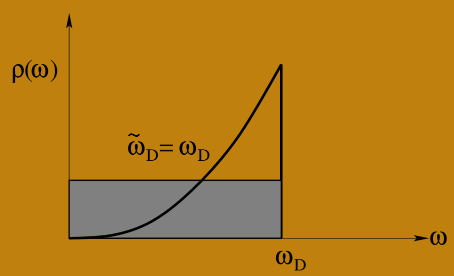
Use EFT: What gives $\rho \sim T$? Nothing!

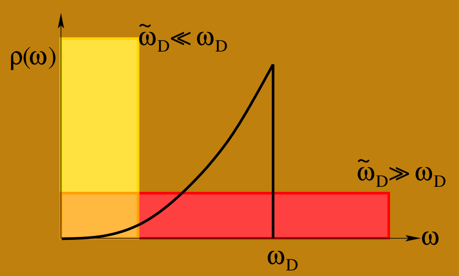
- Electron-phonon interactions (the main mechanism for pairing in BCS): $\rho \sim T^5$
- Electron-electron interactions: $\rho \sim T^2$
- Electron-impurities: $\rho \sim \mathrm{const}$

A Simple Model of Strange Metals

Phonons are NG modes of SSB of space translations.

Consider Debye model with multicritical phonons: $\omega = \zeta |\mathbf{k}|^z$.
Phonon spectrum cut off at the Debye frequency $\tilde{\omega}_{\mathrm{D}}$.
Lower critical dimension: $D = z$. Density of states:





Couple to the Fermi surface, minimally:

$$g \int dt\, d^D\mathbf{x}\, Q \Psi^\dagger \Psi \equiv g \int dt\, d^D\mathbf{x}\, \partial_i Q_i \Psi^\dagger \Psi.$$

This coupling breaks polynomial shift, generates relevant deformations, and a natural pairing mechanism.

Resistivity in Strange Metals

Transport: Use Bloch-Boltzmann theory. In metals, this gives the Bloch-Grüneisen formula (with $\tau(\varepsilon)$ the relaxation time):

$$\rho \sim \frac{1}{\tau(\varepsilon_{\mathrm{F}})} \sim \int_0^{\varepsilon_{\mathrm{F}}/T} |g_k|^2 n(k) k^2 \, k dk$$

with $n(k) = \dfrac{1}{\exp(\omega_k/T) - 1}$ the phonon distribution function,

and $g_k = g\dfrac{k}{\sqrt{\omega_k}}$ the electron-phonon vertex.

$3+1$ dimensions, $z=1$ phonons: $\rho \sim T^5$.

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$D+1$ dimensions, general z phonons: $\rho \sim T^{(3+D-z)/z}$.