New Approaches to the Hierarchy Problem

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CosmoGrav 2018

21 Increasingly Crazy Approaches to the Hierarchy Problem

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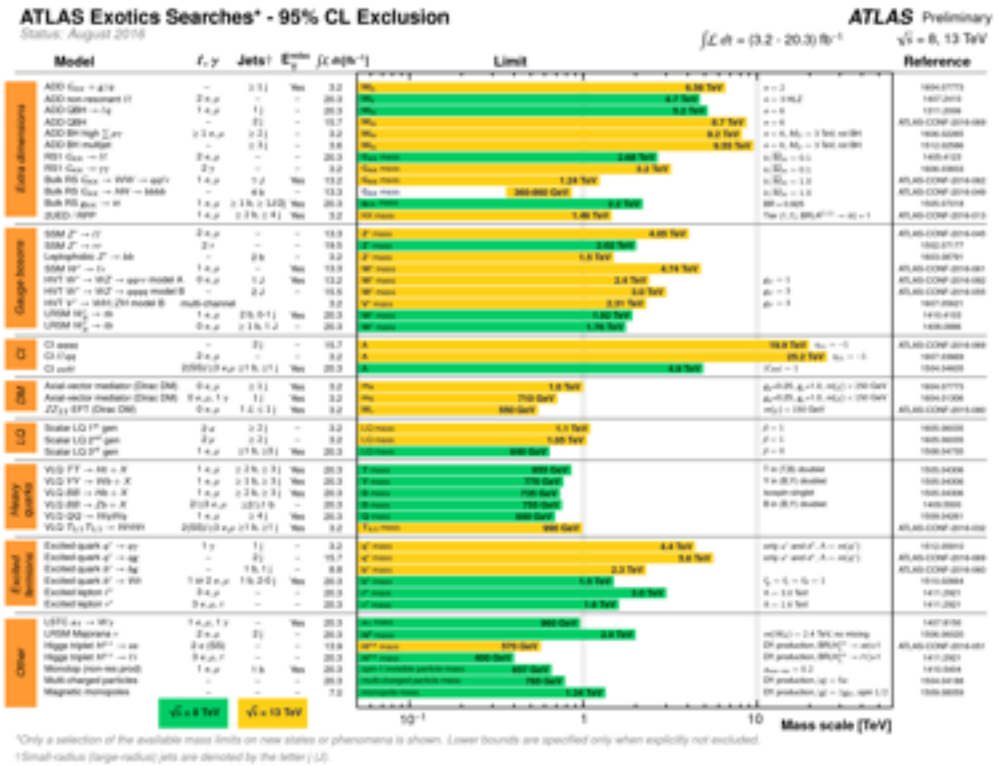
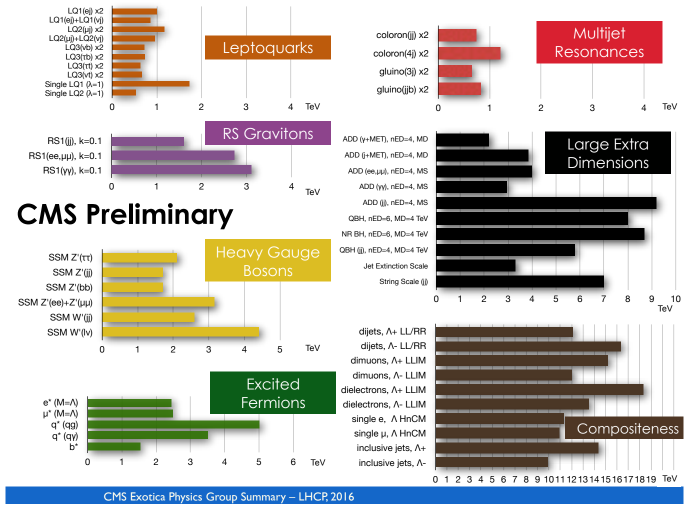
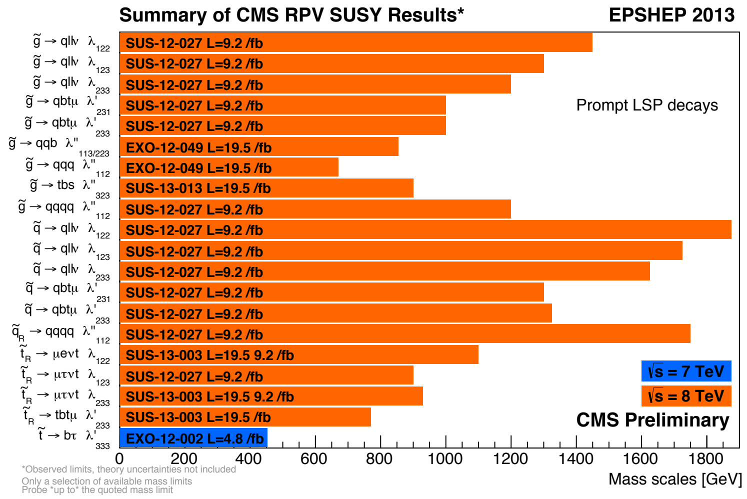
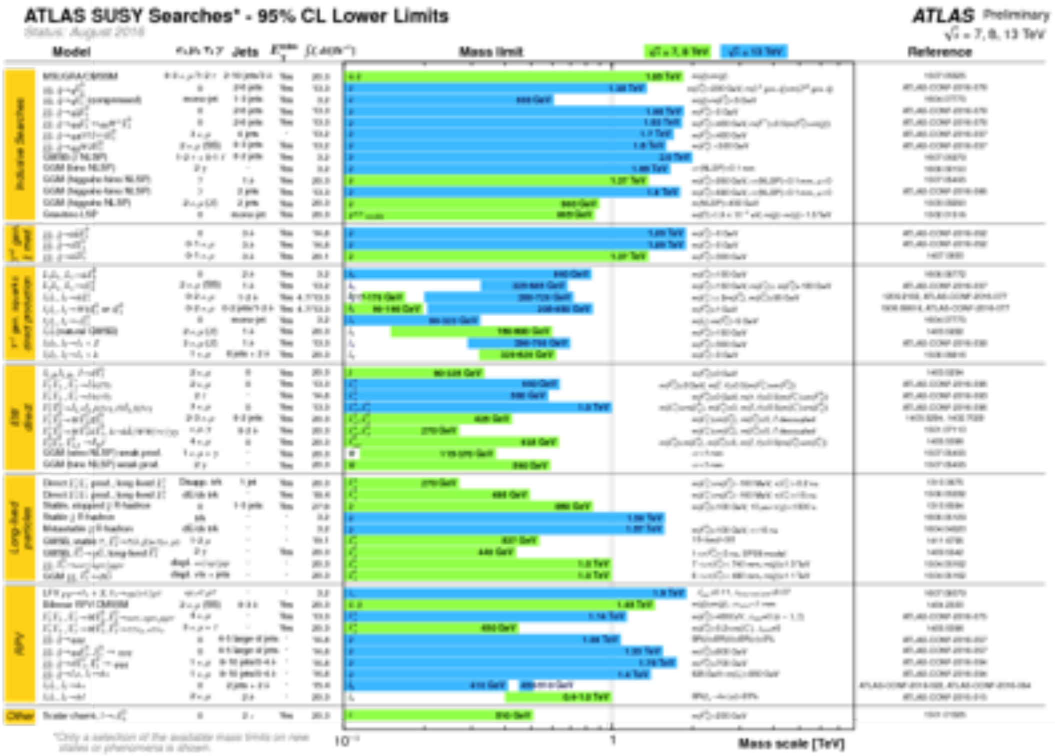
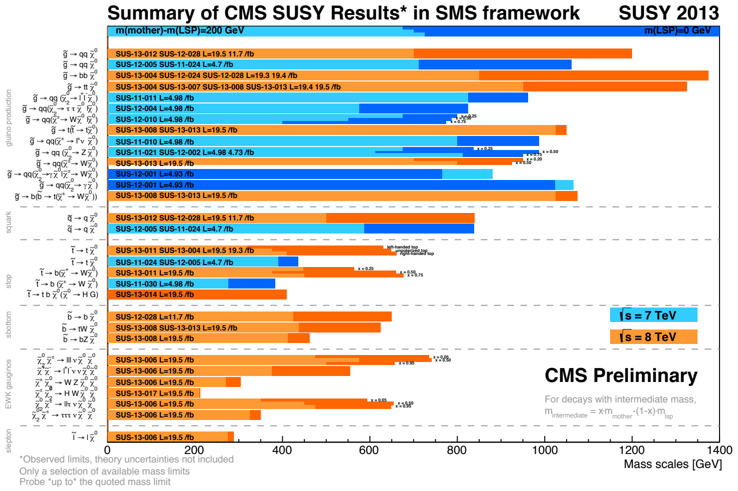


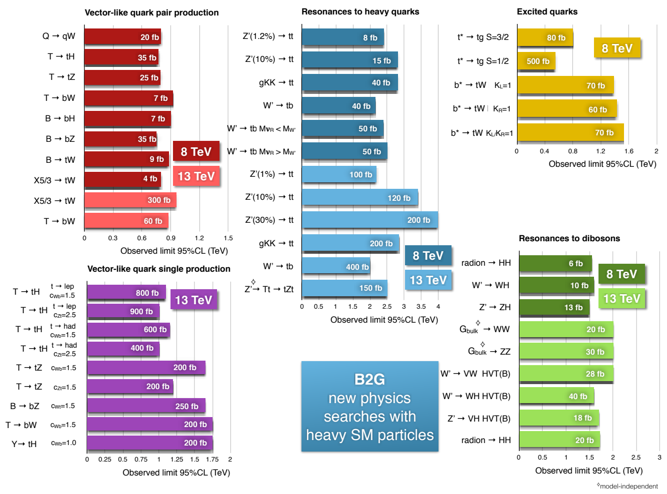
We (collectively) spend most of our time looking for solutions to the hierarchy problem









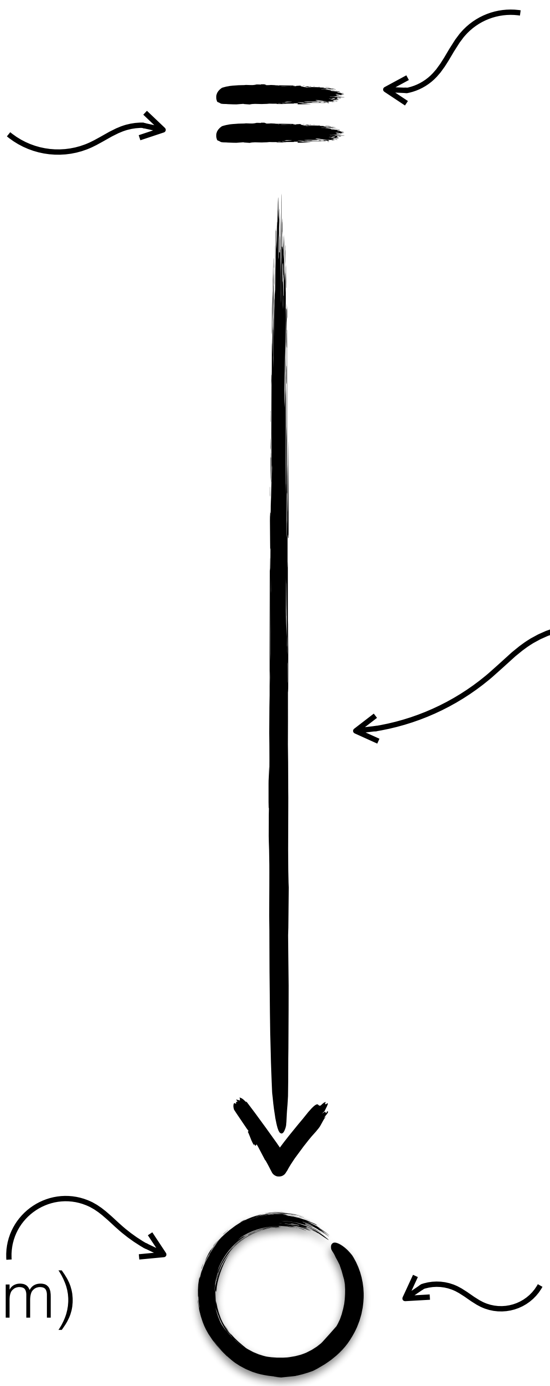


We have yet to find evidence for these solutions.

(not for lack of outstanding experimental effort)

Natural question: *have we exhausted the solutions?*

The Hierarchy Problem



Quantum gravity cutoff

Higgs sector cutoff

Uninteresting RG
flow to IR

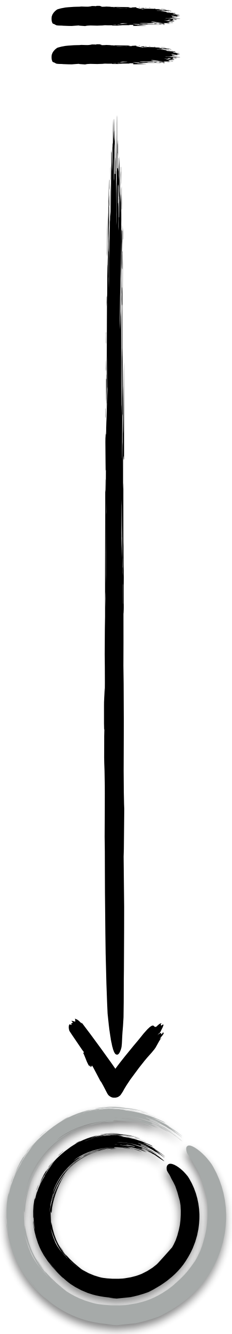
(~unique vacuum)

Standard Model

m_H is not technically natural ⇒ hierarchy problem

Adding a symmetry

…and breaking it softly

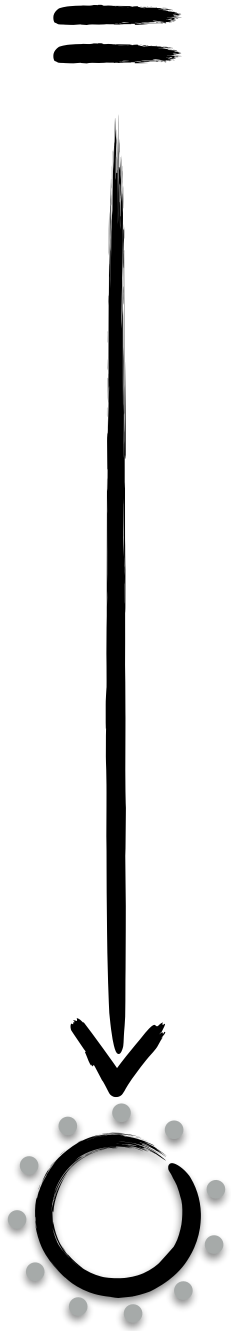


1. Supersymmetry
2. Global symmetry
3. Discrete symmetry

Experimental signals: partner particles

- The familiar host of prompt signals (with or without missing energy)
- Rich variety of displaced decays (RPV, fraternal twin higgs, folded SUSY, …)

Discrete Symmetries



Consider a scalar H transforming as a fundamental under a global SU(4):

$$V(H) = -m^2|H|^2 + \lambda|H|^4$$

Potential leads to spontaneous symmetry breaking,

$$|\langle H \rangle|^2 = \frac{m^2}{2\lambda} \equiv f^2$$

$SU(4) \rightarrow SU(3)$ yields seven goldstone bosons.

Discrete Symmetries

Now gauge $\underset{\boxed{\text{Us}}}{\underset{\uparrow}{SU(2)_A}} \times \underset{\boxed{\text{Twins}}}{\underset{\uparrow}{SU(2)_B}} \subset SU(4)$, w/ $H = \begin{pmatrix} H_A \\ H_B \end{pmatrix}$

Then 6 goldstones are eaten, leaving one behind.

Explicitly breaks the SU(4); expect radiative corrections.

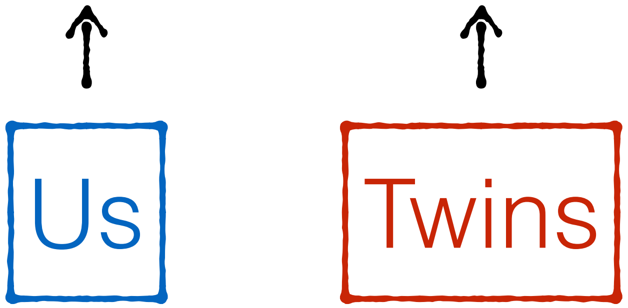
$$V(H) \supset \frac{9}{64\pi^2}\left(g_A^2\Lambda^2|H_A|^2 + g_B^2\Lambda^2|H_B|^2\right)$$

But these become SU(4) symmetric if $g_A=g_B$ from a Z_2

Quadratic potential has accidental SU(4) symmetry.

Discrete Symmetries

Now gauge $SU(2)_A$ x $SU(2)_B \subset SU(4)$, w/ $H = \begin{pmatrix} H_A \\ H_B \end{pmatrix}$



Us

Twins

Then 6 goldstones are eaten, leaving one behind.

Explicitly breaks the SU(4); expect radiative corrections.

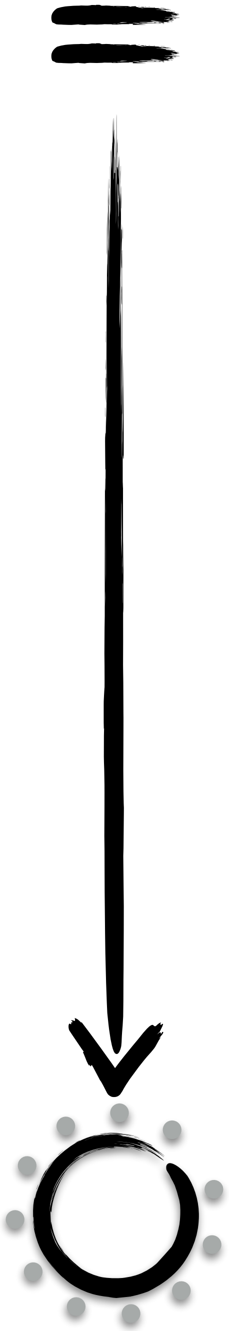
$$V(H) \supset \frac{9}{64\pi^2} g^2 \Lambda^2 \left(|H_A|^2 + |H_B|^2\right)$$

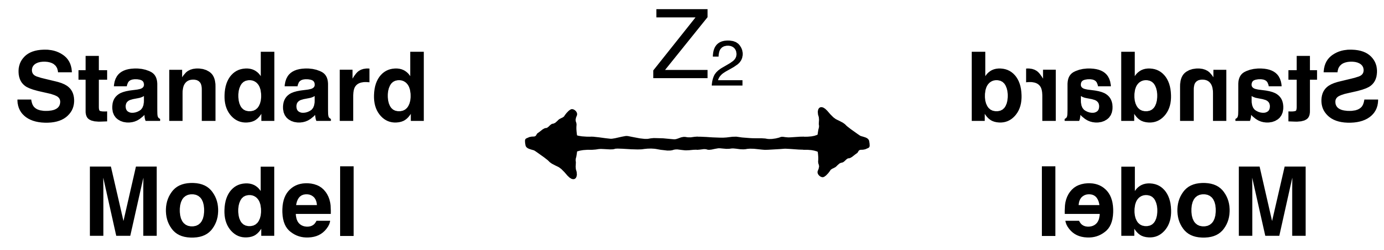
But these become SU(4) symmetric if $g_A = g_B$ from a Z_2

Quadratic potential has accidental SU(4) symmetry.

Twin Higgs

[Chacko, Goh, Harnik ‘05]



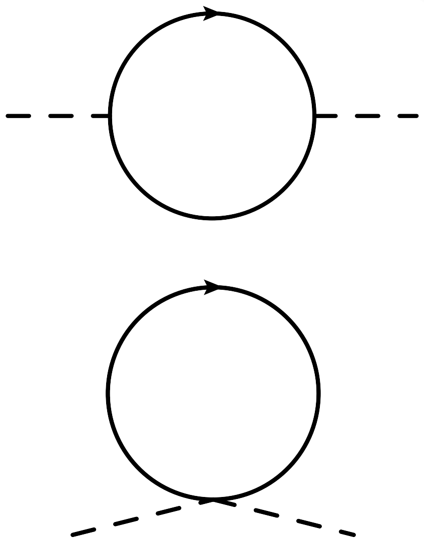


Radiative corrections to the Higgs mass are SU(4) symmetric thanks to Z₂:

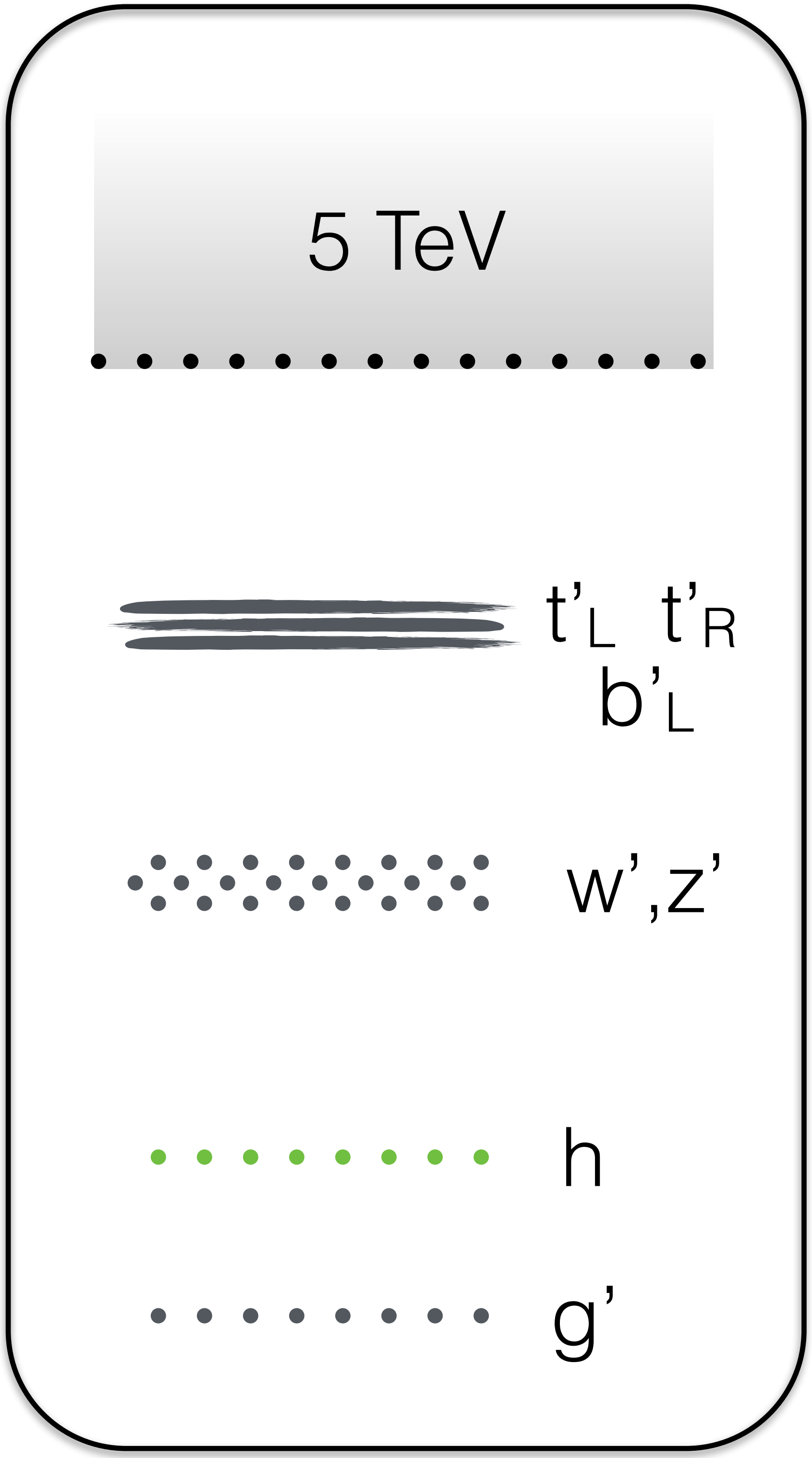
$$V(H) \supset \frac{\Lambda^2}{16\pi^2}\left(-6y_t^2 + \frac{9}{4}g^2 + \dots\right)\left(|H_A|^2 + |H_B|^2\right)$$

Higgs is a PNGB of ~SU(4), but partner states neutral under SM.

$$\mathcal{L} \supset -y_t \underset{\substack{\downarrow \\ h + \dots}}{H_A} Q_3^A \bar{u}_3^A - y_t \underset{\substack{\downarrow \\ f - \frac{h^2}{2f} + \dots}}{H_B} Q_3^B \bar{u}_3^B$$



“Neutral” naturalness



Simplest theory: exact mirror copy of SM

[Chacko, Goh, Harnik ’05]

But this is more than you need, and mirror 1st, 2nd gens lead to cosmological challenges

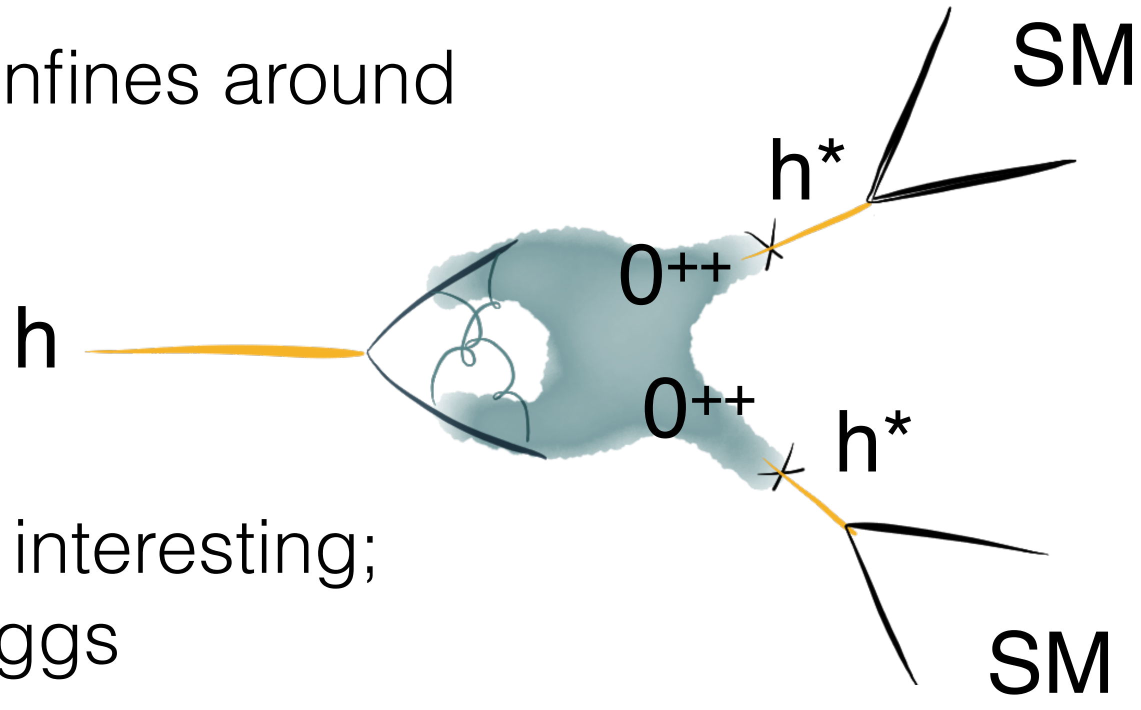
Many more options where symmetry is approximate, e.g. a good symmetry for heaviest SM particles.

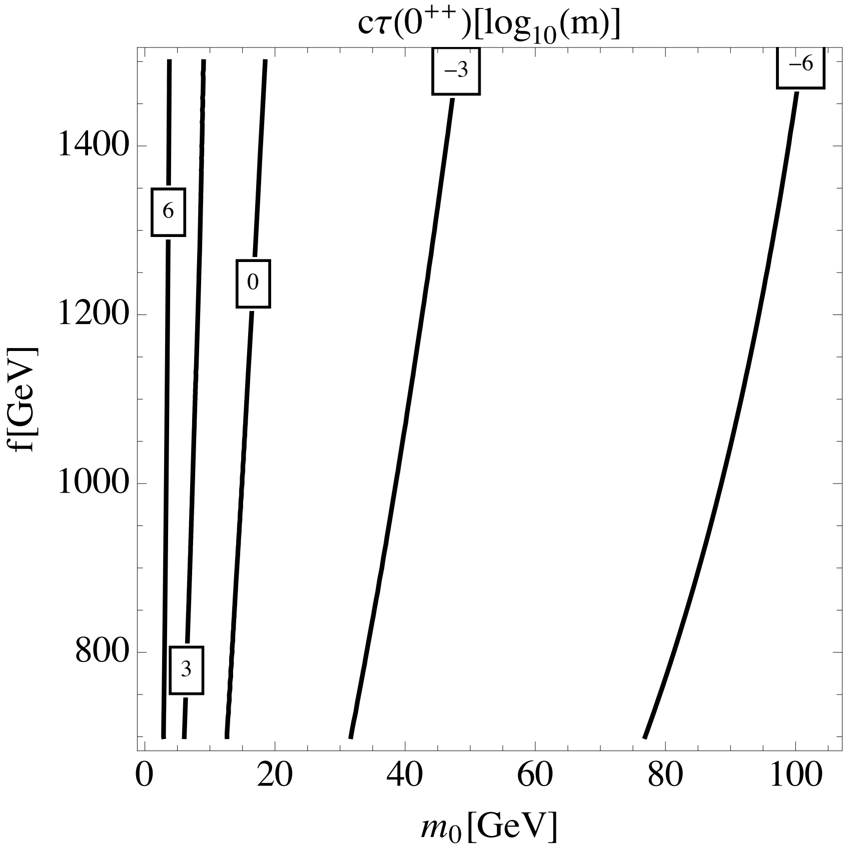
[NC, Knapen, Longhi ’14; Geller, Telem ’14; NC, Katz, Strassler, Sundrum ’15; Barbieri, Greco, Rattazzi, Wulzer ’15; Low, Tesi, Wang ’15, NC, Knapen, Longhi, Strassler ‘16]

[NC, Katz, Strassler, Sundrum '15]

Exotic Higgs Decays

- Twin sector must have twin QCD, confines around QCD scale
- Higgs boson couples to bound states of twin QCD
- Various possibilities. Glueballs most interesting; lightest have same quantum # as Higgs





$$\mathcal{L} \supset -\frac{\alpha_3'}{6\pi}\frac{v}{f}\frac{h}{f}G'^{a}_{\mu\nu}G'^{\mu\nu}_{a}$$

Produce in rare Higgs decays (BR~10^{-3}-10^{-4})

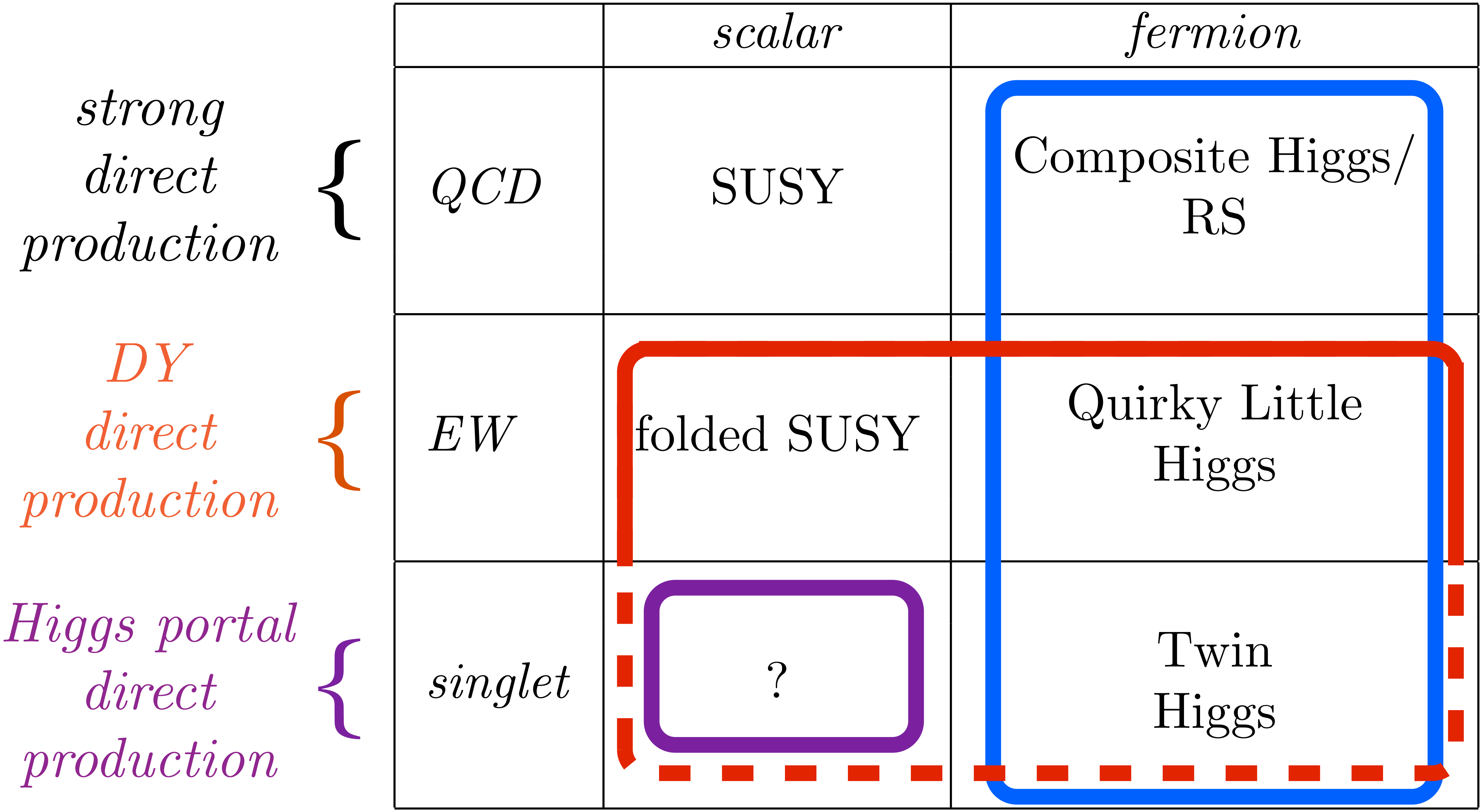
$$gg \to h \to 0^{++} + 0^{++} + \ldots$$

Decay back to SM via Higgs

$$0^{++} \to h^* \to f\bar{f}$$

Long-lived, length scale ~ LHC detectors

[Curtin, Verhaaren '15]



		scalar	*fermion*
strong direct production	*QCD*	SUSY	Composite Higgs/ RS
DY direct production	*EW*	folded SUSY	Quirky Little Higgs
Higgs portal direct production	*singlet*	?	Twin Higgs

Mirror Glueballs

Higgs portal observables

Higgs coupling shifts $\sim$ tuning

Hyperbolic Higgs

[Cohen, NC, Giudice, McCullough '18]

Is there a singlet scalar top partner theory, *a la* supersymmetry?

Instead of accidental SU(4) from Z_2, what about "accidental SU(2,2)?"
(NB, not a symmetry of the full quadratic action)

- Take 2 copies of the MSSM, related by exchange:

$$\textbf{MSSM} \quad \xleftrightarrow{\quad Z_2 \quad} \quad {}_{\mathscr{H}}\textbf{MƧƧM}$$

- Introduce SU(2,2) symmetric tree-level potential:

$$V(H, H_{\mathcal{H}}) = \lambda \left(|H|^2 - |H_{\mathcal{H}}|^2\right)^2$$

- Lift scalars in MSSM, fermions in $\text{MSSM}_{\mathscr{H}}$ (e.g. via 5D SSSB)

$$\delta V(H, H_{\mathcal{H}}) = -c\Lambda^2 \left(|H|^2 - |H_{\mathcal{H}}|^2\right) + \ldots$$

Hyperbolic Higgs

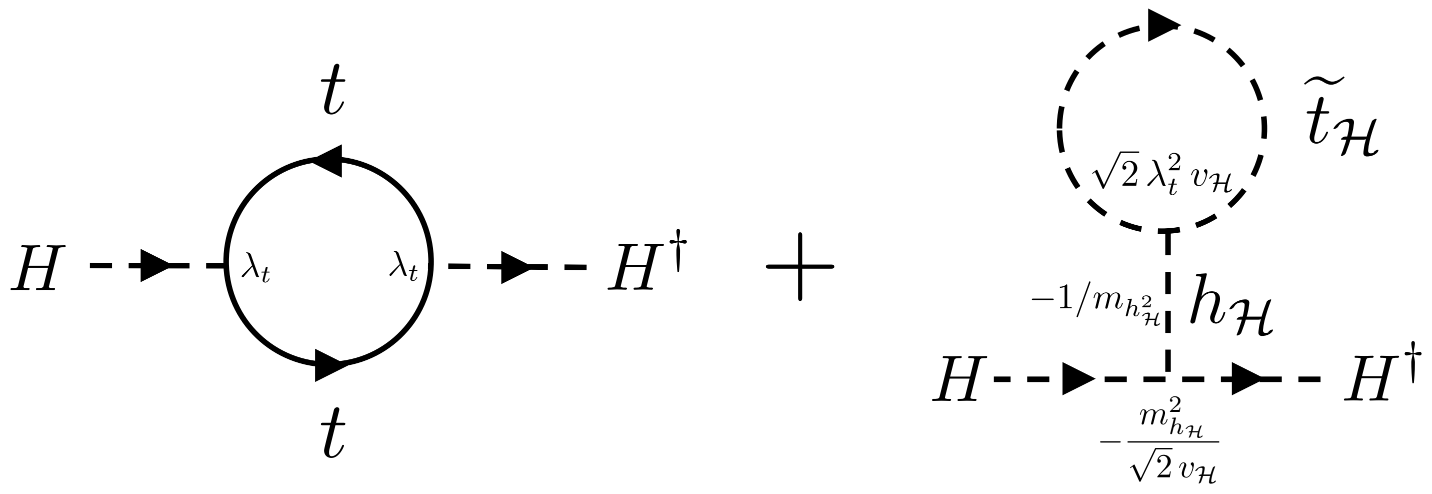
Flat direction (“goldstone” of spontaneously broken SU(2,2))

$$H = H_0 \sinh \frac{H_{\text{flat}}}{f}\,, \qquad H_{\mathcal{H}} = H_0 \cosh \frac{H_{\text{flat}}}{f}$$

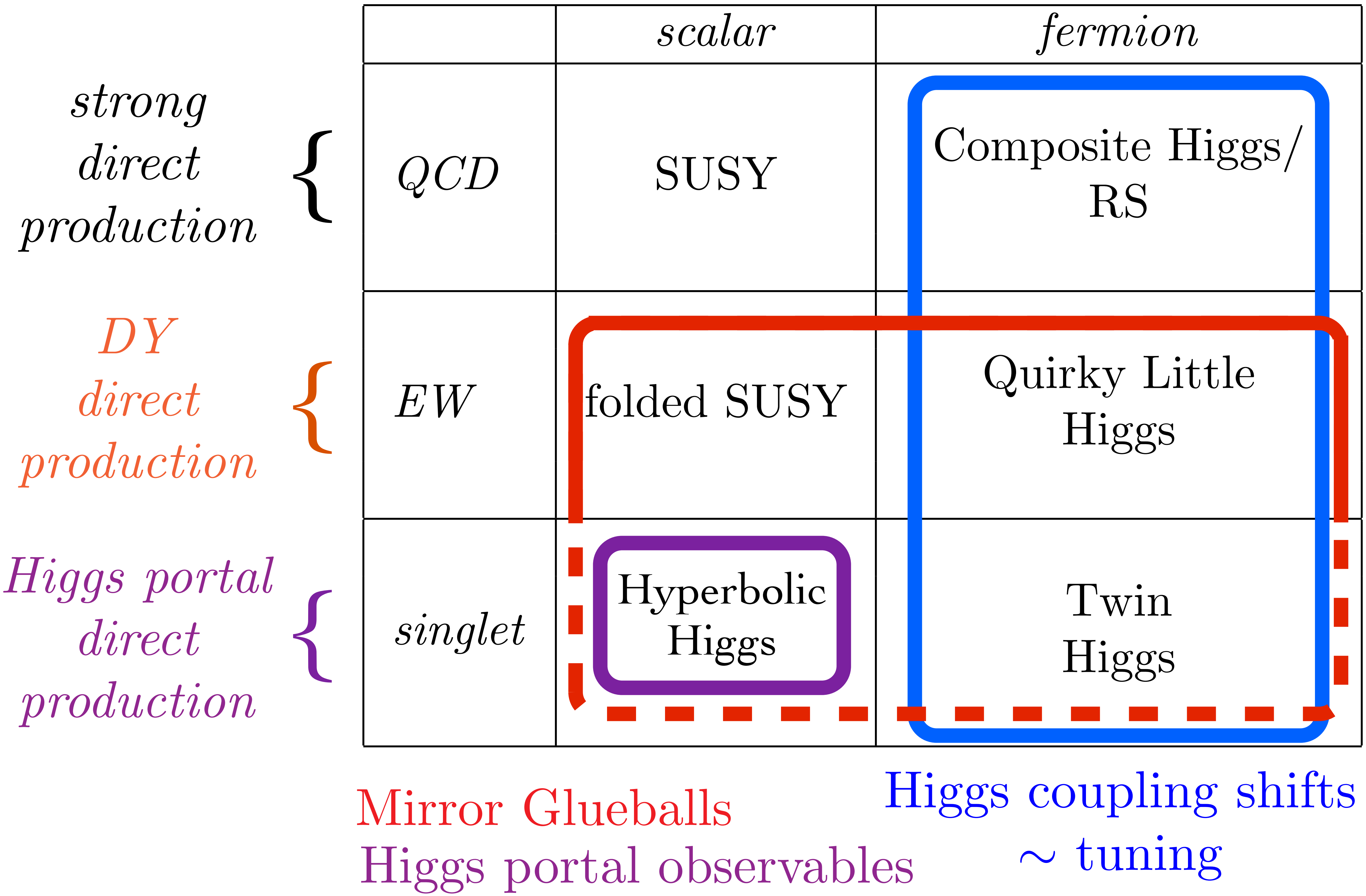
Identification w/ SM-like Higgs provided vacuum alignment,

$$h_{\text{SM}} = h\cos\theta + h_{\mathcal{H}}\sin\theta\,, \qquad \tan\theta = \frac{v}{v_{\mathcal{H}}}$$

Light top partner is SM-neutral stop of MSSM$_{\mathscr{H}}$



Novel dark sector phenomenology, especially if there are hyperbolic charge- and color-breaking minima



strong direct production

DY direct production

Higgs portal direct production

	scalar	*fermion*
QCD	SUSY	Composite Higgs/ RS
EW	folded SUSY	Quirky Little Higgs
singlet	Hyperbolic Higgs	Twin Higgs

Mirror Glueballs

Higgs portal observables

Higgs coupling shifts $\sim$ tuning

Lowering the cutoff

…in diverse dimensions

4. RS / Technicolor
[Randall, Sundrum ’99; Weinberg ’79; Susskind ‘79]

5. LED / 10^{32} x SM
[Arkani-Hamed, Dimopoulos, Dvali ’98; Antoniadis + ibid. ’98; Dvali, Redi ‘09]

6. LST / Clockwork
[Antoniadis, Dimopoulos, Giveon ’01; Kaplan, Rattazzi ’15; Giudice, McCullough ‘16]

7. Classicalization
[Dvali, Giudice, Gomez, Kehagias ‘10]

8. Disorder
[Rothstein ‘12]

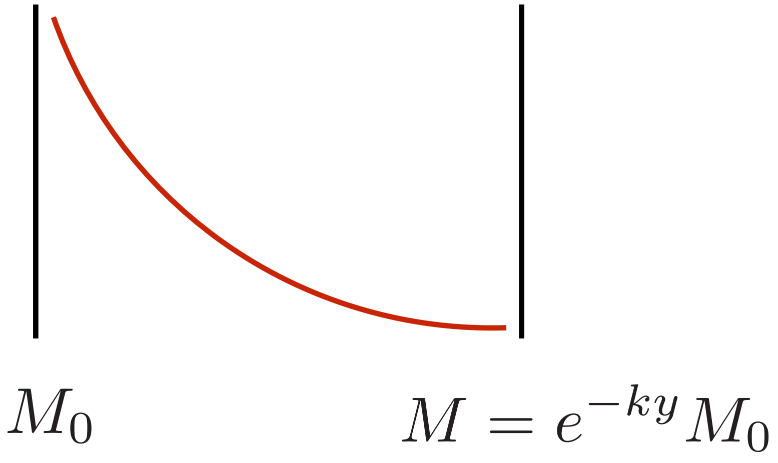
Experimental signals: resonances, …

- Primary distinctions are in spacing & coupling of resonances
- Potential goldmine of unexplored signals for LST — e.g. perturbative string excitations

A Cutoff Solution?: Disorder

How does RS solve hierarchy problem?
Curvature localizes the graviton zero mode.

→ Fields localized at different points in 5th dimension see different fundamental scales



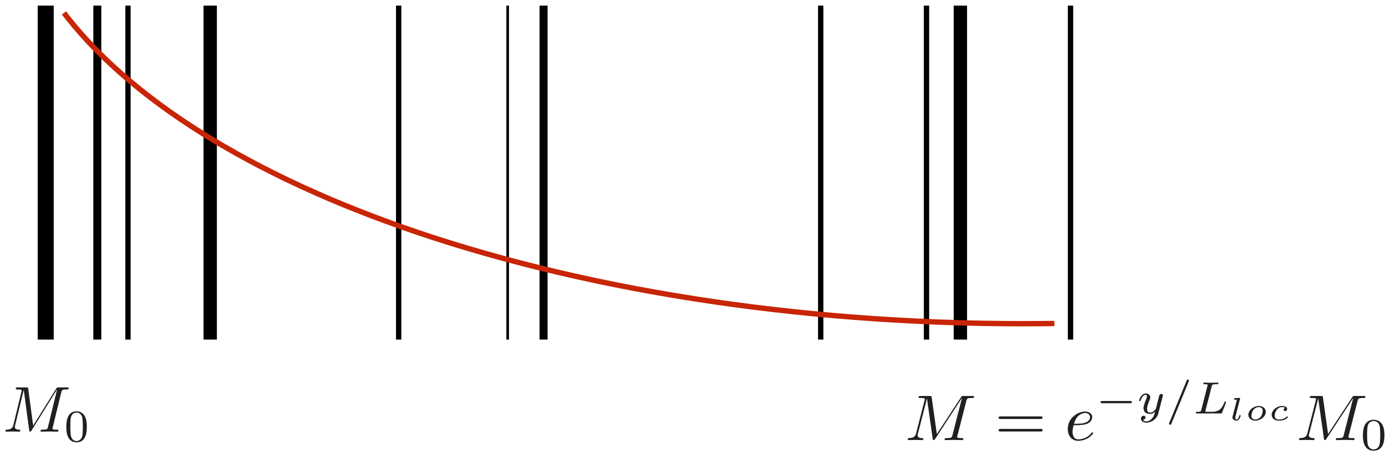
[Rothstein '12]: *Can achieve the same outcome in a flat fifth dimension by localizing graviton w/ disorder*

$$S = -\int d^5x\sqrt{G}(M_\star^3\mathcal{R}) + \sum_{\langle ij\rangle} M_\star^4 V(|X_i - X_j|) - \sum_i \int d^4x\sqrt{g} f_i$$

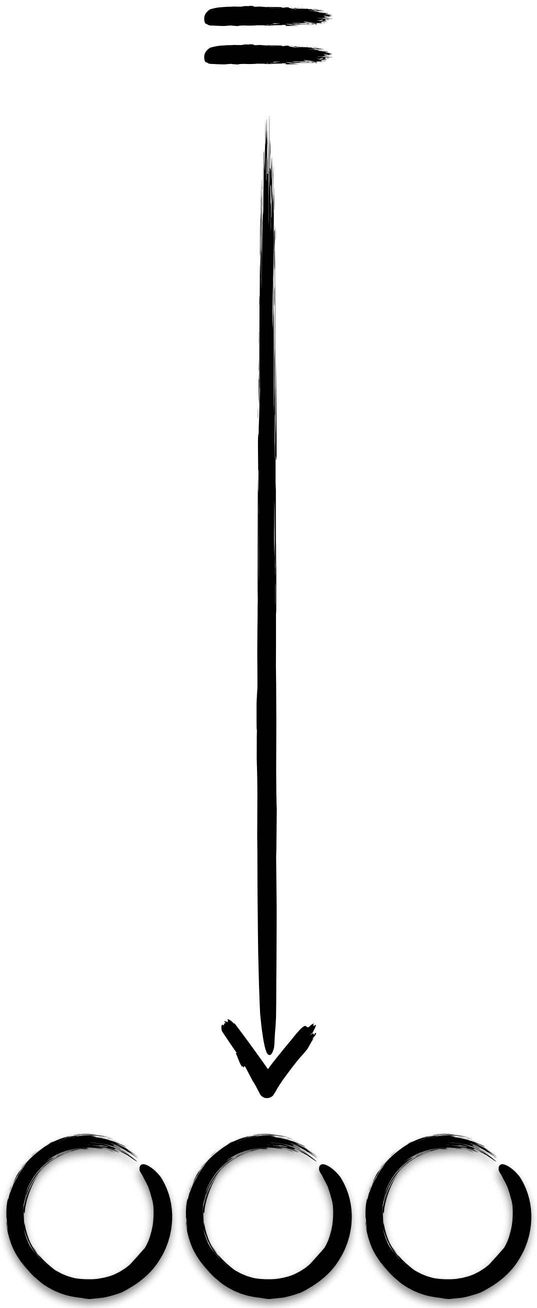
In this case disorder = randomly spaced & tensioned branes

But: not obvious that it works in detail

An interesting source of exponential hierarchies for scalars [NC, Sutherland '17]



Selecting a vacuum



Vacuum is one of many; end up in observed vacuum through dynamical process or anthropic constraint.

9. Anthropics (pressure)
10. Relaxation (dynamics) [Graham, Kaplan, Rajendran ‘15]
11. NNaturalness (reheating) [Arkani-Hamed et al ‘16]

Experimental signals: Diverse, but typically

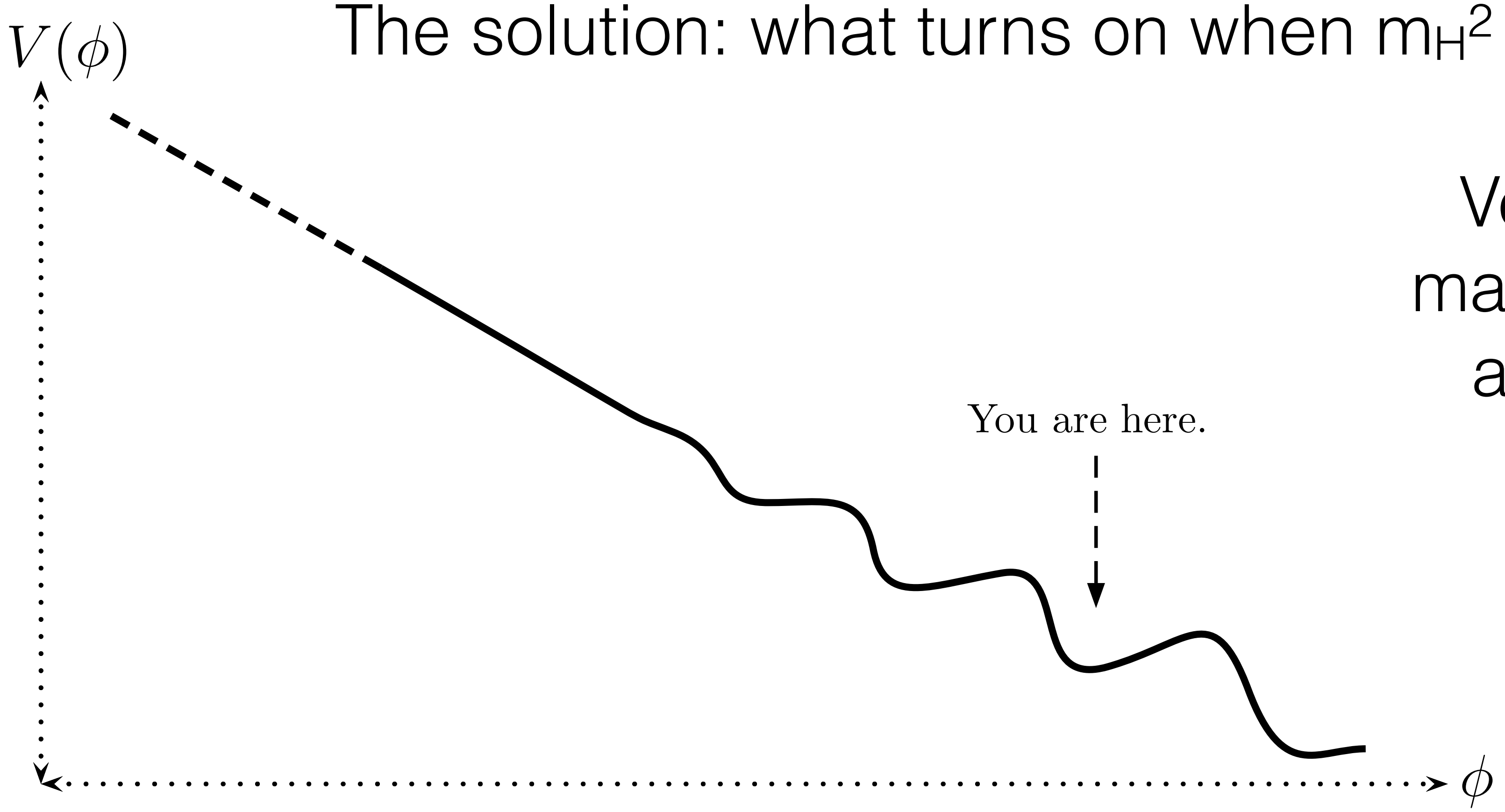
- Cosmology (Bubble collisions; axions; contributions to N_{eff} and $\sum m_\nu$)
- Exotic lab signals (displaced decays, hidden sector confinement, intensity frontier, …)

Relaxion

What if the weak scale is selected by dynamics, not symmetries?

The idea: couple Higgs to field whose minimum sets $m_H=0$
The problem: How to make $m_H=0$ a special point of potential?

The solution: what turns on when m_H^2 goes negative?



Vev gives quark masses which give axion potential.

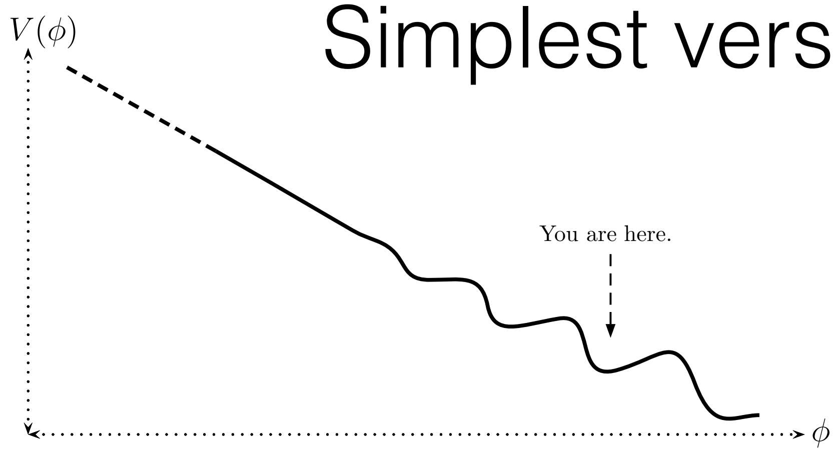
“Relaxion”

[Graham, Kaplan, Rajendran ‘15]

But: immense energy stored in evolving field, need dissipation.

[Graham, Kaplan, Rajendran '15]

Relaxion



Simplest version: an axion coupled to QCD during inflation.

$$(-M^2 + g\phi)|H|^2 + V(g\phi) + \frac{1}{32\pi^2}\frac{\phi}{f}\tilde{G}^{\mu\nu}G_{\mu\nu}$$

$$\Rightarrow \left(-M^2 + g\phi\right)|H|^2 + V(g\phi) + \Lambda^4(H)\cos\left(\phi/f\right)$$

Viable for Higgs + non-compact axion + inflation w/

- Very low Hubble scale (≪Λ_{QCD})
- 10 Giga-years of inflation

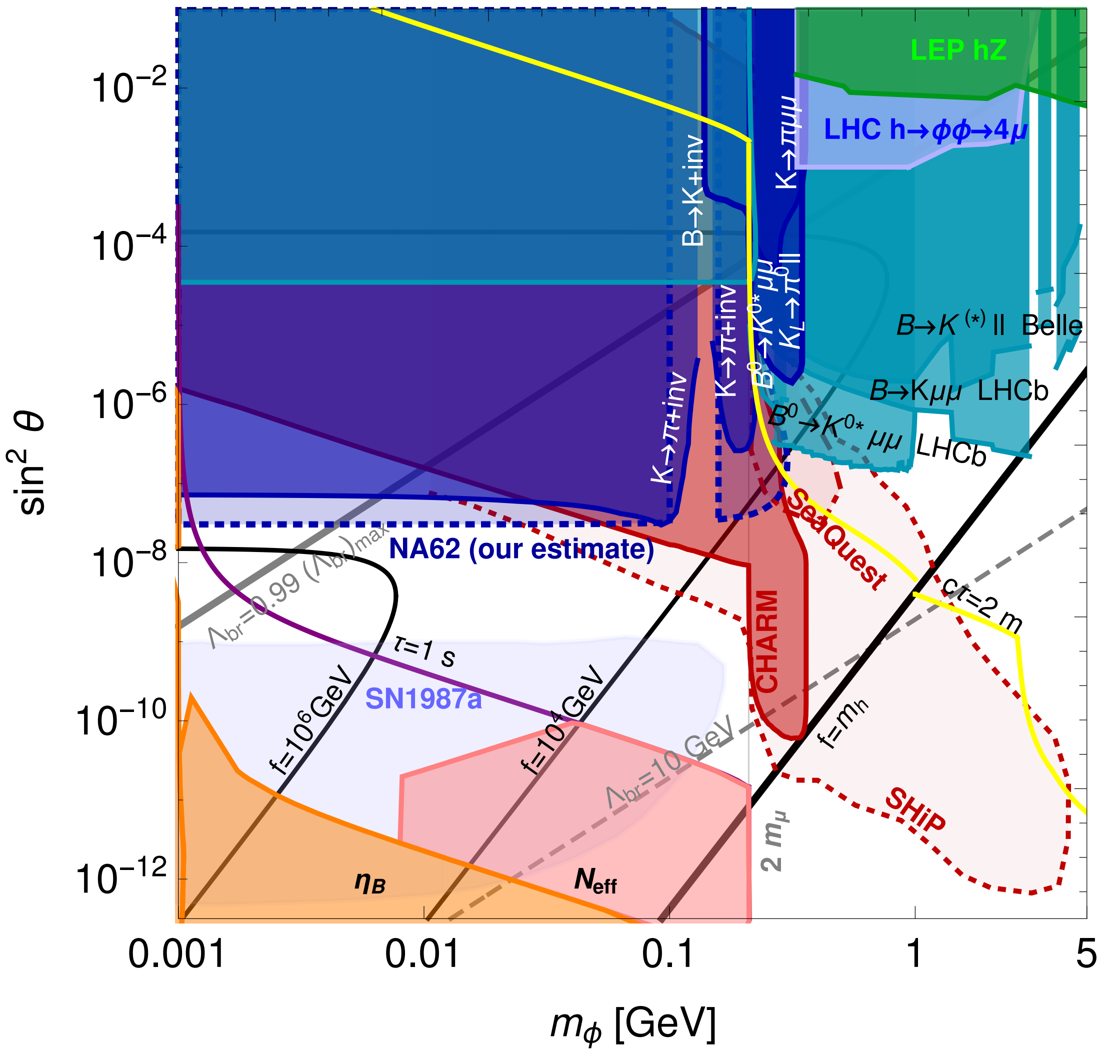
Various other subtleties regarding technical naturalness, CC, avoidance of fine-tuning to inflationary sector; need to solve strong CP problem

Extensive development, e.g. [Espinosa et al. '15; Hardy '15; Gupta et al '15; Batell, Giudice, McCullough '15; Choi, Im '15; Kaplan, Rattazzi '15; Di Chiara et al. '15; Ibanez et al. '15; Hook, Marques-Tavares '16; Nelson, Prescod-Weinstein '17; …]

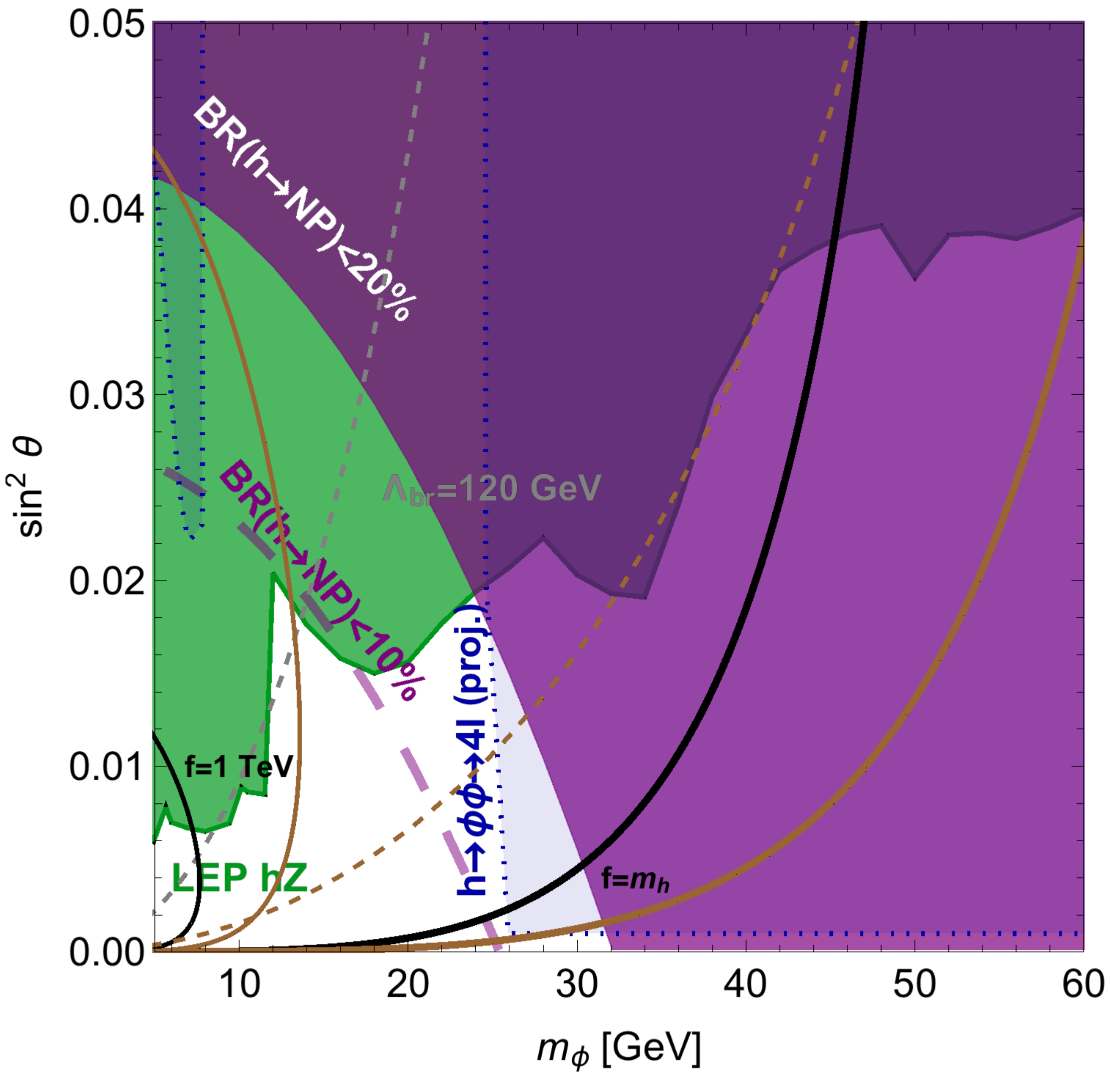
See talks by D. Kaplan, T. Gherghetta

Higgs is SM-like, but there is a new singlet Higgs coupled via $g\phi|H|^2$ and $\Lambda^4(H)\cos(\phi/f)$

$\Lambda^4(H)\cos(\phi/f)$ gives ϕ - H mixing* w/ $\sin\theta \simeq \frac{y_u f_\pi^3}{m_h^2 f}\sin\left(\frac{\langle\phi\rangle}{f}\right)$



[Flacke, Friguele, Fuchs, Gupta, Perez ‘16]



[Flacke, Friguele, Fuchs, Gupta, Perez ‘16]

+5th force for m_ϕ < eV & cosmology for eV < m_ϕ < MeV

**assuming ⟨ϕ⟩ breaks CP*

[Hook, Marques-Tavares ’16; Fonseca, Morgante, Servant ‘18]

Particle production relaxion

Alternative possibility: keep bumps across entire potential, turn on dissipation at a special point of potential.

Novel source of dissipation: particle production

Consider axion-like couplings to massive gauge field: $\mathcal{L} \supset -\frac{\phi}{4f} F\tilde{F}$

E.O.M. for transverse polarizations: $\ddot{A}_\pm + \left(k^2 + m_A^2 \pm \frac{k\dot{\phi}}{f} \right) A_\pm = 0$

For $\dot{\phi} \approx \text{constant}$ $\quad A_\pm(k) \propto e^{i\omega_\pm t} \qquad \omega_\pm^2 = k^2 + m_A^2 \pm \frac{k\dot{\phi}}{f}$

Exponentially growing solution for $\omega_\pm^2 < 0 \Rightarrow |\dot{\phi}| \gtrsim 2fm_A$

Growing mode drains energy from $\dot{\phi}$

Particle production relaxion

Apply to relaxion: use electroweak gauge fields

Instead of

$$\frac{\phi}{f} G\tilde{G}$$

+ inflation

$\Rightarrow$

Use coupling to EWK gauge bosons:

$$\frac{\phi}{f}\left(g^2 W\widetilde{W} - g'^2 B\widetilde{B}\right) + \Lambda_c^4 \cos\frac{\phi}{f'}$$

Exponential production of EWK gauge bosons around h~v slows evolution

Important subtlety: can't couple to pairs of photons!

(*Not a tuning, can be made natural with symmetries, e.g., SU(2)$_L$ x SU(2)$_R$*)

Requiring sub-Planckian field excursions & avoiding overshoot bounds cutoff

$$\Lambda \lesssim (M_{Pl} v^5)^{1/6} \sim 50\,\mathrm{TeV}$$

Corresponding decay constant

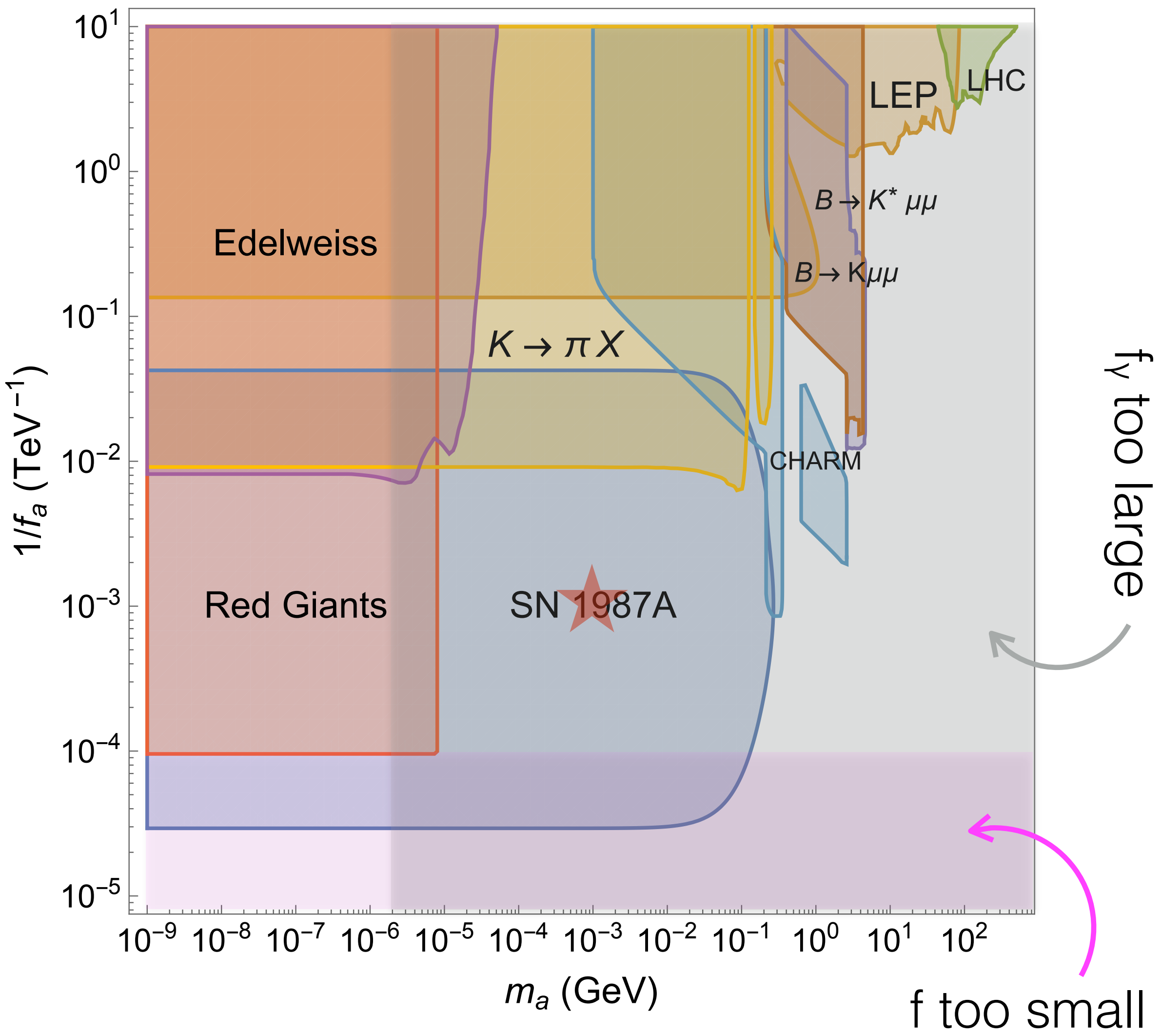
$$f \sim \frac{\dot{\phi}}{v} \sim \frac{\Lambda^2}{v} \lesssim 10^4\,\mathrm{TeV}$$

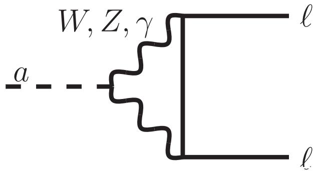
Particle production relaxion

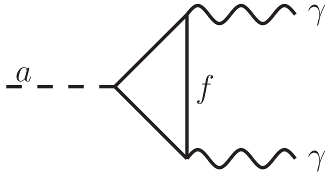
Even if tree-level relaxion couplings to SM states are engineered to be $\frac{\phi}{f}(g^2 W\tilde{W} - g'^2 B\tilde{B})$ in the UV…

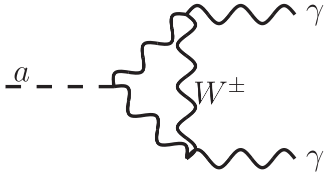
…radiative couplings to fermions induced at one loop, photon pairs at one & two loops [Bauer, Neubert, Thamm '17; NC, Hook, Kasko '18]

[NC, Hook, Kasko '18]









$$f_\gamma \sim 16\pi^2 \frac{m_W^2}{m_a^2} f_a,\ (16\pi^2)^2 \frac{m_f^2}{m_a^2} f_a$$

Requiring e.g. time scale for γ production > Hubble time

$$\frac{\dot{\phi}}{f_\gamma} \sim \frac{v f_a}{f_\gamma} < H \Rightarrow f_\gamma \gtrsim \frac{M_{Pl} v}{\Lambda^2} f_a$$

This + experimental constraints places much of parameter space under tension [NC, Hook, Kasko '18]

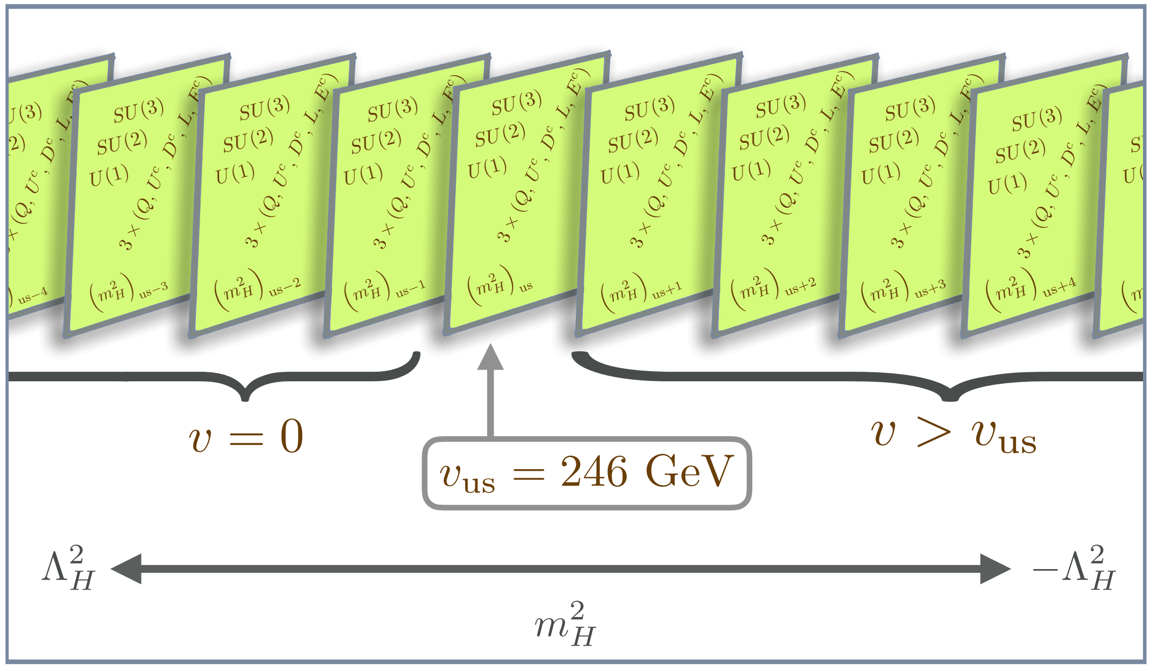
[Arkani-Hamed, Cohen, D'Agnolo, Hook, Kim, Pinner '16]

NNaturalness

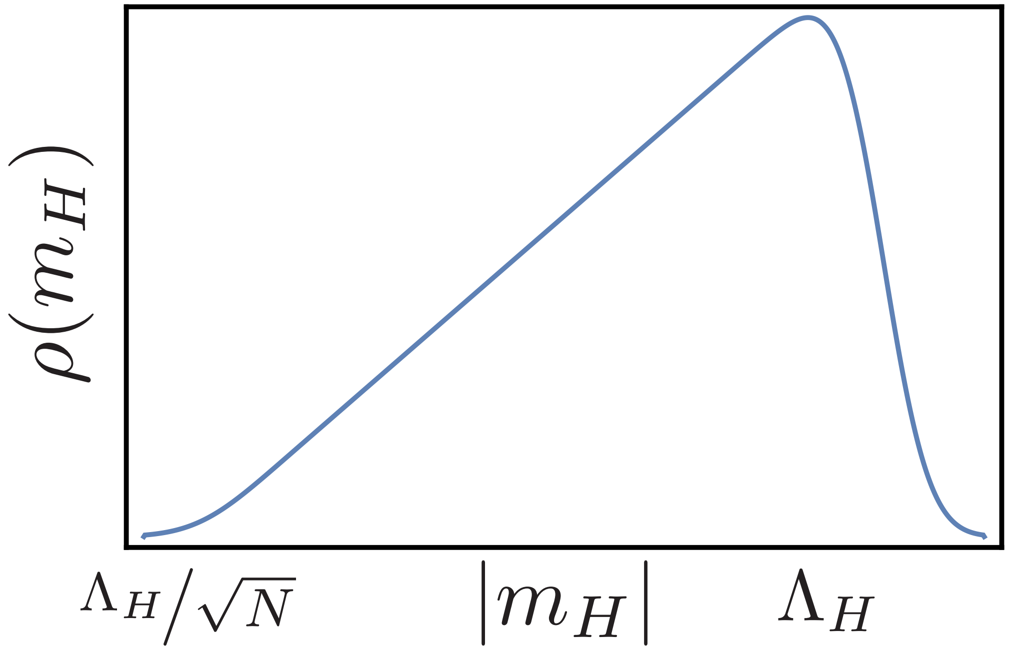
N copies of the SM

High Higgs cutoff Λ_H, high gravity cutoff Λ_G

Two effects:

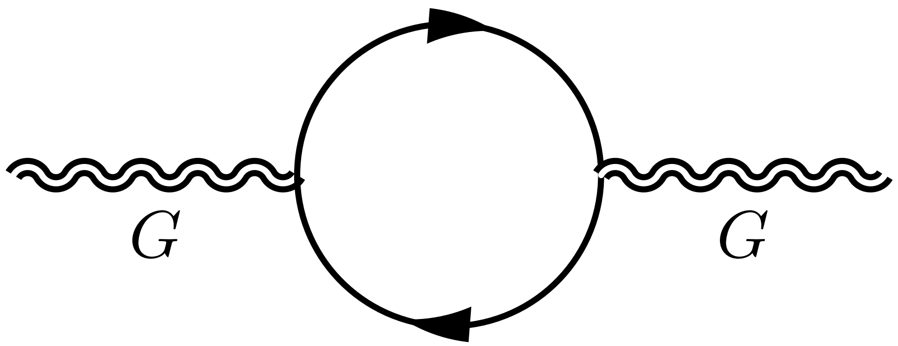


1. Random UV contributions → flat distribution of m_H^2 between $\pm\Lambda_H^2$



At least 1 copy w/ $|m_H| \sim \Lambda_H/\sqrt{N}$

2. Large number of species renormalizes Planck scale (e.g. graviton wavefunction renorm.)

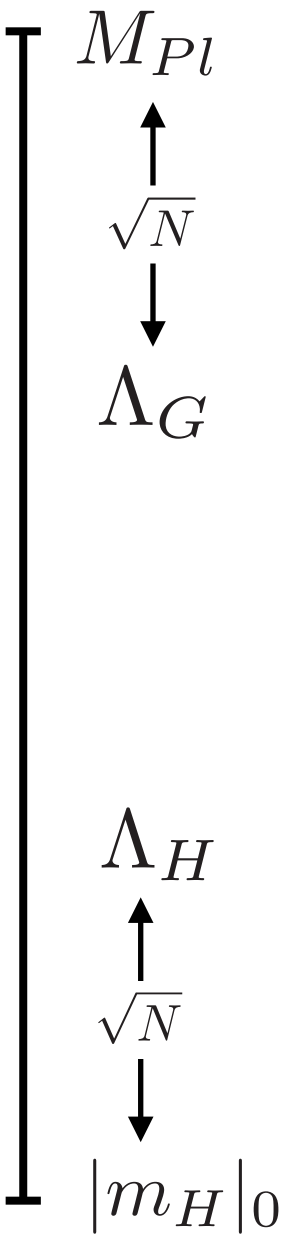


Gravitational strong coupling scale Λ_G below M_{Pl}

$$M_{Pl}^2 \sim N\Lambda_G^2$$

NNaturalness

Scale separation from large N:



For example:
One copy w/ weak-scale Higgs for

N=10^{16}:
$\Lambda_H=10^{10}$ GeV
$\Lambda_G=10^{10}$ GeV
(That's it.)

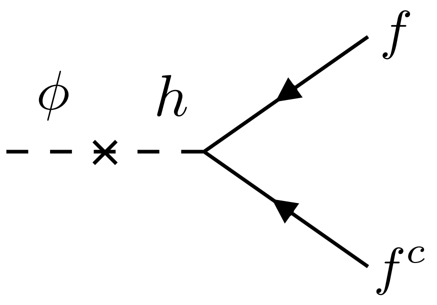
N=10^{4}:
$\Lambda_H=10^{4}$ GeV
$\Lambda_G=10^{16}$ GeV
(SUSY or compositeness at Λ_H)

Now…why does the copy with the smallest m_H dominate?
Cosmology.

Reheaton ϕ starts universe via $\phi\,|H_i|^2$ couplings

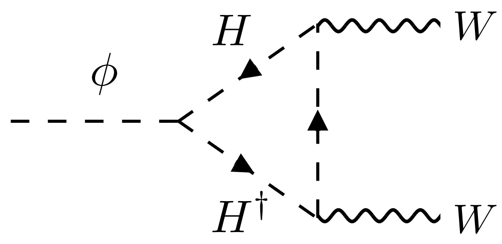
Decays (provided $m_\phi < |m_{H_i}|$)

$$m^2_{H,i} < 0$$



$$\Gamma \propto \frac{1}{m^2_{H,i}}$$

$$m^2_{H,i} \geq 0$$



$$\Gamma \propto \frac{1}{m^4_{H,i}}$$

Preferentially reheats copy w/ smallest $|m_H|$ & $m_H^2<0$

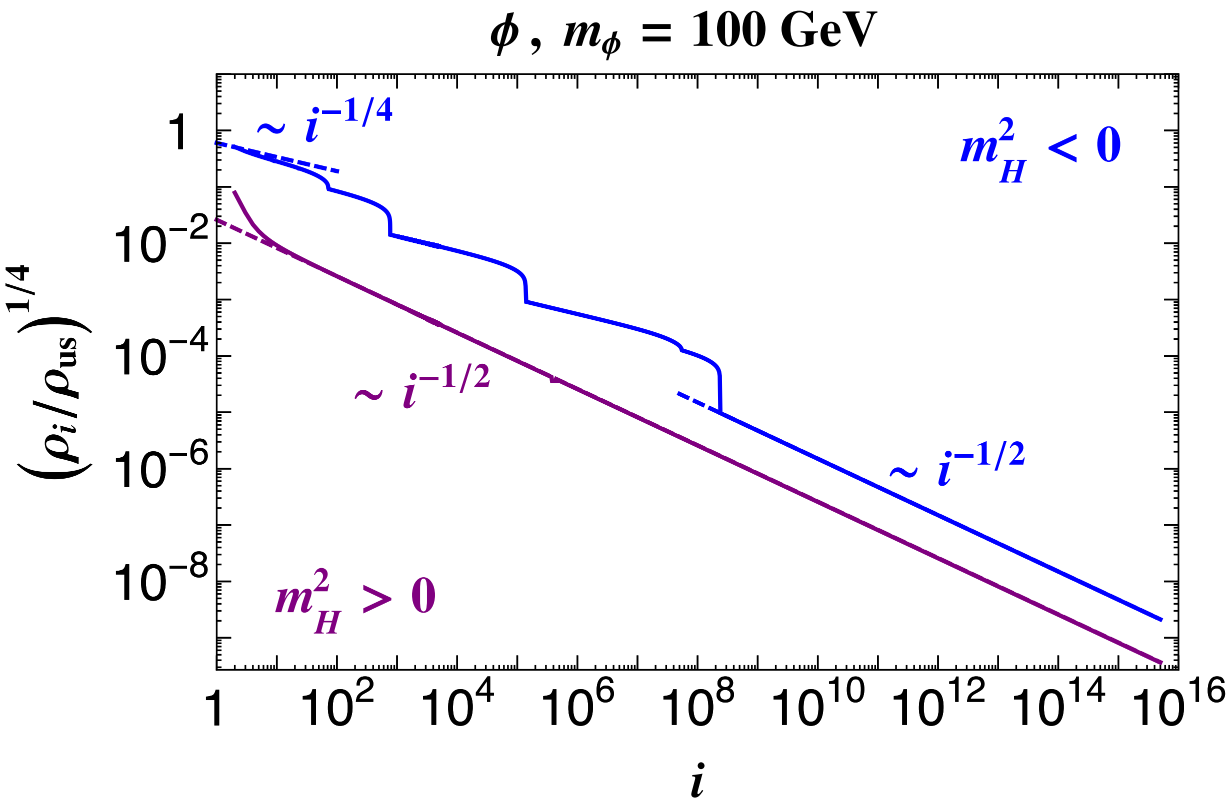
N Higgses…in the sky

All sectors reheated by some amount ⇒ dark radiation

$$\frac{\rho_i}{\rho_{\rm us}} = \frac{\Gamma_i}{\Gamma_{\rm us}}$$

Dominated by sectors with similar scales

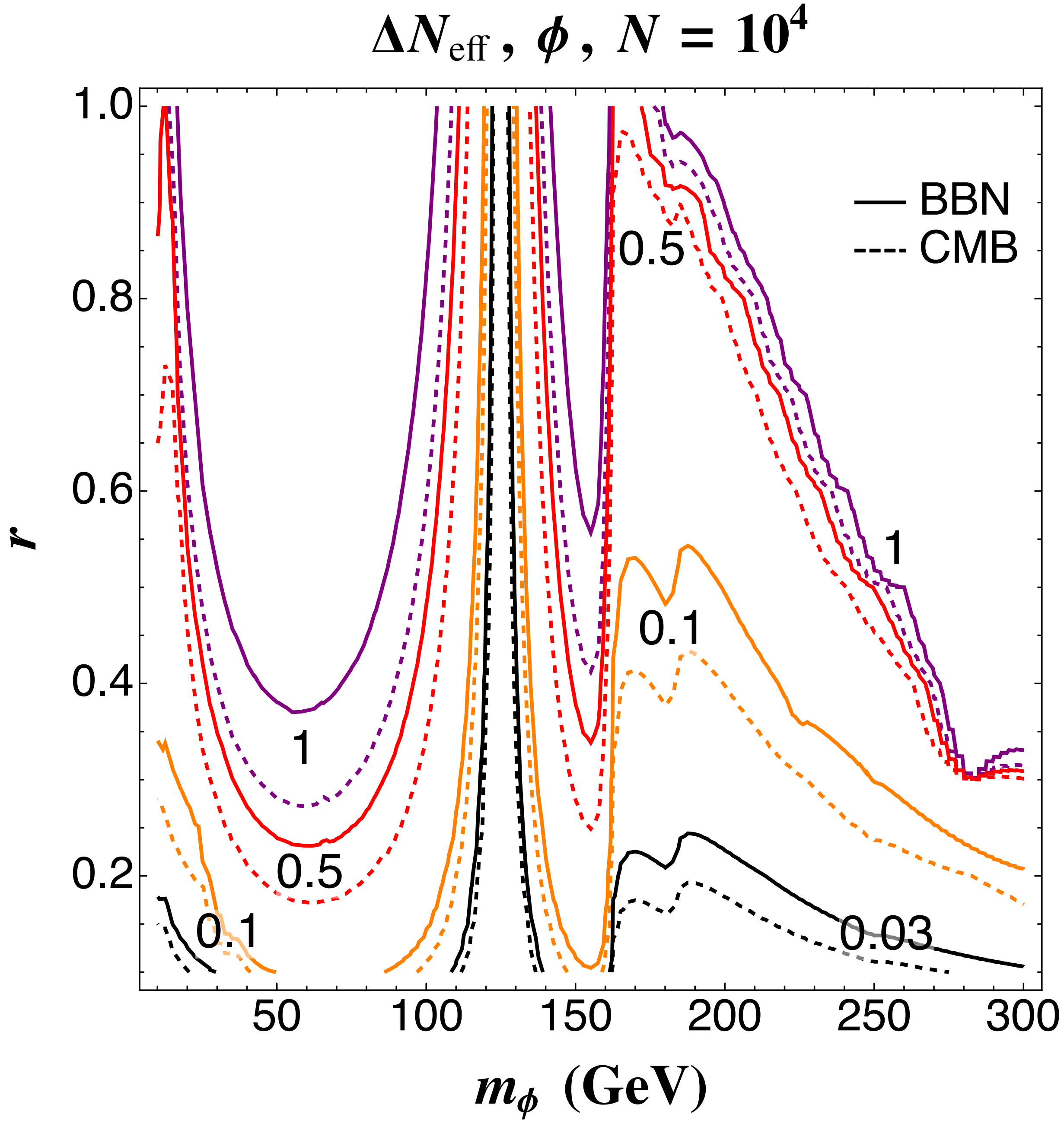
[Arkani-Hamed, Cohen, D'Agnolo, Hook, Kim, Pinner '16]



ϕ , m_ϕ = 100 GeV

Primary signals in dark radiation, extensive coverage by CMB-S4

[Arkani-Hamed, Cohen, D'Agnolo, Hook, Kim, Pinner '16]

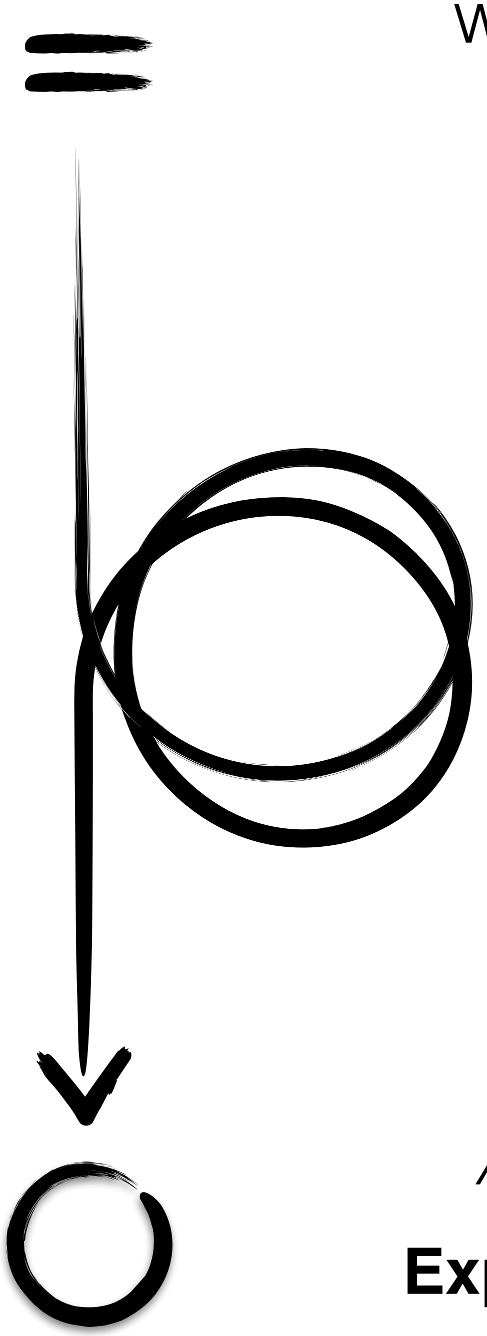


$\Delta N_{\rm eff}$, ϕ , $N = 10^4$

(r=1 ↔ flat m_H^2; r<1 ↔ larger splitting)

NB, similar mechanism for Twin Higgs cosmology
[NC, Koren, Trott '16; Chacko, NC, Fox, Harnik '16]

Complicating the flow



SM is reached from some intermediate fixed point
where, say, a generalized Veltman condition is satisfied

$$\delta m_H^2 = \sum_i c_i \frac{g_{i,\star}^2}{16\pi^2} \Lambda_i^2 = 0$$

This is a sense in which

12. Conformal symmetry

could address the hierarchy problem

Top-down: Embed SM in orbifold of N=4 SYM
[Frampton, Vafa ’99; Csaki, Skiba, Terning ‘99]

Bottom-up: “Little conformal symmetry”
[Houtz, Colwell, Terning ’16]

A challenge: how do fixed point couplings know about UV scale?

Experimental signals: Not fully understood, but expect new particles w/ SM quantum numbers around the TeV scale. Novelty is that their statistics, representations & couplings differ from more familiar solutions.

Exploding the cutoff

Gravity doesn't provide a UV scale & the SM takes care of itself

13. Asymptotic fragility
[Dubovsky, Gorbenko, Mirbabayi '13]

14. Agravity [Salvio, Strumia '14]

Scale M_{Pl} not associated with relevant operator becoming strong, not "felt" by non-grav physics.

In IR, looks like CFT perturbed by irrelevant operators.

In UV, no UV fixed point; cannot define local observables.

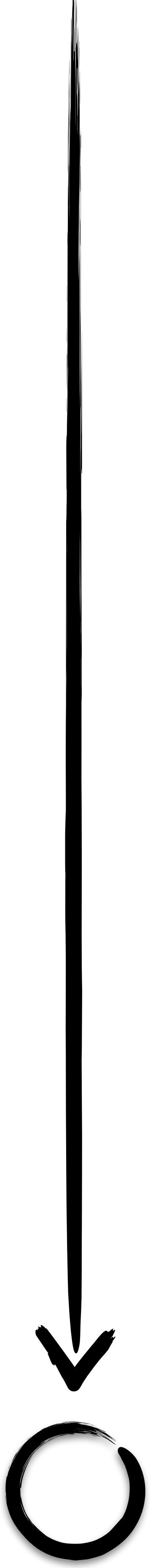
Example in 2d, no proposal for 4d.

Gravity has no intrinsic length scale and is "renormalizable"

$$S \sim \int d^4x \sqrt{g} \left(\frac{R^2}{f_1^2} + \frac{\frac{1}{3}R^2 - R_{\mu\nu}^2}{f_2^2} + \ldots \right)$$

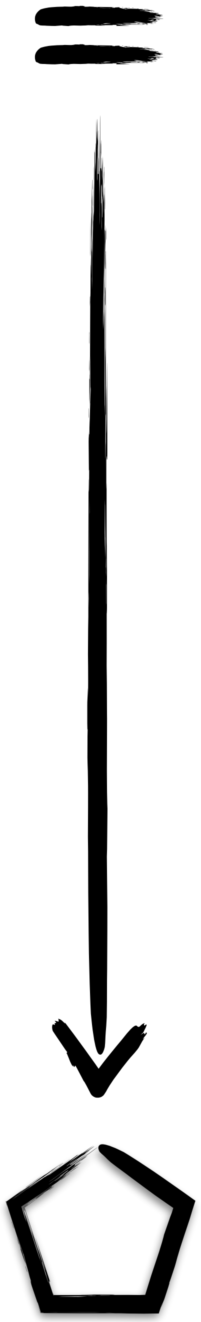
(E-H term via vev of some field)

Can be re-written in terms of 2-deriv fields w/ ghosts.



Experimental signals: Details of gravity sector might be irrelevant. Crucially, must render SM couplings asymptotically free. Not a property of the SM itself, so entails low-scale unification (~10 TeV)

Not actually the SM



15. Lee-Wick (higher derivative scalar)
[Grinstein, O'Connell, Wise '06]

16. Non-semisimple gauge groups?

Lee-Wick: higher-derivative theory

$$\sim \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2M^2}(\partial^2\phi)^2 + \dots$$

Expressible as normal field plus new field with wrong-sign quadratic action

$$-\frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 + \dots$$

Improves UV convergence of diagrams, introduce for every SM field

$$\frac{1}{p^2-m^2} - \frac{1}{p^2-M^2} = \frac{m^2-M^2}{(p^2-m^2)(p^2-M^2)}$$

Can be defined in a unitary, Lorentz-invariant manner with only microscopic acausality. *But* who ordered that?

See talk by D. Anselmi

Non-semisimple gauge group?

Trained from birth to study gauge theories of compact simple subalgebras & U(1)'s to guarantee positive-norm states.

But [Tseytlin '95] perhaps pathologies of non-semisimple groups are not fatal.

Simplest example: E_2^C

$$[e_3, e_i] = \epsilon_{ij} e_j \qquad [e_i, e_j] = \epsilon_{ij} e_4$$

$$[e_4, e_i] = [e_4, e_3] = 0 \qquad i, j = 1, 2$$

Non-degenerate invt bilinear form

$$\Omega_{ab} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 2\gamma & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Degenerate Killing form

$$g_{ab} \equiv f^c_{ad} f^d_{bc} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Suggestively related to algebra of SU(2) x U(1) by λ → 0 scaling limit

$$\tilde{e}_i = \lambda^{-1} e_i \qquad \tilde{e}_3 = e_3 + \lambda^{-2} e_4 \qquad \tilde{e}_4 = e_4 \qquad \lambda \to 0$$

Non-semisimple gauge group?

Can construct a YM theory based on E_2^c

$$S_{YM} = \frac{1}{4}\Omega_{ab}\int d^4x\, F^a_{\mu\nu}F^b_{\mu\nu}$$

with the following intriguing properties:

1. One-loop divergence as in semisimple YM:

$$\Gamma^{1-\text{loop}}_{div.} = \frac{1}{\epsilon}\beta_1 g_{ab}\int d^4x F^a_{\mu\nu}F^b_{\mu\nu} \qquad \beta_1 = -\frac{11}{6(4\pi)^2}$$

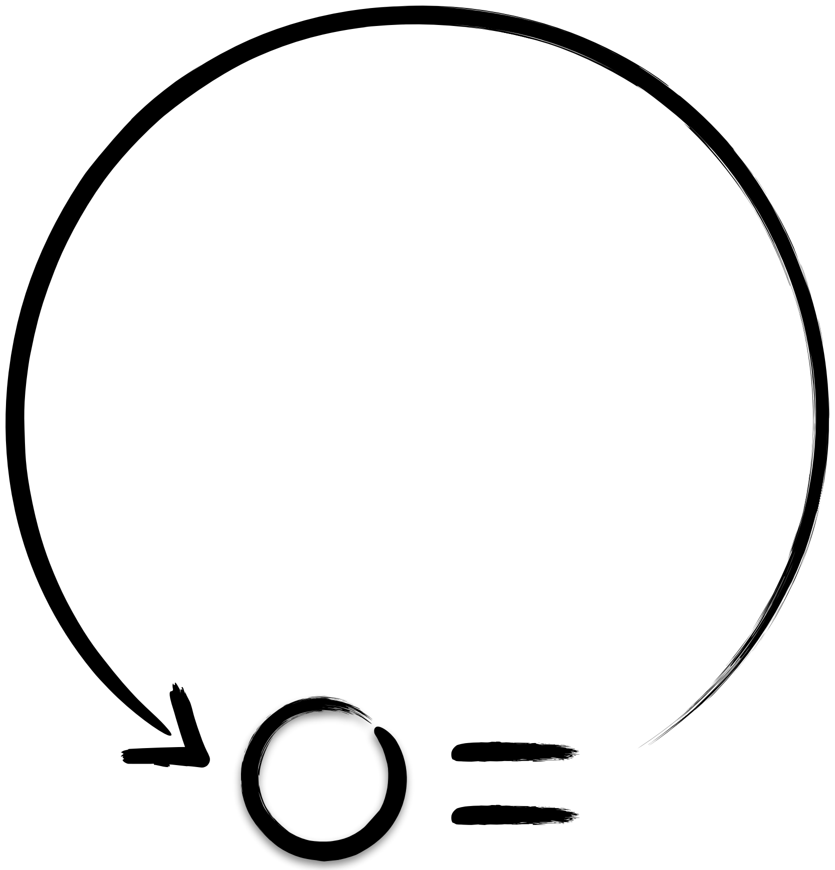
$$\Omega^{1-\text{loop}}_{ab} = \Omega_{ab} + \frac{1}{\epsilon}\beta_1 g_{ab}$$

2. No additional divergences at higher loops:

$$\Omega^{2-\text{loop}}_{ab} = \Omega_{ab} + \frac{1}{\epsilon}\beta_1 g_{ab} + \frac{1}{\epsilon}\beta_2 \cancelto{0}{g_{ac}\Omega^{cd}} g_{db}$$

3. Looks like λ → 0 scaling limit of an SU(2) x $U(1)_{\text{ghost}}$ gauge theory where non-trivial part of S-matrix is just the unitary SU(2) S-matrix

Connecting UV & IR



Essential feature of the hierarchy problem is
that the UV doesn’t know about the IR…
unless it does?

Two “theories” exhibiting UV/IR mixing:
Quantum gravity & non-commutative field theory

QG (cartoon version): probe spacetime with sufficiently energetic particles, make a black hole. More energetic particles → bigger black hole.

NCQFT (cartoon version): non-commutativity of the form $[x^\mu,x^\nu]=i\theta^{\mu\nu}$, qualitatively a position-position uncertainty principle $\Delta x^\mu \, \Delta x^\nu \geq \theta/2$

Two ways to put this to work for hierarchy problem:

17. Indirect UV/IR mixing

18. Direct UV/IR mixing

Indirect UV/IR Mixing

Don’t know the detailed theory of quantum gravity, but can try to ride the coattails of its UV/IR mixing. *For example…*

BH thermodynamics inspires conjecture [Bekenstein ’73-’93] that entropy in box of vol L^3 is non-extensive, **$S \leq \pi M_{Pl}^2 L^2$**

[Cohen, Kaplan, Nelson ’98]: In EFT, **$S \sim L^3 \Lambda^3$.** Bound satisfied by EFT if size of box is bounded, i.e., UV & IR cutoffs correlated:

$$L^3\Lambda^3 \lesssim \pi L^2 M_{Pl}^2 \Rightarrow L \sim \pi M_{Pl}^2/\Lambda^3$$

Many states w/ Schwarzschild radius > L. Conjecture stronger IR cutoff to exclude all states within their Schwarzschild radius,

$$L^3\Lambda^4 \lesssim L M_{Pl}^2 \Rightarrow L \sim M_{Pl}/\Lambda^2$$

Implications: Deviations in radiative corrections. Possible cc explanation, but deeply unsatisfying: L~horizon→ $\Lambda \sim 10^{-2.5}$ eV)

More recently: WGC

(Electric) weak gravity conjecture: an abelian gauge theory must contain a state of charge *q* and mass *m* satisfying

$$q > \frac{m}{M_{Pl}}$$

[Arkani-Hamed, Motl, Nicolis, Vafa '06]

Ride the coattails [Cheung, Remmen '14]: Charge SM fermions under weakly gauged (unbroken) $U(1)_{B-L}$ (bounds currently $q \lesssim 10^{-24}$). Cancel anomalies with RHN ν_R

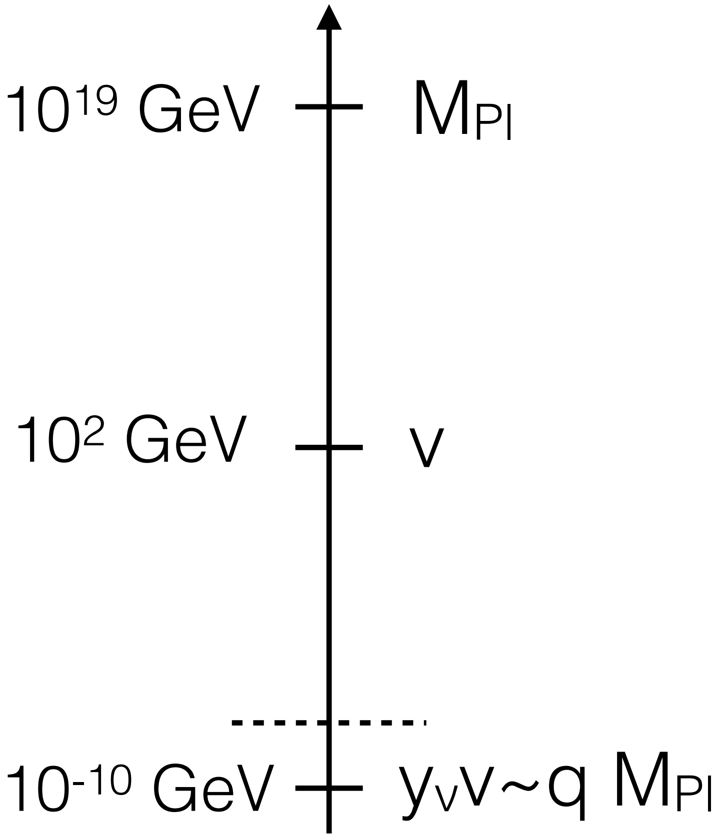
Neutrino mass is

$$y_\nu H \bar{L} \nu_R \rightarrow m_\nu \sim y_\nu v$$

so $m_\nu \sim 0.1$ eV, $q \gtrsim 10^{-29}$

For fixed yukawa, WGC violated if *v* any larger

But: *magnetic* WGC implies cutoff of U(1) field theory is $\Lambda \sim g M_{Pl}$



See talks by A. Hebecker, G. Remmen, L. Ibáñez

Direct UV/IR Mixing

Take the bull by the horns…

Study field theories with UV/IR mixing

Canonical example:

QFT on non-commutative spacetime $[\hat{x}^\mu, \hat{x}^\nu] = i\theta^{\mu\nu}$

UV/IR mixing from “uncertainty principle” $\Delta\hat{x}^\mu \Delta\hat{x}^\nu \geq \frac{1}{2}|\theta^{\mu\nu}|$

Caveats: Lorentz violating; Minkowski NCQFT unitary only for space-space non-commutativity (i.e. $\theta^{0i}=0$).

Not the theory of our universe, but a useful toy model. (See e.g. [Heckman & Verlinde ’14])

NCQFT

Two common approaches:

1. QFT on commutative coordinates w/ star product:

$$f(x) \star g(x) = \exp\left(\frac{i}{2}\theta_{\mu\nu}\partial_y^\mu \partial_z^\nu\right) f(y)g(z)\Big|_{y=z=x}$$

2. Seiberg-Witten map [Seiberg, Witten '99]:

I.e., $f \star g = f \cdot g + \frac{i}{2}\theta^{\mu\nu}\partial_\mu f \cdot \partial_\nu g + \mathcal{O}(\theta^2)$ and e.g.

$$\hat{A}_\mu[A] = A_\mu + \frac{1}{4}\theta^{\rho\sigma}\{A_\sigma, \partial_\rho A_\mu\} + \frac{1}{4}\theta^{\rho\sigma}\{F_{\rho\mu}, A_\sigma\} + \mathcal{O}(\theta^2)$$

Equivalent to any finite order in θ (i.e., option (2) defines a low-energy effective action), but UV/IR mixing only apparent in (1).

[Filk ‘96]

NCQFT: ϕ^4

Consider just ϕ^4 in Euclidean d=4:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}g^2\phi \star \phi \star \phi \star \phi$$

Quadratic terms identical to commutative theory

Interactions associated w/ additional phases:

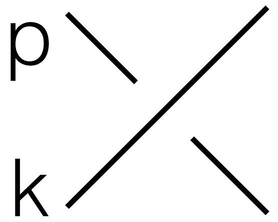
$$V(k_1, k_2, k_3, k_4) = e^{-\frac{i}{2}\sum_{i<j} k_{i\mu}\theta^{\mu\nu}k_{j\nu}}$$

Not invariant under arbitrary permutations of k

Planar graphs: reduces to an overall phase involving external momenta

Nonplanar graphs: additional phases from crossing lines

Feynman rules as usual modulo phases in nonplanar diagrams:

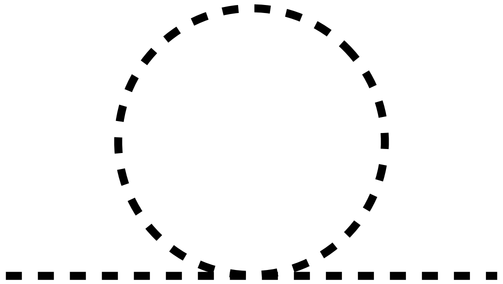


$$\sim e^{ip_\mu \theta^{\mu\nu} k_\nu}$$

[Minwalla, Seiberg, Van Raamsdonk '99]

NCQFT: ϕ^4

Compute one-loop radiative corrections to scalar 2-pt function.
Both "planar" and "non-planar" diagrams at one loop:

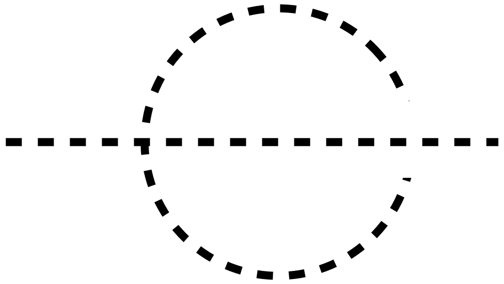


$$\sim \int \frac{d^4k}{k^2}$$

UV divergent

$$\Gamma_1^{2,p} = \frac{g^2}{48\pi^2}\left(\Lambda^2 - m^2 \log\left(\frac{\Lambda^2}{m^2}\right) + \dots\right)$$

Akin to commutative theory



$$\sim \int \frac{d^4k}{k^2} e^{ip\theta k} \sim \frac{1}{p\theta^2 p}$$

IR divergence!

$$\Gamma_1^{2,np} = \frac{g^2}{96\pi^2}\left(\Lambda_{\text{eff}}^2 - m^2 \log\left(\frac{\Lambda_{\text{eff}}^2}{m^2}\right) + \dots\right)$$

Likewise, but with $\Lambda \rightarrow \Lambda_{\text{eff}}$

Appearance of a new "scale":

$$\Lambda_{\text{eff}}^2 = \frac{1}{1/\Lambda^2 + p \circ p}$$

where $p \circ k = |p^\mu \theta_{\mu\nu}^2 k^\nu|$

$$\Lambda_{\text{eff}}^2 \rightarrow \begin{cases} \Lambda^2 & p \circ p \rightarrow 0 \\ 1/p \circ p & \Lambda \rightarrow \infty \end{cases}$$

[Minwalla, Seiberg, Van Raamsdonk '99]

New poles

1-loop 1PI quadratic effective action

$$\frac{1}{2}\left(p^2+M^2+\frac{g^2}{96\pi^2(p\circ p+1/\Lambda^2)}+\dots\right)\phi(p)\phi(-p)$$

w/ renormalized mass M:

$$M^2=m^2+\frac{g^2\Lambda^2}{48\pi^2}-\frac{g^2m^2}{48\pi^2}\ln\frac{\Lambda^2}{m^2}$$

Action @ infinite cutoff (or dim reg):

$$\frac{1}{2}\left(p^2+M^2+\frac{g^2}{96\pi^2 p\circ p}+\dots\right)\phi(p)\phi(-p)$$

Two poles in Λ → ∞ action:

1. Usual one (ϕ quanta) at $p^2+m^2=\mathcal{O}(g^2)$
2. New one at $p\circ p=-\frac{g^2}{96\pi^2}\frac{1}{p_c^2+m^2}+\dots$

Second pole signals existence of new light particle arising from high-momentum modes of ϕ

[Minwalla, Seiberg, Van Raamsdonk '99]

Wilsonian interpretation

Normally require renormalizable Wilsonian action to satisfy

1. Correlation functions well-defined as Λ→∞
2. Correlation functions at finite Λ differ from limiting value by O(1/Λ) at all momenta

Badly violated here *at small p.*

$$\frac{1}{2}\left(p^2 + M^2 + \frac{g^2}{96\pi^2(p \circ p + 1/\Lambda^2)} + \ldots\right)\phi(p)\phi(-p)$$

Restore Wilsonian interpretation w/ new particle χ:

$$\delta\mathcal{L} = \frac{1}{2}\partial\chi \circ \partial\chi + \frac{1}{2}\Lambda^2(\partial \circ \partial\chi)^2 + i\frac{1}{\sqrt{96\pi^2}}g\chi\phi$$

Quadratic, so integrate out:

$$+\frac{1}{2}\frac{1}{96\pi^2}\left(\frac{g^2}{p \circ p} - \frac{g^2}{p \circ p + 1/\Lambda^2}\right)\phi(p)\phi(-p)$$

What have we learned?

High-momentum modes of massive fields in a non-commutative scalar theory are “dual” to additional (peculiar) light fields

4d fields in case of quadratic divergences, 5d for linear divergences, 6d for logarithmic divergences

In a fantasy application to the hierarchy problem, apparently light scalars are the χ fields, not the ϕ fields

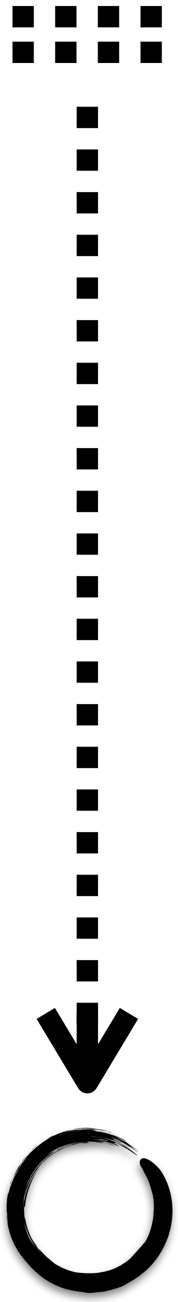
$$\delta\mathcal{L} = \frac{1}{2}\partial\chi\circ\partial\chi + \frac{1}{2}\Lambda^2(\partial\circ\partial\chi)^2 + i\frac{1}{\sqrt{96\pi^2}}g\chi\phi$$

Note, this looks like a Lee-Wick field (?!?)

Such a fantasy is still remote; still need to understand some basic features in more realistic theories. [NC, Koren, *in progress*]

????????????

Things I can’t (yet) cleanly compartmentalize



19. Tune the CC to set the weak scale

[Arvanitaki, Dimopoulos, Gorbenko, Huang, Van Tilburg ‘16]

20. Massless moduli from explicitly broken SUSY

[Dong, Freedman, Zhao ’14, ’15]

21. Self-organized criticality

Example: explicit marginal SUSY breaking involving U(1)R gauge fields on bdy of AdS3

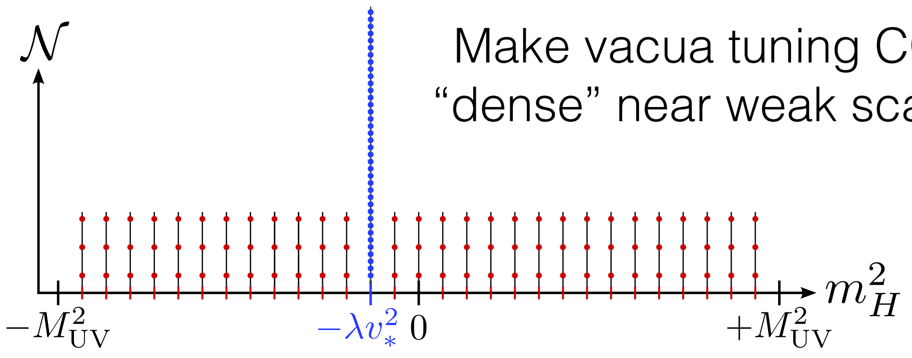
$$\delta S \sim \int_{bdy} A \wedge \tilde{A} \sim \int d^2 z J(z) \tilde{J}(\bar{z})$$

Induces splitting in R-charged multiplets.
Feed to R-neutral multiplets w/ yukawa

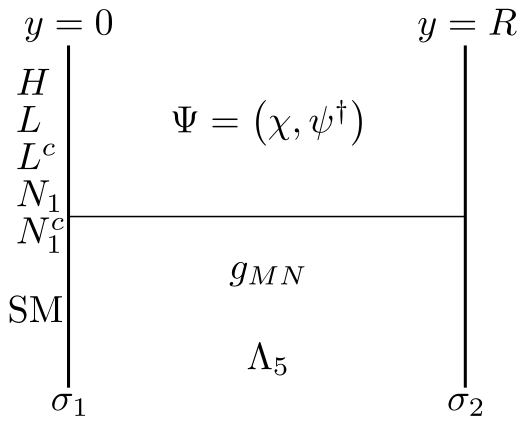
$$\lambda \phi_N \phi_R^\dagger \phi_R$$

R-neutral scalars are massless to all orders

Analogous to $y_t^2 m_t^2 - y_{\tilde{t}}^2 m_{\tilde{t}}^2 = 0$

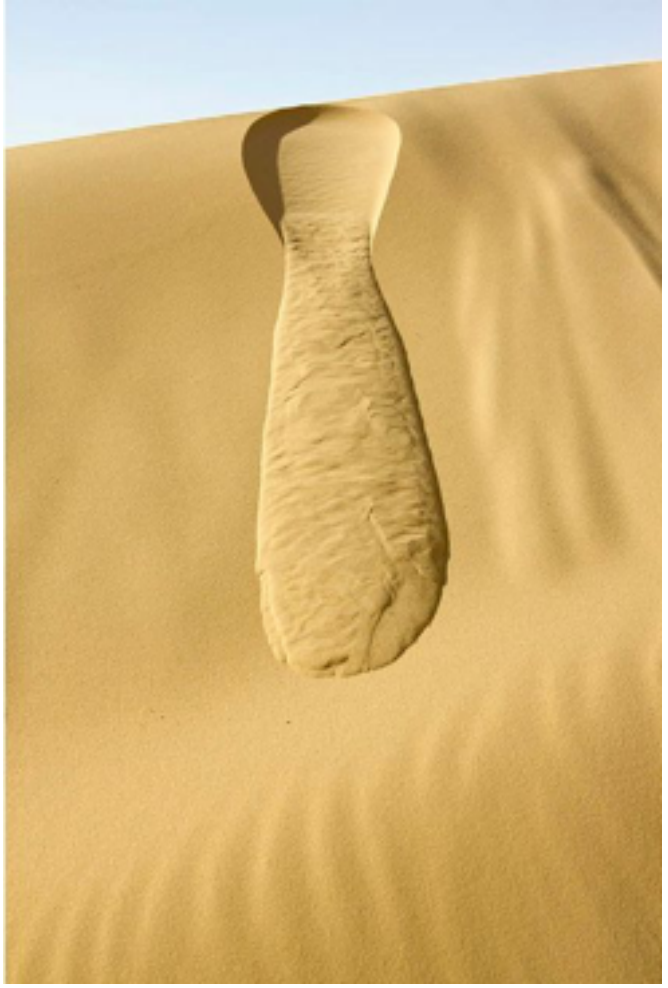


Make vacua tuning CC “dense” near weak scale



Signals

- Vector-like leptons (direct search, Higgs invisible width, precision electroweak)
- Super-light radion ($\mathcal{O}(10^{-10}$ eV))



A ??? Solution: Self-Organized Criticality

Some systems evolve into critical states on their own.
Wouldn't that be nice?

Canonical example: Sandpile. Initially dynamics of individual grains. Critical slope → one grain causes avalanche; correlations far larger than individual grains.

The QFT analog of SOC has been called:

- A free scalar field
- The (2,0) theory in 6d
- A classical FT w/ dissipation
- Soft gluons
- The relaxion
- "A terrible idea"

All of these in some sense true, but it's time to figure out which senses give novel, functional solutions to the hierarchy problem

1. Supersymmetry
2. Global symmetry
3. Discrete symmetry
4. RS/Technicolor
5. LED/10^{32}xSM
6. LST/Clockwork
7. Classicalization
8. Disorder
9. Anthropics
10. Relaxation
11. NNaturalness
12. Conformal symmetry
13. Asymptotic fragility
14. Agravity
15. Lee-Wick Theory
16. Non-compact SM
17. Weak gravity conjecture
18. Non-commutative QFT
19. Weak scale from CC
20. AdS magic
21. Self-organized criticality
22. …

Conclusions

- Electroweak hierarchy problem remains one of the biggest motivations for physics beyond the SM.
- Close to comprehensively understanding conventional solutions & searching accordingly. Should obviously keep searching for these as hard as possible, but…
- …at some point **data** tips the balance towards truly unconventional solutions. *Many of these are a way of making sense of the failure of Wilsonian EFT.*
- Promising places to look: conformal symmetry; naive IR pathologies; UV/IR mixing. But who am I to say? Lots to explore. Lively intersection of QFT, cosmology, quantum gravity.
- Experimental possibilities vast once we understand the theories…

Thank you!