# Massive Gravity with a Higher Strong Scale

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# Massive gauge and gravitational fields:

- Mass due to "matter" in a lab

  Plasma, superconductors, charged condensates,..

- Mass due to a relativistic Higgs vacuum

  Our universe permiated by the Higgs VEV

  Degrees of freedom of the massive gauge boson/graviton: often there are more degrees of freedom, beyond the longitudinal modes of massive gauge bosons, e.g., the ion acoustic wave in plasma, Higgs boson in a relativistic systems

  For a relativistic theory the Higgs boson needed for a weakly coupled UV completion

## Minimal number of degrees of freedom:

- $SU(2)$ massive gauge fields (Vainshtein, Khriplovich '71):

$$\frac{\partial\pi^a\partial\pi^a}{(1+\pi^a\pi^a/v^2)^2}$$

  the strong interaction scale is $v = m/g$

- In pure massive gravity (de Rham, GG, Tolley), the strong scale similar to DGP as derived in (Luty, Porrati, Rattazzi)

$$\Lambda_3 = (M_{\rm P}m^2)^{1/3}$$

- Vainshtein mechanism (Vainshtein 72; Deffayet, Dvali, Gabadadze, Vainshtein)

## Massive graviton plus new degrees of freedom (this talk)

$$\Lambda_* \sim (M_{\rm P} m \bar{H})^{1/3} >> \Lambda_3$$

## Plan:

- Massive GR: metric formulation, symmetries and their breaking
- Decoupling limit and the strong scale
- Cosmological solutions and need for further extension
- Extensions: quasidilaton, possible SL(4) symmetry
- Raising the strong scale: Embedding into higher dimensions
- Warped massive gravity
- Outlook
- *Time permitting: cost to cancel the big* $\Lambda$

## GR Extended by Mass and Potential Terms

Previous no-go statements invalid: *de Rham, GG, '10*

The Lagrangian of the theory: *de Rham, GG, Tolley, '11*

Using $g_{\mu\nu}(x)$ and 4 scalars $\phi^a(x)$, $a = 0, 1, 2, 3$, define

$$\mathcal{K}^\mu_\nu(g,\phi) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}\tilde{f}_{\alpha\nu}} \qquad \tilde{f}_{\alpha\nu} \equiv \partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

The Lagrangian is written using notation $tr(\mathcal{K}) \equiv [\mathcal{K}]$:

$$\mathcal{L} = M_{\rm P}^2\sqrt{g}\left(R + m^2\left(\mathcal{U}_2 + \alpha_3\,\mathcal{U}_3 + \alpha_4\,\mathcal{U}_4\right)\right)$$

$$\mathcal{U}_2 = [\mathcal{K}]^2 - [\mathcal{K}^2] \sim det_2(\mathcal{K})$$

$$\mathcal{U}_3 = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3] \sim det_3(\mathcal{K})$$

$$\mathcal{U}_4 = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4] \sim det_4(\mathcal{K})$$

## Symmetries and loopholes in the no-go theorems:

Constant fields were assumed to preserve Poinceré symmetry.
This might be too restrictive: coordinate dependent background fields. Naively, this would break Poincareé invariance, however, one could think of cases when there is still remaining $ISO(3,1)$:
For example, a symmetry breaking pattern

$$ISO(3,1)_1 \times ISO(3,1)_2 \to ISO(3,1)_{\mathrm{Observ}}$$

The background fields:

$$g_{\mu\nu} = \eta_{\mu\nu}, \qquad \partial_\mu \phi^a = \delta^a_\mu$$

Another (similar) example with Galileon symmetry:

$$Gal_{\mathrm{int}} \to ISO(3,1)_{\mathrm{Observ}}$$

Galileons have coordinate dependent backgrounds.

## General considerations (Arkani-Hamed, Georgi, Schwartz)

The longitudinal mode gets kinetic term via mixing with a tensor:

$$\mathcal{L}_2 = (\partial h)^2 + m^2 h\partial\partial\pi + hT$$

Mass scale is irrelevant in the leading approximation:

$$\mathcal{L}_2 = (\partial \hat{h})^2 - m^4(\partial\pi)^2 + \hat{h}T + m^2\pi T$$

and rescale, $\pi \to \pi/m^2$.

$$\Lambda_5 = (M_{\mathrm{P}} m^4)^{1/5} \quad \textit{versus} \quad m^4\pi(\partial\partial\pi)^2 \to \frac{\pi(\partial\partial\pi)^2}{M_{\mathrm{P}} m^2}\,.$$

## Poincaré+Galileons: *de Rham, GG*

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{2} h^{\mu\nu} \hat{\mathcal{E}}^{\alpha\beta}_{\mu\nu} h_{\alpha\beta} + h^{\mu\nu} \left( a X^{(1)}_{\mu\nu} + \frac{\alpha}{\Lambda_3^3} X^{(2)}_{\mu\nu} + \frac{\beta}{\Lambda_3^6} X^{(3)}_{\mu\nu} \right) \\
X^{(1)}_{\mu\nu} &= \epsilon_{\mu\alpha}\epsilon_{\nu\beta}\Pi^{\alpha\beta} \\
X^{(2)}_{\mu\nu} &= \epsilon_{\mu\alpha\rho}\epsilon_{\nu\beta\sigma}\Pi^{\alpha\beta}\Pi^{\rho\sigma} \\
X^{(3)}_{\mu\nu} &= \epsilon_{\mu\alpha\rho\gamma}\epsilon_{\nu\beta\sigma\delta}\Pi^{\alpha\beta}\Pi^{\rho\sigma}\Pi^{\gamma\delta} \\
\Pi_{\mu\nu} &= \partial_\mu\partial_\nu\pi, \qquad \text{background}\pi \sim x_\mu^2
\end{aligned}
$$

$Gal_{int} \to ISO(3,1)_{\text{Observ}}$

Diagonalization leads to the galileon theory, with a strog scale $\Lambda_3$

Cosmology of pure massive gravity. No flat FRW solution: *D'Amico, de Rham, Dubovsky, GG, Pirtskhalava, Tolley, '11*
Exception: Open FRW selfaccelerated universe, *Gumrukcuoglu, Lin, Mykohyama 11*; (in)stability
Pseudo-homogeneous selfaccelerated solutions: In the dec limit: *de Rham, GG, Heisenberg, Pirtskhalava*. Exact solution: *Koyama, Niz, Tasinato (1,2,3), M. Volkov; L. Berezhiani, et al; ...*
For instance, *Koyama,Niz,Tasinato*:

$$ds^2 = -d\tau^2 + e^{m\tau}(d\rho^2 + \rho^2 d\Omega^2)$$

while, $\phi^0$ and $\phi^\rho$, are inhomogeneous functions. Selfacceleration is a generic feature of this theory, however, vanishing of the kinetic terms for some of the 5 modes is also a common feature of these solutions – too bad! Anisotropic solutions and fluctuations: Gumrukcuoglu, Lin, Mukohyama, '12.

More complex solutions are OK (Mukohyama et al.), or else extensions beyond pure massive gravity are needed for cosmology.

# Brief summary:

Linearized theory: 3 NG Bosons eaten up by the tensor field that becomes massive. The theory guarantees unitary 5 degrees of freedom on (nearly) Minkowski backgrounds.

Nonlinear interactions are such that there are 5 degrees of freedom on any background. However, there is no guarantee that some of these 5 degrees of freedom aren't bad on certain backgrounds for certain values of the two free parameters (instabilities, superluminalities).

The theory is not finished – further completion needed. No IR obstruction to UV completion for certain values of the parameters of the theory – C. Cheung, G. Remmen (see also de Rham, Melville, Tolley).

## Theory of Quasi-Dilaton: *D'Amico, GG, Hui, Pirtskhalava, '12*

Invariance of the action to rescaling of the reference frame coordinates $\phi^a$ w.r.t. the physical space coordinates, $x^a$, requires a field $\sigma$. In the Einstein frame:

$$\phi^a \to e^{\alpha}\ \phi^a, \qquad \sigma \to \sigma - \alpha M_{\rm Pl}$$

Hence we can construct the invariant action by replacing $\mathcal{K}$ by $\bar{K}$

$$\bar{K}^{\mu}_{\nu} = \delta^{\mu}_{\nu} - \sqrt{g^{\mu\alpha}\bar{f}_{\alpha\nu}} \qquad\qquad \bar{f}_{\alpha_\nu} = e^{2\sigma/M_{\rm Pl}}\partial_\alpha\phi^a\partial_\nu\phi^b\eta_{ab}$$

and adding the sigma kinetic term

$$\mathcal{L} = \mathcal{L}_{dRGT}\left(\mathcal{K} \to \bar{K}\right) - \omega\sqrt{g}(\partial\sigma)^2 + \text{Galileons of } \sigma$$

and the term $\int d^4x\sqrt{-det\bar{f}}$ can also be added. In the Einstein frame $\sigma$ does not couple to matter, but it does in the Jordan frame

Extensions of massive gravity (subjective and incomplete list):

Extended Quasidilaton: De Felice, Mukohyama, '13; Mukohyama, '13; De Felice, Gümrükcüoglu, Mukohyama, '13, Mukohyama, 14; GG, Kimura, Pirtskhalava, '14,'15

Bigravity: Hassan, R.A. Rosen, '11, ... . Cosmology e.g., De Felice, Gümrükc|uoglu, Mukohyama, Tanahashi, Tanaka, 14, ....

Extended and Generalized Massive Gravities: GG, Hinterbichler, Khoury, Pirtskhalava, Trodden, 13; Gümrükcüoglu, Hinterbichler, Lin, Mukohyama, Trodden 13; de Rham, Keltner, Tolley, 14, ...

The vierbein formulation: Hinterbichler and R. A. Rosen '12 Fully Diffeomorphism and Local Lorentz Invariant vierbein formulation: GG, Hinterbichler, Pirtskhalava, Y. Shang, '13

$$\mathcal{L}_\Lambda \sim M_{\rm P}^2 \Lambda \, \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd} \, e_\mu{}^a e_\nu{}^b e_\alpha{}^c e_\beta{}^d$$

The mass and potentials

$$\mathcal{L}_2 \sim M_{\rm P}^2 m^2 \, \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd} \, e_\mu{}^a e_\nu{}^b k_\alpha{}^c k_\beta{}^d$$
$$\mathcal{L}_3 \sim \alpha_3 M_{\rm P}^2 m^2 \, \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd} \, e_\mu{}^a k_\nu{}^b k_\alpha{}^c k_\beta{}^d$$
$$\mathcal{L}_4 \sim \alpha_4 M_{\rm P}^2 m^2 \, \epsilon^{\mu\nu\alpha\beta}\epsilon_{abcd} \, k_\mu{}^a k_\nu{}^b k_\alpha{}^c k_\beta{}^d$$

where $k_\mu{}^a \equiv e_\mu{}^a - \lambda^a{}_{\bar a}\partial_\mu \phi^{\bar a}$, and $\lambda^a_{\bar a}$ transforms w.r.t. $SO(3,1)$'s.

Hamiltonian construction: *Hassan, R. A. Rosen, '11,'12; Deffayet, Mourad, Zahariade, '12* Other proofs: *Mirbabayi, '12; Hinterbichler, R.A. Rosen, '12; Golovnev, 12; Kugo, Ohta, '13*

The mass terms can be promoted to the locally $SL(4)$ symmetric structures by promoting $\lambda$'s to $SL(4)$! Hence the mass terms can have a larger local symmetry group than the EH term does.

## On curved backgrounds, e.g., AdS:

*(Porrati; Kogan, Mouslopoulos, Papazoglou)*

On curved backgrounds, e.g., on $AdS_4$, with $-\Lambda < 0$, one obtains, $-\Lambda m^2(\partial\pi)^2$. This would raise the strong scale as long as the magnitude of the cosmological constant is large, $\Lambda >> m^2$.

A way to use the above while being in Minkowski, GG

4D massive gravity is embedded into a $D$-dimensional ($D = 4 + n > 4$) massive gravity with a large curvature scale, $\bar{\Lambda}$; $D$-dimensional kinetic term for the $D$-dimensional longitudinal mode, $\Pi(x^\mu, z^1, z^2, ..., z^n)$,

$$-M_D^{2+n}\bar{m}^2\bar{\Lambda}\left(\partial_D\Pi(x^\mu, z^1, z^2, ..., z^n)\right)^2$$

where $M_D$ and $\bar{m}$ are the higher-dimensional Planck and graviton mass respectively.

## The action and coupling to the matter:

$$\tilde{\mathcal{L}}_2 = (\partial\hat{h})^2 - m^4(\partial\pi)^2 - M_D^{2+n}L^n\bar{m}^2\bar{\Lambda}(\partial\pi)^2 + \hat{h}T + m^2\pi T$$

As long as, $M_D^{2+n}L^n\bar{m}^2\bar{\Lambda} >> m^4$, (in $M_{\rm P} = 1$ units), all OK

However, the general argument must be incomplete

below the scale of new physics still the old theory; it has to be that

$$m_{KK} < \Lambda_3$$

# Example: warped massive gravity, GG

4D massive gravity embedded in 5D AdS massive gravity:
The 5D massive action just a generalization of the 4D action

$$S_5 = M_5^3 \int d^4x\, dz\, \sqrt{-\bar{g}} \left( \bar{R}(\bar{g}) + 2\bar{\Lambda} + 2\bar{m}^2 \mathcal{V}(\bar{\mathcal{K}}^M_N) \right)$$

where

$$\mathcal{V}(\bar{\mathcal{K}}) = \det_2(\bar{\mathcal{K}}) + \beta_3 \det_3(\bar{\mathcal{K}}) + \beta_4 \det_4(\bar{\mathcal{K}}) + \beta_5 \det_5(\bar{\mathcal{K}})$$

with the definition

$$\bar{\mathcal{K}}^A{}_B = \delta^A_B - \sqrt{\bar{g}^{AM} \bar{f}_{MB}}, \qquad \bar{f}_{MN} = \partial_M \Phi^I \partial_N \Phi^J \tilde{f}_{IJ}(\Phi)$$

and $\Phi^J(x^\mu, z)$, $(I, J = 0, 1, 2, 3, 5)$, five scalar Stückelberg fields.
*(F. Hassan, R. A. Rosen, arbitrary fiducial metric, bigravity)*

The total action:

$$S_{total} = S_5 + S_4 + S_{Boundary}\ ,$$

with $S_5$ and $S_4$ defined similarly in 5D and 4D respectively; $S_{Boundary}$ is the Gibbons-Hawking plus additional boundary terms (see below).

Bulk boundary connection

$$\bar{g}_{\mu\nu}(x,z)|_{z=0} = g_{\mu\nu}(x)$$

$$\delta^a_J \Phi^J(x,z)|_{z=0} = \varphi^a(x)$$

$$\delta^I_a \delta^J_b \tilde{f}_{IJ}(\Phi)|_{\Phi^z=0} = \eta_{ab}$$

# Classical solutions:

The fiducial metric is assumed to be AdS (justified a posteriori)

$$ds^2_{Fid} = \tilde{f}_{IJ} d\Phi^I d\Phi^J = \frac{L^2}{(\Phi^z + L)^2} \left[\eta_{ab} d\Phi^a d\Phi^b + (d\Phi^z)^2\right]$$

Then, the physical metric has a solution ($z > 0$, $\Phi^z > 0$)

$$ds^2 = \bar{g}_{AB} dx^A dx^B = A^2(z) \left[\eta_{\mu\nu} dx^\mu dx^\nu + dz^2\right], \quad A(z) \equiv \frac{L}{z + L}$$

This could be obtained in bigravity, with a weak coupling of the massless graviton

$$\tilde{M}_5^3 \int d^5\Phi \sqrt{\tilde{f}(\Phi)} \left(R(\tilde{f}(\Phi)) + 2\bar{\Lambda}\right)$$

# Linearized theory – the action:

$$\mathcal{L}_{5D} = M_5^3 \sqrt{\bar{g}^{AdS}} \left( -\bar{h}_{AB} \bar{\mathcal{E}}^{ACBD} \bar{h}_{CD} - \frac{\bar{m}^2}{2} (\tilde{h}_{AB}^2 - \tilde{h}^2) \right)$$

$\bar{\mathcal{E}}^{ACBD}$ is the Einstein operator on $AdS_5$; the Stückelberg fields, $\Phi^J \delta_J^A = x^A + \frac{1}{\bar{m}} V^A$, enter the Lagrangian via

$$\tilde{h}_{AB} \equiv \bar{h}_{AB} - \frac{1}{\bar{m}} (\nabla_A V_B + \nabla_B V_A) ,$$

The quadratic part of the 4D Lagrangian reads as follows:

$$\mathcal{L}_{4D} = M_4^2 \left( -h_{\mu\nu} \mathcal{E}^{\mu\alpha\nu\beta} h_{\alpha\beta} - \frac{m^2}{2} ({h'}_{\mu\nu}^2 - {h'}^2) \right) + h_{\mu\nu} T^{\mu\nu}$$

$\mathcal{E}^{\mu\alpha\nu\beta}$ is the Einstein operator for 4D Minkowski space, $h'$ is

$${h'}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{\bar{m}} (\partial_\mu v_\nu + \partial_\nu v_\mu)$$

all contractions by $\eta^{\mu\nu}$.

# Additional boundary terms (GG; GG, Pirtskhalava,..)

There are total derivative terms for Stückelbergs in massive gravity. These total derivatives induce nonzero surface terms when a boundary is present.

The latter would ruin the theory since they lead to more than two time derivatives acting on a field on the boundary.

One needs to introduced a new boundary term in the action, proportional to

$$\int d^4x \, (V^A \nabla_A V_z - V_z \nabla^C V_C)|$$

to cancel the surface term generated from the bulk.
With this boundary term the theory for the vector Stückelberg is just the bulk Maxwell theory! Similar approach at nonlinear level.

This very boundary term removes the bad induced surface terms for the field $\Pi$; as a result, the quadratic action for $\Pi$, in the decoupling limit will contain only its first derivatives.

# Linearized theory – the spectrum:

The linearized theory is continuous in the following massless limit:

$$m \to 0, \quad \bar{m} \to 0, \quad m/\bar{m} \to 0, \quad \bar{\Lambda} = \mathit{fixed}$$

In the above limit the spectrum consists of:
RS zero mode, KK gravitons, KK vectors and scalars

Away from the limit: the RS zero mode disappears, a resonance in the KK tower (similar to a scalar, Dubovsky, Rubakov, Tinyakov)

The strong coupling is due to the longitudinal mode of the resonance graviton; the latter gets a large kinetic term due to the background

$$-M_5^3\bar{\Lambda}(\partial\Pi)^2, \quad \Pi = \frac{\Pi^c}{\sqrt{M_5^3\bar{\Lambda}}} = \frac{\Pi^c}{M_5^{3/2}\bar{H}}$$

## Nonlinear interactions:

Bulk generic (C. de Rham, GG, 10)

$$M_5^3 \bar{m}^2 \bar{h} \left( \left( \frac{\nabla\nabla\Pi}{\bar{m}} \right)^2 + \left( \frac{\nabla\nabla\Pi}{\bar{m}} \right)^3 + \left( \frac{\nabla\nabla\Pi}{\bar{m}} \right)^4 \right)$$

Bulk special (related to total derivatives)

$$M_5^3 \bar{m}^2 \bar{\Lambda} \left( \left( \frac{\nabla\Pi}{\bar{m}} \right) \left( \frac{\nabla\Pi}{\bar{m}} \right) \left( \frac{\nabla\nabla\Pi}{\bar{m}} \right) \ldots + \left( \frac{\nabla\Pi}{\bar{m}} \right) \left( \frac{\nabla\Pi}{\bar{m}} \right) \left( \frac{\nabla\nabla\Pi}{\bar{m}} \right)^3 \right)$$

The $\Pi$ field needs to be canonically normalized

As a result, the 5D strong scale is

$$\Lambda_{5D} \simeq (M_5^{3/2}\bar{m}\bar{H})^{2/7} = \Lambda_{7/2}\left(\frac{\bar{H}}{\bar{m}}\right)^{2/7} >> \Lambda_{7/2}$$

How about the 4D strong scale?

Effective kinetic term of the longitudinal mode, captures all KK's

$$-\frac{M_5^3\bar{\Lambda}}{2}\,\pi(x)\,\sqrt{-\Box_4}\,\frac{K_1(L\sqrt{-\Box_4})}{K_2(L\sqrt{-\Box_4})}\,\pi(x) \qquad (1)$$

In the low energy approximation $L\sqrt{-\Box_4} << 1$

$$L\,\frac{M_5^3\bar{\Lambda}}{2}\,\pi(x)\,\Box_4\,\pi(x) \qquad (2)$$

## The 4D strong scale:

$$-LM_5^3\bar{\Lambda}(\partial\pi)^2 + LM_5^3\bar{m}^2 h\left(\Sigma_{n=1}^3\left(\frac{\partial\partial\pi}{\bar{m}}\right)^n\right) + \frac{LM_5^3\bar{\Lambda}}{\bar{m}}(\partial\pi)^2(\partial\partial\pi)^2\cdots$$

$$\Lambda_* \simeq (M_5^{3/2}\bar{m}\bar{H}^{1/2})^{1/3}$$

$$\Lambda_* \sim (M_{\rm P}\bar{m}\bar{H})^{1/3} = (\Lambda_2^2\bar{H})^{1/3}$$

For: $\bar{H} \sim 10^{16}\ GeV$, $M_5 \sim 10^{18}\ GeV$, and $\bar{m} \sim m \sim 10^{-42}\ GeV$

$$\Lambda_{5D} \sim GeV, \quad \Lambda_* \sim MeV$$

is some 19 orders of magnitude greater than $\Lambda_3 \sim 10^{-19}\ MeV$.

## Outlook

- Can the strong scale be raised even further?

  Tuning between free parameters of the theory in the bulk and on the brane might lead to cancellation of some of the leading nonlinearities and give a higher scale (GG, Pirtskhalava, in progress).

  Inducing the bulk graviton mass by quantum corrections in the 5D AdS bulk (Porrati; Duff, Liu); additional states in the 5D bulk with special boundary conditions. Union of AdS spaces as dual of product of CFT's (Kiritsis; Aharony, Clark, Karch)) This could lead to a theory with the strong scale at $M_5$, which would be an ultimate goal (GG, Older, Pirtskhalava, in progress).

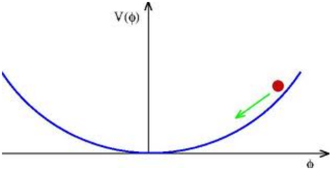
## The cost to eliminate the big CC *Tseytlin '90*

The modified action principle:

$$\bar{S} = \frac{S}{V_g} = \frac{1}{V_g}\int d^4x\sqrt{g}\left(\frac{1}{2}R + L(g,\psi_n)\right)$$

where $V_g = \int d^4x\sqrt{g}$. Any constant shift, $L \to L + \Lambda$, gives rise to a shift of the new action by the same constant, $\bar{S} \to \bar{S} + \Lambda$, that does not affect equations of motion.

Subtracts a constant from a scalar potential

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## Eliminates the "future value" of the stress tensor

The Einstein equations:

$$R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R = T_{\mu\nu} - \frac{1}{4} g_{\mu\nu} T \,, \qquad R + T = 0 \,.$$

Tseytlin's proposal for the trace equation:

$$R + T = \langle T \rangle - 2 \langle g^{\mu\nu} \frac{\partial L}{\partial g^{\mu\nu}} \rangle$$

where $< \cdots >$ denotes a certain space-time average defined as follows:

$$\langle \cdots \rangle \equiv \frac{\int d^4x \sqrt{g} (\cdots)}{\int d^4x \sqrt{g}} \equiv \frac{[\cdots]}{V_g} \,. \tag{3}$$

Local quantities are affected by global ones – non-locality

This non-locality is operative only for vacuum energy, nothing else

# Problems with the loops, Tseytlin '90

The $1/V_g$ factor gives an effective rescaling of the Planck's constant, $\hbar \to \hbar V_g$

$$\bar{S}_{Ren} = \frac{1}{V_g} \int d^4x \sqrt{g} \left( \frac{1}{2} R + L(g, \psi_n) + V_g L_1(g, \psi_n) + \mathcal{O}(V_g^2) \right)$$

where $L_1, L_2, ..$ contain all possible terms consistent with diffeomorphism and internal symmetries. This ruins the solution!

Same could be seen by defining an extended action:

$$\bar{S}_{q,\lambda} = \frac{1}{q} \int d^4x \sqrt{g} \left( \frac{1}{2} R + L \right) + \lambda (V_g - q) \,,$$

and writing down the path integral for gravity as follows

$$Z_g = \mathrm{const} \int d\mu(g)\, dq\, d\lambda \exp\left( \frac{i}{\hbar} \bar{S}_{q,\lambda} \right) \,,$$

# Dealing with the loop problems: *GG '14*

The main idea – global bigravity:

$$A = \frac{V_f}{V_g} S + \int d^D y \sqrt{f} \left( \frac{M_f^{D-2}}{2} R(y) + c_0 M^D \cdots \right)$$

where $f_{AB}(y)$ is another metric, and $V_f = \int d^D y \sqrt{f(y)}$.
The CC of our universe renormalizes CC in the other universe

$$\Delta A_{CC} = \frac{V_f}{V_g} \int d^4 x \sqrt{g} \Lambda = \int d^D y \sqrt{f} \Lambda$$

1. Our vacuum energy curves the other space-time; hence no old CC problem in our universe

2. If $V_f >> V_g$, then, $\hbar \to \hbar (V_g / V_f)$ loop effects suppressed

## Defining the path integral for quantized SM:

$$Z(g, J_n) \sim \int d\mu(\tilde{\psi}_n)\exp\left(i\int d^4x\sqrt{g}\left(\mathcal{L}(g,\tilde{\psi}_n) + J_n\tilde{\psi}_n\right)\right)$$

The metric $g$ is an external field, and so are the sources, $J_n$'s. Then, the effective Lagrangian $L(g, \psi_n)$ is defined as a Legendre transform of $W(g, J_n) = -i\ln Z(g, J_n)$

$$\int d^4x\sqrt{g}L(g,\psi_n) \equiv Re\left(W(g, J_n) - \int d^4x\sqrt{g}J_n\psi_n\right)$$

where $\sqrt{g}\psi_n \equiv -i\delta\ln Z(g, J_n)/\delta J_n$. The obtained quantum effective action is a 1PI action. Thus, all the quantum corrections due to non-gravitational interactions are already taken into account in the effective Lagrangian $L$.

Thus, we define an effective generating functional

$$Z_{\mathrm{SM}}(g,\psi_n) \equiv \exp\left(i \int d^4x \sqrt{g} L(g,\psi_n)\right)$$

that includes all the SM loops, but not quantized gravity.

In the end, $g_{\mu\nu}$ should also be quantized. Corrections are large at scales $M_{QG}$, and they should be taken care of by a putative UV completion of the theory. However, irrespective of a UV completion the quantum gravity corrections should be small at scales well below $M_{QG}$ for our approximations above to be meaningful. Hence one first needs to define the rules of calculation of the gravity loops given that the classical action has an unusual form.

We define an *extended* action:

$$\bar{A}_{q,\lambda} = \frac{1}{q}\int d^4x\sqrt{g}\left(\frac{1}{2}R + L\right) + \lambda\left(\frac{V_g}{V_f} - q\right) + S_f$$

and the path integral for gravity as follows

$$Z_g \sim \int d\mu(g)\, d\mu(f)\, dq\, d\lambda \exp\left(i\bar{A}_{q,\lambda}\right)$$

where we also integrates w.r.t. the *parameters* $q$ and $\lambda$. This can be rewritten in terms of the path integral for the SM fields $Z_{\mathrm{SM}}$:

$$Z_g \sim \int d\mu(g) d\mu(f)\, dq\, d\lambda \left(e^{iS_{\mathrm{EH}}}\, Z_{\mathrm{SM}}(g, \psi_n)\right)^{\frac{1}{q}}\; e^{i\lambda\left(\frac{V_g}{V_f} - q\right) + iS_f}$$

The SM loops done in a conventional way, gravity loops via an unconventional prescription specified above.

How do we achieve the condition $V_f/V_g >> 1$?

Assume that the $g$-universe has supersymmetry broken at some high scale, and therefore, there is a natural value of its vacuum energy density proportional to $E_{vac}^4$.

The scale $E_{vac}$ can be anywhere between a few $TeV$ and the GUT scale, $\mu_{GUT} \sim 10^{16}\ GeV$.

As to the $f$-universe, $M_f \sim M_{\mathrm{P}}$, but also we'd need the scale $M$ to be somewhat higher than $E_{vac}$.

The latter condition should be natural, since without special arrangements one would expect $M \sim M_f \sim M_{\mathrm{P}}$, and since $E_{vac} << M_{\mathrm{P}}$, one would also get $E_{vac} < M$. If so, then the vacuum energy of the $g$-universe, $E_{vac}^4$, would make a small contribution to the pre-existing vacuum energy of the $f$-universe.

The $f$-universe can be exactly supersymmetric, described, for instance, by unbroken AdS supergravity.

The new terms do not affect the trace equations, except that they just introduce a overall multiplier $V_f$. Thus, the cosmological constant is eliminated from the g-universe. There is, however, a new equation due to variation w.r.t. $f$:

$$M_f^{D-2}(R_{AB}(y) - \frac{1}{2} f_{AB} R(y)) = f_{AB}(\bar{S} + c_0 M^D) + \cdots . \qquad (4)$$

The right hand side contains a vacuum energy generated in our universe, $\bar{S} = \frac{[E_{vac}^4]}{V_g} = E_{vac}^4$, as well as that of the $f$-universe.

According to our construction, the net energy density is negative, so that the $f$-universe has an AdS curvature. If so, then $V_f = \infty$.

Still need to produce $V_f / V_g >>> 1$; use massive gravity – and its extensions with quasidilaton – instead of GR in the $g$-universe:

Could $f$ and the fiducial metric, $\tilde{f}$, be related?
GG and Siqing Yu, '15: The f-universe as $AdS_5$

$$ds^2 = f_{AB} dy^A dy^B = \frac{l^2}{z^2}\left(\eta_{ab} dy^a dy^b + dz^2\right), \quad a = 0, 1, 2, 3; A = a, 5$$

The AdS boundary coordinates $x^\mu$, $\mu = 0, 1, 2, 3$. Parametrization of the boundary located at $z = 0$, $y^a = \phi^a(x)$

$$ds^2 = \frac{l^2}{z^2}\left(\tilde{f}_{\mu\nu} dx^\mu dx^\nu + dz^2\right), \quad \tilde{f}_{\mu\nu} = \partial_\mu \phi^a \partial_\nu \phi^b \eta_{ab}$$

The fiducial metric, $\tilde{f}_{\mu\nu}$, as a non-dynamical pullback of the 5D AdS metric

$$A = \frac{V_f}{V_g} S_{mGR}(g, \tilde{f}) + S_{AdS_5}(f)$$

This removes our CC into the 5D AdS space (need a small hierarchy between 5D and 4D CC's, as before), and gives rise to dark energy via massive gravity or it extensions.

## Conclusions:

- The strong scale of massive gravity can be raised due to new light degree of freedom – an example is warped massive gravity. 4D "Higgs" mechanism via the 5D AdS "Higgs mechanism"?
- The big cosmological constant can be eliminated via a nonlocal mechanism. The cost is exceedingly high –space-time nonlocality. The proposed action is stable w.r.t. quantum gravity loop corrections.